Developmental Math—An Open Program Unit
Measurement
First Edition

Lesson 1 – U.S. Customary Units of Measurement

TOPICS

1.1.1 Length
1. Define units of length and convert from one to another.
2. Perform arithmetic calculations on units of length.
3. Solve application problems involving units of length.

1.1.2 Weight
1. Define units of weight and convert from one to another.
2. Perform arithmetic calculations on units of weight.
3. Solve application problems involving units of weight.

1.1.3 Capacity
1. Define units of capacity and convert from one to another.
2. Perform arithmetic calculations on units of capacity.
3. Solve application problems involving units of capacity.

Lesson 2 – Metric Units of Measurement

TOPICS

1.2.1 The Metric System
1. Describe the general relationship between the U.S. customary units and metric units of length, weight/mass, and volume.
2. Define the metric prefixes and use them to perform basic conversions among metric units.

Lesson 2 – Metric Units of Measurement

TOPICS (continued)

1.2.2 Converting within the Metric System

1 Perform arithmetic calculations on metric units of length, mass, and volume.

1.2.3 Using Metric Conversions to Solve Problems

1 Solve application problems involving metric units of length, mass, and volume.

Lesson 3 – Temperature

TOPICS

1.3.1 Temperature Scales

1 State the freezing and boiling points of water on the Celsius and Fahrenheit temperature scales.
2 Convert from one temperature scale to the other, using conversion formulas.

1.1.1 Length

Learning Objective(s)
1 Define units of length and convert from one to another.
2 Perform arithmetic calculations on units of length.
3 Solve application problems involving units of length.

Introduction

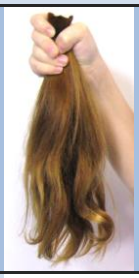
Measurement is a number that describes the size or amount of something. You can measure many things like length, area, capacity, weight, temperature and time. In the United States, two main systems of measurement are used: the **metric system** and the **U.S. customary measurement system**. This topic addresses the measurement of length using the U.S. customary measurement system.

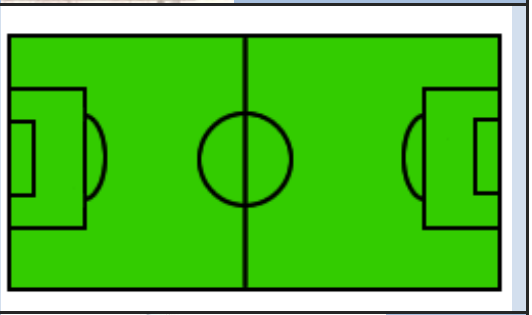
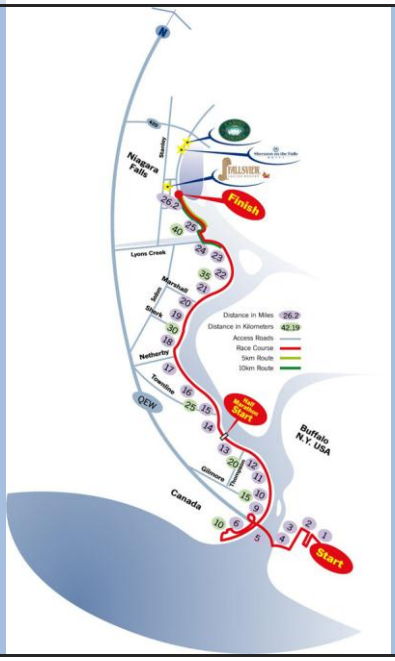
Suppose you want to purchase tubing for a project, and you see two signs in a hardware store: *$1.88 for 2 feet* of tubing and *$5.49 for 3 yards* of tubing. If both types of tubing will work equally well for your project, which is the better price? You need to know about two **units of measurement**, yards and feet, in order to determine the answer.

Units of Length

Objective 1

Length is the distance from one end of an object to the other end, or from one object to another. For example, the length of a letter-sized piece of paper is 11 inches. The system for measuring length in the United States is based on the four customary units of length: **inch**, **foot**, **yard**, and **mile**. Below are examples to show measurement in each of these units.

Unit	Description	Image
Inch/Inches	Some people donate their hair to be made into wigs for cancer patients who have lost hair as a result of treatment. One company requires hair donations to be at least 8 inches long.	
	Frame size of a bike: the distance from the center of the crank to the top of the seat tube. Frame size is usually measured in inches. This frame is 16 inches.	

Foot/Feet	Rugs are typically sold in standard lengths. One typical size is a rug that is 8 feet wide and 11 feet long. This is often described as an 8 by 11 rug.	
Yard/Yards	Soccer fields vary some in their size. An official field can be any length between 100 and 130 yards.	
Mile/Miles	A marathon is 26.2 miles long. One marathon route is shown in the map to the right.	

You can use any of these four U.S. customary measurement units to describe the length of something, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the length of a rug in feet rather than miles, and to describe a marathon in miles rather than inches.

You may need to convert between units of measurement. For example, you might want to express your height using feet and inches (5 feet 4 inches) or using only inches (64 inches). You need to know the unit equivalents in order to make these conversions between units.

The table below shows equivalents and conversion factors for the four customary units of measurement of length.

Unit Equivalents	Conversion Factors (longer to shorter units of measurement)	Conversion Factors (shorter to longer units of measurement)
1 foot = 12 inches	$\frac{12\text{ inches}}{1\text{ foot}}$	$\frac{1\text{ foot}}{12\text{ inches}}$
1 yard = 3 feet	$\frac{3\text{ feet}}{1\text{ yard}}$	$\frac{1\text{ yard}}{3\text{ feet}}$
1 mile = 5,280 feet	$\frac{5{,}280\text{ feet}}{1\text{ mile}}$	$\frac{1\text{ mile}}{5{,}280\text{ feet}}$

Note that each of these conversion factors is a ratio of equal values, so each conversion factor equals 1. Multiplying a measurement by a conversion factor does not change the size of the measurement at all since it is the same as multiplying by 1; it just changes the units that you are using to measure.

Converting Between Units of Length

You can use the conversion factors to convert a measurement, such as feet, to another type of measurement, such as inches.

Note that there are many more inches for a measurement than there are feet for the same measurement, as feet is a longer unit of measurement. You could use the conversion factor $\frac{12\text{ inches}}{1\text{ foot}}$.

If a length is measured in feet, and you'd like to convert the length to yards, you can think, "I am converting from a shorter unit to a longer one, so the length in yards will be less than the length in feet." You could use the conversion factor $\frac{1\text{ yard}}{3\text{ feet}}$.

If a distance is measured in miles, and you want to know how many feet it is, you can think, "I am converting from a longer unit of measurement to a shorter one, so the number of feet would be greater than the number of miles." You could use the conversion factor $\frac{5{,}280\text{ feet}}{1\text{ mile}}$.

You can use the **factor label method** to convert a length from one unit of measure to another using the conversion factors. In the factor label method, you multiply by unit fractions to convert a measurement from one unit to another. Study the example below to see how the factor label method can be used to convert $3\frac{1}{2}$ feet into an equivalent number of inches.

Example

Problem **How many inches are in $3\frac{1}{2}$ feet?**

$3\frac{1}{2}\text{ feet} = \_\_\_\text{ inches}$ — Begin by reasoning about your answer. Since a foot is longer than an inch, this means the answer would be greater than $3\frac{1}{2}$.

$3\frac{1}{2}\text{ feet} \bullet \frac{12\text{ inches}}{1\text{ foot}} = \_\_\_\text{ inches}$ — Find the conversion factor that compares inches and feet, with "inches" in the numerator, and multiply.

$\frac{7\text{ feet}}{2} \bullet \frac{12\text{ inches}}{1\text{ foot}} = \_\_\_\text{ inches}$ — Rewrite the mixed number as an improper fraction before multiplying.

$\frac{7\ \cancel{\text{feet}}}{2} \bullet \frac{12\text{ inches}}{1\ \cancel{\text{foot}}} = \_\_\_\text{ inches}$ — You can cancel similar units when they appear in the numerator *and* the denominator. So here, cancel the similar units "feet" and "foot." This eliminates this unit from the problem.

$\frac{7}{2} \bullet \frac{12\text{ inches}}{1} = \_\_\_\text{ inches}$

$\frac{7 \bullet 12\text{ inches}}{2 \bullet 1} = \_\_\_\text{ inches}$ — Rewrite as multiplication of numerators and denominators.

$\frac{84\text{ inches}}{2} = \_\_\_\text{ inches}$ — Multiply.

$\frac{84\text{ inches}}{2} = 42\text{ inches}$ — Divide.

Answer There are 42 inches in $3\frac{1}{2}$ feet.

Notice that by using the factor label method you can cancel the units out of the problem, just as if they were numbers. You can only cancel if the unit being cancelled is in both the numerator and denominator of the fractions you are multiplying.

In the problem above, you cancelled *feet* and *foot* leaving you with *inches*, which is what you were trying to find.

$$\frac{7 \text{ feet}}{2} \bullet \frac{12 \text{ inches}}{1 \text{ foot}} = \_\_\_ \text{ inches}$$

What if you had used the wrong conversion factor?

$$\frac{7 \text{ feet}}{2} \bullet \frac{1 \text{ foot}}{12 \text{ inches}} =$$

You could not cancel the feet because the unit is not the same in *both* the numerator and the denominator. So if you complete the computation, you would still have both feet and inches in the answer and no conversion would take place.

Here is another example of a length conversion using the factor label method.

Example		
Problem	**How many yards is 7 feet?**	
	$7 \text{ feet} = \_\_\_ \text{ yards}$	Start by reasoning about the size of your answer. Since a yard is longer than a foot, there will be fewer yards. So your answer will be less than 7.
	$7 \text{ feet} \bullet \frac{1 \text{ yard}}{3 \text{ feet}} = \_\_\_ \text{ yards}$	Find the conversion factor that compares feet and yards, with yards in the numerator.
	$\frac{7 \text{ feet}}{1} \bullet \frac{1 \text{ yard}}{3 \text{ feet}} = \_\_\_ \text{ yards}$	Rewrite the whole number as a fraction in order to multiply.
	$\frac{7 \text{ feet}}{1} \bullet \frac{1 \text{ yard}}{3 \text{ feet}} = \_\_\_ \text{ yards}$	Cancel the similar units “feet” and “feet” leaving only yards.
	$\frac{7}{1} \bullet \frac{1 \text{ yard}}{3} = \_\_\_ \text{ yards}$	
	$\frac{7 \bullet 1 \text{ yard}}{1 \bullet 3} = \_\_\_ \text{ yards}$	Multiply.
	$\frac{7 \text{ yards}}{3} = 2\frac{1}{3} \text{ yards}$	Divide, and write as a mixed number.
Answer	7 feet equals $2\frac{1}{3}$ yards.	

Note that if the units do not cancel to give you the answer you are trying to find, you may not have used the correct conversion factor.

Self Check A

How many feet are in $2\frac{1}{2}$ miles?

A) 10,560 feet

B) 30 feet

C) 2,112 feet

D) 13,200 feet

Applying Unit Conversions

Objective 2

Objective 3

There are times when you will need to perform computations on measurements that are given in different units. For example, consider the tubing problem given earlier. You must decide which of the two options is a better price, and you have to compare prices given in different unit measurements.

In order to compare, you need to convert the measurements into one single, common unit of measurement. To be sure you have made the computation accurately, think about whether the unit you are converting to is smaller or larger than the number you have. Its relative size will tell you whether the number you are trying to find is greater or lesser than the given number.

Example	
Problem	**An interior decorator needs border trim for a home she is wallpapering. She needs 15 feet of border trim for the living room, 30 feet of border trim for the bedroom, and 26 feet of border trim for the dining room. How many yards of border trim does she need?**
15 feet + 30 feet + 26 feet = 71 feet	You need to find the total length of border trim that is needed for all three rooms in the house. Since the measurements for each room are given in feet, you can add the numbers.
71 feet =___ yards	How many yards is 71 feet? Reason about the size of your answer. Since a yard is longer than a foot, there will be fewer yards. Expect your answer to be less than 71.

$\frac{71 \text{ feet}}{1} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} = \_\_\_ \text{ yards}$ Use the conversion factor $\frac{1 \text{ yard}}{3 \text{ feet}}$.

$\frac{71 \cancel{\text{feet}}}{1} \cdot \frac{1 \text{ yard}}{3 \cancel{\text{feet}}} = \_\_\_ \text{ yards}$ Since "feet" is in the numerator and denominator, you can cancel this unit.

$\frac{71}{1} \cdot \frac{1 \text{ yard}}{3} = \_\_\_ \text{ yards}$

$\frac{71 \cdot 1 \text{ yard}}{1 \cdot 3} = \_\_\_ \text{ yards}$ Multiply.

$\frac{71 \text{ yards}}{3} = \_\_\_ \text{ yards}$

$\frac{71 \text{ yards}}{3} = 23\frac{2}{3} \text{ yards}$ Divide, and write as a mixed number.

Answer The interior decorator needs $23\frac{2}{3}$ yards of border trim.

The next example uses the factor label method to solve a problem that requires converting from miles to feet.

Example

Problem **Two runners were comparing how much they had trained earlier that day. Jo said, "According to my pedometer, I ran 8.3 miles." Alex said, "That's a little more than what I ran. I ran 8.1 miles." How many more feet did Jo run than Alex?**

8.3 miles – 8.1 miles = 0.2 mile

$0.2 \text{ mile} = \frac{2}{10} \text{ mile}$ You need to find the difference between the distance Jo ran and the distance Alex ran. Since both distances are given in the same unit, you can subtract and keep the unit the same.

$\frac{2}{10} \text{ mile} = \_\_\_ \text{ feet}$ Since the problem asks for the difference in *feet*, you must convert from miles to feet. How many feet is 0.2 mile? Reason about the size of your answer. Since a mile is longer than a foot, the distance when expressed as feet will be a number greater than 0.2.

$$\frac{2\text{ mile}}{10}\cdot\frac{5{,}280\text{ feet}}{1\text{ mile}} = \_\_\_\text{ feet}$$ Use the conversion factor $\frac{5{,}280\text{ feet}}{1\text{ mile}}$.

$$\frac{2\ \cancel{\text{mile}}}{10}\cdot\frac{5{,}280\text{ feet}}{1\cancel{\text{mile}}} = \_\_\_\text{ feet}$$

$$\frac{2}{10}\cdot\frac{5{,}280\text{ feet}}{1} = \_\_\_\text{ feet}$$

$$\frac{2\cdot 5{,}280\text{ feet}}{10\cdot 1} = \_\_\_\text{ feet}$$ Multiply.

$$\frac{10{,}560\text{ feet}}{10} = \_\_\_\text{ feet}$$

$$\frac{10{,}560\text{ feet}}{10} = 1{,}056\text{ feet}$$ Divide.

Answer Jo ran 1,056 feet further than Alex.

Now let's revisit the question from earlier.

Example

Problem **You are walking through a hardware store and notice two sales on tubing.**

- **3 yards of Tubing A costs $5.49.**
- **Tubing B sells for $1.88 for 2 feet.**

Either tubing is acceptable for your project. Which tubing is less expensive?

Tubing A

3 yards = $5.49 Find the unit price for each tubing. This will make it easier to compare.

$$\frac{\$5.49\div 3}{3\text{ yards}\div 3} = \frac{\$1.83}{1\text{ yard}}$$ Find the cost per yard of Tubing A by dividing the cost of 3 yards of the tubing by 3.

Tubing B

2 feet = $1.88 Tubing B is sold by the foot. Find the cost per foot by dividing $1.88 by 2 feet.

$$\frac{\$1.88\div 2}{2\text{ feet}\div 2} = \frac{\$0.94}{1\text{ foot}}$$

$$\frac{\$0.94}{1\,\text{foot}} \cdot \frac{3\ \text{feet}}{1\,\text{yard}} = \frac{\$\ \_\_\_}{\_\_\_\ \text{yard}}$$ To compare the prices, you need to have the same unit of measure.

$$\frac{\$0.94}{1\ \cancel{\text{foot}}} \cdot \frac{3\ \cancel{\text{feet}}}{1\,\text{yard}} = \frac{\$2.82}{1\,\text{yard}}$$ Use the conversion factor $\frac{3\ \text{feet}}{1\,\text{yard}}$, cancel and multiply.

$2.82 per yard

Tubing A: $1.83 per yard
Tubing B: $2.82 per yard
Compare prices for 1 yard of each tubing.

Answer Tubing A is less expensive than Tubing B.

In the problem above, you could also have found the price per foot for each kind of tubing and compared the unit prices of each per foot.

Self Check B

A fence company is measuring a rectangular area in order to install a fence around its perimeter. If the length of the rectangular area is 130 yards and the width is 75 feet, what is the total length of the distance to be fenced?

A) 410 yards

B) 930 feet

C) 710 feet

D) 465 feet

Summary

The four basic units of measurement that are used in the U.S. customary measurement system are: inch, foot, yard, and mile. Typically, people use yards, miles, and sometimes feet to describe long distances. Measurement in inches is common for shorter objects or lengths.

You need to convert from one unit of measure to another if you are solving problems that include measurements involving more than one type of measurement. Each of the units can be converted to one of the other units using the table of equivalents, the conversion factors, and/or the factor label method shown in this topic.

1.1.1 Self Check Solutions

Self Check A

How many feet are in $2\frac{1}{2}$ miles?

A) 10,560 feet

B) 30 feet

C) 2,112 feet

D) 13,200 feet

A) 10,560 feet

Incorrect. There are 5,280 feet in a mile, so multiply $2\frac{1}{2}$, not 2, by 5,280. The correct answer is 13,200 feet.

B) 30 feet

Incorrect. A mile is much longer than 30 feet. There are 5,280 feet in a mile, so multiply 5,280 by $2\frac{1}{2}$, not $2\frac{1}{2}$ by 12, to find the number of feet in $2\frac{1}{2}$ miles. The correct answer is 13,200 feet.

C) 2,112 feet

Incorrect. Multiply, don't divide, 5,280 by $2\frac{1}{2}$. The correct answer is 13,200 feet.

D) 13,200 feet

Correct. There are 5,280 feet in a mile, so multiply $2\frac{1}{2}$ by 5,280 to get 13,200 feet.

Self Check B

A fence company is measuring a rectangular area in order to install a fence around its perimeter. If the length of the rectangular area is 130 yards and the width is 75 feet, what is the total length of the distance to be fenced?

A) 410 yards

B) 930 feet

C) 710 feet

D) 465 feet

A) 410 yards
Incorrect. The distance around the rectangle is two times the length plus two times the width, but you cannot perform this computation unless the units are the same. Convert yards to feet and then compute. The correct answer is 930 feet.

B) 930 feet
Correct. 130 yards is equivalent to 390 feet. To find the perimeter, add length + length + width + width: 390 feet + 390 feet + 75 feet + 75 feet = 930 feet.

C) 710 feet
Incorrect. Convert 130 yards to feet by multiplying by 3. Then, double to get the distance of the two long sides of the rectangle in feet. Distance for the width is already given in feet, so multiply by 2 to get the length of both short sides of the rectangle. The correct answer is 930 feet.

D) 465 feet
Incorrect. Fencing is needed for 4 sides, not just two sides. The correct answer is 930 feet.

1.1.2 Weight

Learning Objective(s)
1 Define units of weight and convert from one to another.
2 Perform arithmetic calculations on units of weight.
3 Solve application problems involving units of weight.

Introduction

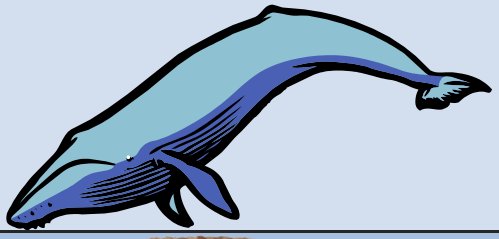
When you mention how heavy or light an object is, you are referring to its weight. In the U.S. customary system of measurement, weight is measured in ounces, pounds, and tons. Like other units of measurement, you can convert between these units and you sometimes need to do this to solve problems.

In 2010, the post office charges $0.44 to mail something that weighs an ounce or less. The post office charges $0.17 for each additional ounce, or fraction of an ounce, of weight. How much will it cost to mail a package that weighs two pounds three ounces? To answer this question, you need to understand the relationship between ounces and pounds.

Units of Weight

Objective 1

You often use the word **weight** to describe how heavy or light an object or person is. Weight is measured in the U.S. customary system using three units: ounces, pounds, and tons. An **ounce** is the smallest unit for measuring weight, a **pound** is a larger unit, and a **ton** is the largest unit.

Whales are some of the largest animals in the world. Some species can reach weights of up to 200 tons--that's equal to 400,000 pounds.	
Meat is a product that is typically sold by the pound. One pound of ground beef makes about four hamburger patties.	

Ounces are used to measure lighter objects. A stack of 11 pennies is equal to about one ounce.	

You can use any of the customary measurement units to describe the weight of something, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the weight of a human being in pounds rather than tons. It makes more sense to describe the weight of a car in tons rather than ounces.

1 pound = 16 ounces

1 ton = 2,000 pounds

Converting Between Units of Weight

Four ounces is a typical serving size of meat. Since meat is sold by the pound, you might want to convert the weight of a package of meat from pounds to ounces in order to determine how many servings are contained in a package of meat.

The weight capacity of a truck is often provided in tons. You might need to convert pounds into tons if you are trying to determine whether a truck can safely transport a big shipment of heavy materials.

The table below shows the unit conversions and conversion factors that are used to make conversions between customary units of weight.

Unit Equivalents	Conversion Factors (heavier to lighter units of measurement)	Conversion Factors (lighter to heavier units of measurement)
1 pound = 16 ounces	$\frac{16 \text{ ounces}}{1 \text{ pound}}$	$\frac{1 \text{ pound}}{16 \text{ ounces}}$
1 ton = 2000 pounds	$\frac{2000 \text{ pounds}}{1 \text{ ton}}$	$\frac{1 \text{ ton}}{2000 \text{ pounds}}$

You can use the *factor label method* to convert one customary unit of weight to another customary unit of weight. This method uses conversion factors, which allow you to "cancel" units to end up with your desired unit of measurement.

Each of these conversion factors is a ratio of equal values, so each conversion factor equals 1. Multiplying a measurement by a conversion factor does not change the size of the measurement at all, since it is the same as multiplying by 1. It just changes the units that you are using to measure it in.

Two examples illustrating the factor label method are shown below.

Example

Problem

How many ounces are in $2\frac{1}{4}$ pounds?

$2\frac{1}{4}\text{ pounds} = \_\_\_\text{ ounces}$

Begin by reasoning about your answer. Since a pound is heavier than an ounce, expect your answer to be a number greater than $2\frac{1}{4}$.

$2\frac{1}{4}\text{ pounds} \bullet \frac{16\text{ ounces}}{1\text{ pound}} = \_\_\_\text{ ounces}$

Multiply by the conversion factor that relates ounces and pounds: $\frac{16\text{ ounces}}{1\text{ pound}}$.

$\frac{9\text{ pounds}}{4} \bullet \frac{16\text{ ounces}}{1\text{ pound}} = \_\_\_\text{ ounces}$

Write the mixed number as an improper fraction.

$\frac{9\ \cancel{\text{pounds}}}{4} \bullet \frac{16\text{ ounces}}{1\ \cancel{\text{pound}}} = \_\_\_\text{ ounces}$

The common unit "pound" can be cancelled because it appears in both the numerator and denominator.

$\frac{9}{4} \bullet \frac{16\text{ ounces}}{1} = \_\_\_\text{ ounces}$

Multiply and simplify.

$\frac{9 \bullet 16\text{ ounces}}{4 \bullet 1} = \_\_\_\text{ ounces}$

$\frac{144\text{ ounces}}{4} = \_\_\_\text{ ounces}$

$\frac{144\text{ ounces}}{4} = 36\text{ ounces}$

Answer

There are 36 ounces in $2\frac{1}{4}$ pounds.

Example

Problem **How many tons is 6,500 pounds?**

$6{,}500 \text{ pounds} = \_\_\_ \text{ tons}$ — Begin by reasoning about your answer. Since a ton is heavier than a pound, expect your answer to be a number less than 6,500.

$6{,}500 \text{ pounds} \cdot \frac{1 \text{ ton}}{2{,}000 \text{ pounds}} = \_\_\_ \text{ tons}$ — Multiply by the conversion factor that relates tons to pounds: $\frac{1 \text{ ton}}{2{,}000 \text{ pounds}}$.

$\frac{6{,}500 \text{ pounds}}{1} \cdot \frac{1 \text{ ton}}{2{,}000 \text{ pounds}} = \_\_\_ \text{ tons}$

$\frac{6{,}500 \ \cancel{\text{pounds}}}{1} \cdot \frac{1 \text{ ton}}{2{,}000 \ \cancel{\text{pounds}}} = \_\_\_ \text{ tons}$ — Apply the Factor Label method.

$\frac{6{,}500}{1} \cdot \frac{1 \text{ ton}}{2{,}000} = \_\_\_ \text{ tons}$ — Multiply and simplify.

$\frac{6{,}500 \text{ tons}}{2{,}000} = \_\_\_ \text{ tons}$

$\frac{6{,}500 \text{ tons}}{2{,}000} = 3\frac{1}{4} \text{ tons}$

Answer 6,500 pounds is equal to $3\frac{1}{4}$ tons.

Self Check A

How many pounds is 72 ounces?

A) $4\frac{1}{2}$ pounds

B) 6 pounds

C) 24 pounds

D) 1,152 pounds

Objective 2

Objective 3

Applying Unit Conversions

There are times when you need to perform calculations on measurements that are given in different units. To solve these problems, you need to convert one of the measurements to the same unit of measurement as the other measurement.

Think about whether the unit you are converting to is smaller or larger than the unit you are converting from. This will help you be sure that you are making the right computation. You can use the factor label method to make the conversion from one unit to another.

Here is an example of a problem that requires converting between units.

Example

Problem **A municipal trash facility allows a person to throw away a maximum of 30 pounds of trash per week. Last week, 140 people threw away the maximum allowable trash. How many tons of trash did this equal?**

$140 \cdot 30 \text{ pounds} = 4{,}200 \text{ pounds}$ — Determine the total trash for the week expressed in pounds. If 140 people each throw away 30 pounds, you can find the total by multiplying.

$4{,}200 \text{ pounds} = \_\_\_ \text{ tons}$ — Then convert 4,200 pounds to tons. Reason about your answer. Since a ton is heavier than a pound, expect your answer to be a number less than 4,200.

$$\frac{4{,}200 \text{ pounds}}{1} \cdot \frac{1 \text{ ton}}{2{,}000 \text{ pounds}} = \_\_\_ \text{ tons}$$

Find the conversion factor appropriate for the situation: $\frac{1 \text{ ton}}{2{,}000 \text{ pounds}}$.

$$\frac{4{,}200 \ \cancel{\text{pounds}}}{1} \cdot \frac{1 \text{ ton}}{2{,}000 \ \cancel{\text{pounds}}} = \_\_\_ \text{ tons}$$

$$\frac{4{,}200}{1} \cdot \frac{1 \text{ ton}}{2{,}000} = \_\_\_ \text{ tons}$$

$$\frac{4{,}200 \cdot 1 \text{ ton}}{1 \cdot 2{,}000} = \_\_\_ \text{ tons}$$

Multiply and simplify.

$$\frac{4{,}200 \text{ ton}}{2{,}000} = \_\_\_ \text{ tons}$$

$$\frac{4{,}200 \text{ ton}}{2{,}000} = 2\frac{1}{10} \text{ tons}$$

Answer The total amount of trash generated is $2\frac{1}{10}$ tons.

Let's revisit the post office problem that was posed earlier. We can use unit conversion to solve this problem.

Example

Problem **The post office charges $0.44 to mail something that weighs an ounce or less. The charge for each additional ounce, or fraction of an ounce, of weight is $0.17. At this rate, how much will it cost to mail a package that weighs 2 pounds 3 ounces?**

2 pounds 3 ounces = ___ ounces

Since the pricing is for ounces, convert the weight of the package from pounds and ounces into just ounces.

$$\frac{2 \text{ pounds}}{1} \bullet \frac{16 \text{ ounces}}{\text{pound}} = \_\_\_ \text{ ounces}$$

First use the factor label method to convert 2 pounds to ounces.

$$\frac{2 \ \cancel{\text{pounds}}}{1} \bullet \frac{16 \text{ ounces}}{\cancel{\text{pound}}} = \_\_\_ \text{ ounces}$$

$$\frac{2}{1} \bullet \frac{16 \text{ ounces}}{1} = 32 \text{ ounces}$$

2 pounds = 32 ounces.

32 ounces + 3 ounces = 35 ounces

Add the additional 3 ounces to find the weight of the package. The package weighs 35 ounces. There are 34 additional ounces, since 35 – 1 = 34.

$0.44 + $0.17(34)

$0.44 + $5.78

Apply the pricing formula. $0.44 for the first ounce and $0.17 for each additional ounce.

$0.44 + $5.78 = $6.22

Answer It will cost $6.22 to mail a package that weighs 2 pounds 3 ounces.

Self Check B

The average weight of a northern bluefin tuna is 1,800 pounds. The average weight of a great white shark is $2\frac{1}{2}$ tons. On average, how much more does a great white shark weigh, in pounds, than a northern bluefin tuna?

A) 5,000 pounds

B) 3,200 pounds

C) $182\frac{1}{2}$ pounds

D) You cannot answer this because the units of weight are different.

Summary

In the U.S. customary system of measurement, weight is measured in three units: ounces, pounds, and tons. A pound is equivalent to 16 ounces, and a ton is equivalent to 2,000 pounds. While an object's weight can be described using any of these units, it is typical to describe very heavy objects using tons and very light objects using an ounce. Pounds are used to describe the weight of many objects and people. Often, in order to compare the weights of two objects or people or to solve problems involving weight, you must convert from one unit of measurement to another unit of measurement. Using conversion factors with the factor label method is an effective strategy for converting units and solving problems.

1.1.2 Self Check Solutions

Self Check A

How many pounds is 72 ounces?

A) $4\frac{1}{2}$ pounds

B) 6 pounds

C) 24 pounds

D) 1,152 pounds

A) $4\frac{1}{2}$ pounds

Correct. There are 16 ounces in one pound, so 72 ounces • $\frac{1 \text{ pound}}{16 \text{ ounces}}$ = $4\frac{1}{2}$ pounds.

B) 6 pounds

Incorrect. There are 16 ounces in a pound, not 12. The correct answer is $4\frac{1}{2}$ pounds.

C) 24 pounds

Incorrect. There are 16 ounces in a pound, not 3. The correct answer is $4\frac{1}{2}$ pounds.

D) 1,152 pounds

Incorrect. Pounds are heavier than ounces, so the answer must be less than 72. Multiply by $\frac{1 \text{ pound}}{16 \text{ ounces}}$, not $\frac{16 \text{ ounces}}{1 \text{ pound}}$. The correct answer is $4\frac{1}{2}$ pounds.

Self Check B

The average weight of a northern bluefin tuna is 1,800 pounds. The average weight of a great white shark is $2\frac{1}{2}$ tons. On average, how much more does a great white shark weigh, in pounds, than a northern bluefin tuna?

A) 5,000 pounds

B) 3,200 pounds

C) $182\frac{1}{2}$ pounds

D) You cannot answer this because the units of weight are different.

A) 5,000 pounds

Incorrect. A great white shark has an average weight of 5,000 pounds, and you have to subtract 1,800 to find the difference in the weights of the shark and the tuna. The correct answer is 3,200 pounds.

B) 3,200 pounds

Correct. $2\frac{1}{2}$ tons = 5,000 pounds. 5,000 pounds – 1,800 pounds = 3,200 pounds.

C) $182\frac{1}{2}$ pounds

Incorrect. You cannot subtract these weights because they are given in different units. To find the difference, first convert the weight of the shark to pounds and then subtract the weight of the tuna in pounds. The correct answer is 3,200 pounds.

D) You cannot answer this because the units of weight are different.
Incorrect. The units are different, but you can express the weights in the same unit and then compute. First convert the weight of the shark to pounds and then subtract the weight of the tuna in pounds. The correct answer is 3,200 pounds.

1.1.3 Capacity

Learning Objective(s)
1 Define units of capacity and convert from one to another.
2 Perform arithmetic calculations on units of capacity.
3 Solve application problems involving units of capacity.

Introduction

Capacity is the amount of liquid (or other pourable substance) that an object can hold when it's full. When a liquid, such as milk, is being described in gallons or quarts, this is a measure of capacity.

Understanding units of capacity can help you solve problems like this: Sven and Johanna were hosting a potluck dinner. They did not ask their guests to tell them what they would be bringing, and three people ended up bringing soup. Erin brought 1 quart, Richard brought 3 pints, and LeVar brought 9 cups. How many cups of soup did they have all together?

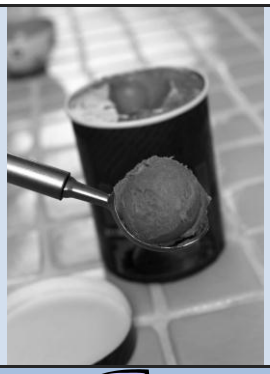
Units of Capacity

Objective 1

There are five main units for measuring capacity in the U.S. customary measurement system. The smallest unit of measurement is a **fluid ounce**. "Ounce" is also used as a measure of weight, so it is important to use the word "fluid" with ounce when you are talking about capacity. Sometimes the prefix "fluid" is not used when it is clear from the context that the measurement is capacity, not weight.

The other units of capacity in the customary system are the **cup**, **pint**, **quart**, and **gallon**. The table below describes each unit of capacity and provides an example to illustrate the size of the unit of measurement.

Fluid Ounce A unit of capacity equal to $\frac{1}{8}$ of a cup. One fluid ounce of water at 62°F weighs about one ounce. The amount of liquid medicine is often measured in fluid ounces.	
Cup A unit equal to 8 fluid ounces. The capacity of a standard measuring cup is one cup.	

Pint A unit equal to 16 fluid ounces, or 2 cups. The capacity of a carton of ice cream is often measured in pints.	
Quart A unit equal to 32 fluid ounces, or 4 cups. You often see quarts of milk being sold in the supermarket.	
Gallon A unit equal to 4 quarts, or 128 fluid ounces. When you fill up your car with gasoline, the price of gas is often listed in dollars per gallon.	

You can use any of these five measurement units to describe the capacity of an object, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the capacity of a swimming pool in gallons and the capacity of an expensive perfume in fluid ounces.

Sometimes you will need to convert between units of measurement. For example, you might want to express 5 gallons of lemonade in cups if you are trying to determine how many 8-fluid ounce servings the amount of lemonade would yield.

The table below shows some of the most common equivalents and conversion factors for the five customary units of measurement of capacity.

Unit Equivalents	Conversion Factors (heavier to lighter units of measurement)	Conversion Factors (lighter to heavier units of measurement)
1 cup = 8 fluid ounces	$\frac{1 \text{ cup}}{8 \text{ fluid ounces}}$	$\frac{8 \text{ fluid ounces}}{1 \text{ cup}}$
1 pint = 2 cups	$\frac{1 \text{ pint}}{2 \text{ cups}}$	$\frac{2 \text{ cups}}{1 \text{ pint}}$
1 quart = 2 pints	$\frac{1 \text{ quart}}{2 \text{ pints}}$	$\frac{2 \text{ pints}}{1 \text{ quart}}$
1 quart = 4 cups	$\frac{1 \text{ quart}}{4 \text{ cups}}$	$\frac{4 \text{ cups}}{1 \text{ quart}}$
1 gallon = 4 quarts	$\frac{1 \text{ gallon}}{4 \text{ quarts}}$	$\frac{4 \text{ quarts}}{1 \text{ gallon}}$
1 gallon = 16 cups	$\frac{1 \text{ gallon}}{16 \text{ cups}}$	$\frac{16 \text{ cups}}{1 \text{ gallon}}$

Converting Between Units of Capacity

Objective 2

As with converting units of length and weight, you can use the factor label method to convert from one unit of capacity to another. An example of this method is shown below.

Example

Problem

How many pints is $2\frac{3}{4}$ gallons?

$2\frac{3}{4}$ gallons =___ pints

Begin by reasoning about your answer. Since a gallon is larger than a pint, expect the answer in pints to be a number greater than $2\frac{3}{4}$.

$$\frac{11 \text{ gallons}}{4} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} = \text{___ pints}$$

The table above does not contain a conversion factor for gallons and pints, so you cannot convert it in one step. However, you can use quarts as an intermediate unit, as shown here.

Set up the equation so that two sets of labels cancel—gallons

$$\frac{11\ \text{gallons}}{4}\cdot\frac{4\ \text{quarts}}{1\ \text{gallon}}\cdot\frac{2\ \text{pints}}{1\ \text{quart}}=\_\_\_\ \text{pints}$$

and quarts.

$$\frac{11}{4}\cdot\frac{4}{1}\cdot\frac{2\ \text{pints}}{1}=\_\_\_\ \text{pints}$$

$$\frac{11\cdot 4\cdot 2\ \text{pints}}{4\cdot 1\cdot 1}=\_\_\_\ \text{pints}$$

Multiply and simplify.

$$\frac{88\ \text{pints}}{4}=22\ \text{pints}$$

Answer $2\frac{3}{4}$ gallons is 22 pints.

Example

Problem **How many gallons is 32 fluid ounces?**

32 fluid ounces = ___ gallons

Begin by reasoning about your answer. Since gallons is a larger unit than fluid ounces, expect the answer to be less than 32.

$$\frac{32\ \text{fl oz}}{1}\cdot\frac{1\ \text{cup}}{8\ \text{fl oz}}\cdot\frac{1\ \text{pt}}{2\ \text{cups}}\cdot\frac{1\ \text{qt}}{2\ \text{pt}}\cdot\frac{1\ \text{gal}}{4\ \text{qt}}=\_\_\_\ \text{gal}$$

The table above does not contain a conversion factor for gallons and fluid ounces, so you cannot convert it in one step. Use a series of intermediate units, as shown here.

$$\frac{32\ \text{fl oz}}{1}\cdot\frac{1\ \text{cup}}{8\ \text{fl oz}}\cdot\frac{1\ \text{pt}}{2\ \text{cups}}\cdot\frac{1\ \text{qt}}{2\ \text{pt}}\cdot\frac{1\ \text{gal}}{4\ \text{qt}}=\_\_\_\ \text{gal}$$

Cancel units that appear in both the numerator and denominator.

$$\frac{32}{1}\cdot\frac{1}{8}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1\ \text{gal}}{4}=\_\_\_\ \text{gal}$$

$$\frac{32\cdot 1\cdot 1\cdot 1\cdot 1\ \text{gal}}{1\cdot 8\cdot 2\cdot 2\cdot 4}=\_\_\_\ \text{gal}$$

Multiply and simplify.

$$\frac{32\ \text{gal}}{128}=\frac{1}{4}\ \text{gal}$$

Answer

32 fluid ounces is the same as $\frac{1}{4}$ gallon.

Self Check A

Find the sum of 4 gallons and 2 pints. Express your answer in cups.

A) 68 cups

B) 6 cups

C) 2 gallons and 4 pints

D) 34 pints

Applying Unit Conversions

Objective 3

There are times when you will need to combine measurements that are given in different units. In order to do this, you need to convert first so that the units are the same.

Consider the situation posed earlier in this topic.

Example

Problem **Sven and Johanna were hosting a potluck dinner. They did not ask their guests to tell them what they would be bringing, and three people ended up bringing soup. Erin brought 1 quart, Richard brought 3 pints, and LeVar brought 9 cups. How much soup did they have total?**

1 quart + 3 pints + 9 cups	Since the problem asks for the total amount of soup, you must add the three quantities. Before adding, you must convert the quantities to the same unit. The problem does not require a particular unit, so you can choose. Cups might be the easiest computation.
1 quart = 4 cups	This is given in the table of equivalents.
$\frac{3 \text{ pints}}{1} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} = \_\_\_ \text{ cups}$	Use the factor label method to convert pints to cups.

$$\frac{3 \text{ pints}}{1} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} = 6 \text{ cups}$$

4 cups + 6 cups + 9 cups = 19 cups Add the 3 quantities.

Answer There are 19 cups of soup for the dinner.

Example

Problem **Natasha is making lemonade to bring to the beach. She has two containers. One holds one gallon and the other holds 2 quarts. If she fills both containers, how many cups of lemonade will she have?**

1 gallon + 2 quarts = ___ cups — This problem requires you to find the sum of the capacity of each container and then convert that sum to cups.

4 quarts + 2 quarts = 6 quarts — First, find the sum in quarts. 1 gallon is equal to 4 quarts.

$$\frac{6 \text{ quarts}}{1} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} = \text{___ cups}$$

Since the problem asks for the capacity in cups, convert 6 quarts to cups.

$$\frac{6 \text{ quarts}}{1} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} = \text{___ cups}$$

Cancel units that appear in both the numerator and denominator.

6 • 2 • 2 = 24 cups — Multiply.

Answer Natasha will have 24 cups of lemonade.

Another way to work the problem above would be to first change 1 gallon to 16 cups and change 2 quarts to 8 cups. Then add: 16 + 8 = 24 cups.

Self Check B

Alan is making chili. He is using a recipe that makes 24 cups of chili. He has a 5-quart pot and a 2-gallon pot and is trying to determine whether the chili will all fit in one of these pots. Which of the pots will fit the chili?

A) The chili will not fit into either of the pots.

B) The chili can fit into either pot.

C) The chili will fit into the 5-quart pot only.

D) The chili will fit into the 2-gallon pot only.

Summary

There are five basic units for measuring capacity in the U.S. customary measurement system. These are the fluid ounce, cup, pint, quart, and gallon. These measurement units are related to one another, and capacity can be described using any of the units. Typically, people use gallons to describe larger quantities and fluid ounces, cups, pints, or quarts to describe smaller quantities. Often, in order to compare or to solve problems involving the amount of liquid in a container, you need to convert from one unit of measurement to another.

1.1.3 Self Check Solutions

Self Check A

Find the sum of 4 gallons and 2 pints. Express your answer in cups.

A) 68 cups

B) 6 cups

C) 2 gallons and 4 pints

D) 34 pints

A) 68 cups
Correct. Each gallon has 16 cups, so $4 \bullet 16 = 64$ will give you the number of cups in 4 gallons. Each pint has 2 cups, so $2 \bullet 2 = 4$ will give you the number of cups in 2 pints. $64 + 4 = 68$ cups.

B) 6 cups
Incorrect. You cannot add measurements expressed in different units. First convert each amount to cups and then add. The correct answer is 68 cups.

C) 2 gallons and 4 pints
Incorrect. This is not the correct amount. First convert each amount to cups and then add them together. Also, notice that this answer is not expressed in cups, as the problem requires. The correct answer is 68 cups.

D) 34 pints
Incorrect. This is the correct amount, but it is expressed in the incorrect unit—pints. Multiply 34 pints by 2 (since there are 2 cups in a pint) to give the amount in cups as required. The correct answer is 68 cups.

Self Check B

Alan is making chili. He is using a recipe that makes 24 cups of chili. He has a 5-quart pot and a 2-gallon pot and is trying to determine whether the chili will all fit in one of these pots. Which of the pots will fit the chili?

A) The chili will not fit into either of the pots.

B) The chili can fit into either pot.

C) The chili will fit into the 5-quart pot only.

D) The chili will fit into the 2-gallon pot only.

A) The chili will not fit into either of the pots.
Incorrect. The 5-quart pot will hold only 20 cups, leaving 4 cups that won't fit into the pot. However, the 2-gallon pot is large enough to fit the chili. The correct answer is the 2-gallon pot only.

B) The chili can fit into either pot.
Incorrect. The 5-quart pot will hold only 20 cups, so the 24 cups of chili cannot fit in there. The correct answer is the 2-gallon pot only.

C) The chili will fit into the 5-quart pot only.
Incorrect. The 5-quart pot will hold only 20 cups. The 2-gallon pot will hold 32 cups. The correct answer is the 2-gallon pot only.

D) The chili will fit into the 2-gallon pot only.
Correct. 5 quarts = 5 • 4 cups = 20 cups, so 24 cups of chili will not fit into the 5-quart pot. 2 gallons = 32 cups, so 24 cups of chili will fit in this pot.

1.2.1 The Metric System

Learning Objective(s)

1 Describe the general relationship between the U.S. customary units and metric units of length, weight/mass, and volume.

2 Define the metric prefixes and use them to perform basic conversions among metric units.

Introduction

Objective 1

In the United States, both the **U.S. customary measurement system** and the **metric system** are used, especially in medical, scientific, and technical fields. In most other countries, the metric system is the primary system of measurement. If you travel to other countries, you will see that road signs list distances in kilometers and milk is sold in liters. People in many countries use words like “kilometer,” “liter,” and “milligram” to measure the length, volume, and weight of different objects. These measurement units are part of the metric system.

Unlike the U.S. customary system of measurement, the metric system is based on 10s. For example, a liter is 10 times larger than a deciliter, and a centigram is 10 times larger than a milligram. This idea of “10” is not present in the U.S. customary system—there are 12 inches in a foot, and 3 feet in a yard…and 5,280 feet in a mile!

So, what if you have to find out how many milligrams are in a decigram? Or, what if you want to convert meters to kilometers? Understanding how the metric system works is a good start.

What is Metric?

The metric system uses units such as **meter**, **liter**, and **gram** to measure length, liquid volume, and mass, just as the U.S. customary system uses feet, quarts, and ounces to measure these.

In addition to the difference in the basic units, the metric system is based on 10s, and different measures for length include kilometer, meter, decimeter, centimeter, and millimeter. Notice that the word “meter” is part of all of these units.

The metric system also applies the idea that units within the system get larger or smaller by a power of 10. This means that a meter is 100 times larger than a centimeter, and a kilogram is 1,000 times heavier than a gram. You will explore this idea a bit later. For now, notice how this idea of “getting bigger or smaller by 10” is very different than the relationship between units in the U.S. customary system, where 3 feet equals 1 yard, and 16 ounces equals 1 pound.

Length, Mass, and Volume

The table below shows the basic units of the metric system. Note that the names of all metric units follow from these three basic units.

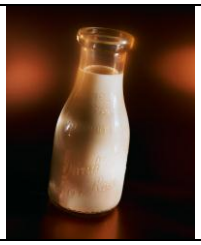
Length	Mass	Volume
basic units		
meter	gram	liter
other units you may see		
kilometer	kilogram	dekaliter
centimeter	centigram	centiliter
millimeter	milligram	milliliter

In the metric system, the basic unit of length is the meter. A meter is slightly larger than a yardstick, or just over three feet.

The basic metric unit of mass is the gram. A regular-sized paperclip has a mass of about 1 gram.

Among scientists, one gram is defined as the mass of water that would fill a 1-centimeter cube. You may notice that the word “mass” is used here instead of “weight.” In the sciences and technical fields, a distinction is made between weight and mass. Weight is a measure of the pull of gravity on an object. For this reason, an object’s weight would be different if it was weighed on Earth or on the moon because of the difference in the gravitational forces. However, the object’s mass would remain the same in both places because mass measures the amount of substance in an object. As long as you are planning on only measuring objects on Earth, you can use mass/weight fairly interchangeably—but it is worth noting that there is a difference!

Finally, the basic metric unit of volume is the liter. A liter is slightly larger than a quart.

		
The handle of a shovel is about 1 meter.	A paperclip weighs about 1 gram.	A medium-sized container of milk is about 1 liter.

Though it is rarely necessary to convert between the customary and metric systems, sometimes it helps to have a mental image of how large or small some units are. The table below shows the relationship between some common units in both systems.

	Common Measurements in Customary and Metric Systems
Length	1 centimeter is a little less than half an inch.
	1.6 kilometers is about 1 mile.
	1 meter is about 3 inches longer than 1 yard.
Mass	1 kilogram is a little more than 2 pounds.
	28 grams is about the same as 1 ounce.
Volume	1 liter is a little more than 1 quart.
	4 liters is a little more than 1 gallon.

Prefixes in the Metric System

Objective **2**

The metric system is a base 10 system. This means that each successive unit is 10 times larger than the previous one.

The names of metric units are formed by adding a prefix to the basic unit of measurement. To tell how large or small a unit is, you look at the **prefix**. To tell whether the unit is measuring length, mass, or volume, you look at the base.

Prefixes in the Metric System						
kilo-	*hecto-*	*deka-*	meter gram liter	*deci-*	*centi-*	*milli-*
1,000 times **larger** than base unit	100 times **larger** than base unit	10 times **larger** than base unit	base units	10 times **smaller** than base unit	100 times **smaller** than base unit	1,000 times **smaller** than base unit

Using this table as a reference, you can see the following:

- A kilogram is 1,000 times larger than one gram (so 1 kilogram = 1,000 grams).
- A centimeter is 100 times smaller than one meter (so 1 meter = 100 centimeters).
- A dekaliter is 10 times larger than one liter (so 1 dekaliter = 10 liters).

Here is a similar table that just shows the metric units of measurement for mass, along with their size relative to 1 gram (the base unit). The common abbreviations for these metric units have been included as well.

Measuring Mass in the Metric System						
kilogram (kg)	hectogram (hg)	dekagram (dag)	gram (g)	decigram (dg)	centigram (cg)	milligram (mg)
1,000 grams	100 grams	10 grams	gram	0.1 gram	0.01 gram	0.001 gram

Since the prefixes remain constant through the metric system, you could create similar charts for length and volume. The prefixes have the same meanings whether they are attached to the units of length (meter), mass (gram), or volume (liter).

Self Check A

Which of the following sets of three units are all metric measurements of length?

A) inch, foot, yard

B) kilometer, centimeter, millimeter

C) kilogram, gram, centigram

D) kilometer, foot, decimeter

Converting Units Up and Down the Metric Scale

Converting between metric units of measure requires knowledge of the metric prefixes and an understanding of the decimal system—that's about it.

For instance, you can figure out how many centigrams are in one dekagram by using the table above. One dekagram is larger than one centigram, so you expect that one dekagram will equal many centigrams.

In the table, each unit is 10 times larger than the one to its immediate right. This means that 1 dekagram = 10 grams; 10 grams = 100 decigrams; and 100 decigrams = 1,000 centigrams. So, 1 dekagram = 1,000 centigrams.

Example

Problem **How many milligrams are in one decigram?**

kg	hg	dag	g	dg	cg	mg
				^		^

Identify locations of milligrams and decigrams.

Decigrams (dg) are larger than milligrams (mg), so you expect there to be many mg in one dg.

×10 ×10

kg	hg	dag	g	dg	cg	mg
				^		^
				→	→	

Dg is 10 times larger than a cg, and a cg is 10 times larger than a mg.

Since you are going from a larger unit to a smaller unit, multiply.

1 dg • 10 • 10 = 100 mg

Multiply: 1 • 10 • 10, to find the number of milligrams in one decigram.

Answer There are 100 milligrams (mg) in 1 decigram (dg).

Example

Problem **Convert 1 centimeter to kilometers.**

km	hm	dam	m	dm	cm	mm
^					^	

Identify locations of kilometers and centimeters.

Kilometers (km) are larger than centimeters (cm), so you expect there to be less than one km in a cm.

	÷10 ÷10 ÷10 ÷10 ÷10 | km | hm | dam | m | dm | cm | mm | ^ (km) ^ (cm) ← ← ← ← ←	Cm is 10 times smaller than a dm; a dm is 10 times smaller than a m, etc. Since you are going from a smaller unit to a larger unit, divide.
	1 cm ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 = 0.00001 km	Divide: 1 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10, to find the number of kilometers in one centimeter.
Answer	1 centimeter (cm) = 0.00001 kilometers (km).	

Once you begin to understand the metric system, you can use a shortcut to convert among different metric units. The size of metric units increases tenfold as you go up the metric scale. The decimal system works the same way: a tenth is 10 times larger than a hundredth; a hundredth is 10 times larger than a thousandth, etc. By applying what you know about decimals to the metric system, converting among units is as simple as moving decimal points.

Here is the first problem from above: How many milligrams are in one decigram? You can recreate the order of the metric units as shown below:

$$kg \quad hg \quad dag \quad g \quad d\underbrace{g \quad c}_{1}\underbrace{g \quad m}_{2}g$$

This question asks you to start with 1 decigram and convert that to milligrams. As shown above, milligrams is two places to the right of decigrams. You can just move the decimal point two places to the right to convert decigrams to milligrams: $1dg = 1\underset{1}{0}\underset{2}{0}.\,mg$.

The same method works when you are converting from a smaller to a larger unit, as in the problem: Convert 1 centimeter to kilometers.

$$k\underbrace{m \quad h}_{5}\underbrace{m \quad d}_{4}\underbrace{am \quad}_{3}\underbrace{m \quad d}_{2}\underbrace{m \quad c}_{1}m \quad mm$$

Note that instead of moving to the right, you are now moving to the left—so the decimal point must do the same: $1cm = 0.\underset{5}{0}\underset{4}{0}\underset{3}{0}\underset{2}{0}\underset{1}{1}km$.

Self Check B
How many milliliters are in 1 liter?

A) 0.001
B) 0.1
C) 100
D) 1,000

Summary

The metric system is an alternative system of measurement used in most countries, as well as in the United States. The metric system is based on joining one of a series of prefixes, including kilo-, hecto-, deka-, deci-, centi-, and milli-, with a base unit of measurement, such as meter, liter, or gram. Units in the metric system are all related by a power of 10, which means that each successive unit is 10 times larger than the previous one. This makes converting one metric measurement to another a straightforward process, and is often as simple as moving a decimal point. It is always important, though, to consider the direction of the conversion. If you are converting a smaller unit to a larger unit, then the decimal point has to move to the left (making your number smaller); if you are converting a larger unit to a smaller unit, then the decimal point has to move to the right (making your number larger).

1.2.1 Self Check Solutions

Self Check A

Which of the following sets of three units are all metric measurements of length?

A) inch, foot, yard

B) kilometer, centimeter, millimeter

C) kilogram, gram, centigram

D) kilometer, foot, decimeter

A) inch, foot, yard
Incorrect. Although these units do measure length, they are all units of measurement from the U.S. customary system. The correct answer is kilometer, centimeter, millimeter.

B) kilometer, centimeter, millimeter
Correct. All of these measurements are from the metric system. You can tell they are measurements of length because they all contain the word “meter.”

C) kilogram, gram, centigram
Incorrect. These measurements are from the metric system, but they are measurements of mass, not length. The correct answer is kilometer, centimeter, millimeter.

D) kilometer, foot, decimeter
Incorrect. Kilometer and decimeter are metric units of length, but foot is not. The correct answer is kilometer, centimeter, millimeter.

Self Check B

How many milliliters are in 1 liter?

A) 0.001

B) 0.1

C) 100

D) 1,000

A) 0.001
Incorrect. One liter is larger than one milliliter, so you would expect there to be more than one milliliter in 1 liter. The correct answer is 1,000.

B) 0.1
Incorrect. One liter is larger than one milliliter, so you would expect there to be more than one milliliter in 1 liter. The correct answer is 1,000.

C) 100
Incorrect. There are 100 milliliters in 1 deciliter. The correct answer is 1,000.

D) 1,000
Correct. There are 10 milliliters in a centiliter, 10 centiliters in a deciliter, and 10 deciliters in a liter. Multiply: 10 • 10 • 10, to find the number of milliliters in a liter, 1,000.

1.2.2 Converting within the Metric System

Learning Objective(s)
1 Perform arithmetic calculations on metric units of length, mass, and volume.

Introduction

While knowing the different units used in the metric system is important, the real purpose behind learning the metric system is for you to be able to use these measurement units to calculate the size, mass, or volume of different objects. In practice, it is often necessary to convert one metric measurement to another unit—this happens frequently in the medical, scientific, and technical fields, where the metric system is commonly used.

If you have a prescription for 5,000 mg of medicine, and upon getting it filled, the dosage reads 5 g of medicine, did the pharmacist make a mistake?

For a moment, imagine that you are a pharmacist. You receive three prescriptions for liquid amoxicillin: one calls for 2.5 centiliters, one calls for 0.3 deciliters, and one calls for 450 milliliters. Amoxicillin is stored in the refrigerator in 1 liter, 1 deciliter, and 1 centiliter containers. Which container should you use to ensure you are not wasting any of the unused drug?

To solve this problem, you need to know how to convert from one measurement to another as well as how to add different quantities together. Let's take a look at how to do this.

Converting from Larger to Smaller Units

Objective 1

Converting between measurements in the metric system is simply a matter of identifying the unit that you have, the unit that you want to convert to, and then counting the number of units between them. A basic example of this is shown below.

Example

Problem **Convert 1 kilometer to decimeters.**

km	hm	dam	m	dm	cm	mm
^				^		

Identify locations of kilometers and decimeters.

Kilometers (km) are larger than decimeters (dm), so you expect there to be more than one dm in a km.

×10 ×10 ×10 ×10

km	hm	dam	m	dm	cm	mm
^				^		

→ → → →

Count the intermediate units, multiplying by 10 as you go.

		(Since you are going from a larger unit to a smaller unit, you multiply.)
	1 km • 10 • 10 • 10 • 10 = 10,000 dm	Multiply to find the number of decimeters in one kilometer.
Answer	1 kilometer = 10,000 decimeters	

This problem is straightforward because you are converting 1 kilometer to another unit. The example below shows how you would solve this problem if you were asked to convert 8.2 *kilometers* to *decimeters*. Notice that most steps are the same; the critical difference is that you multiply by 8.2 in the final step.

Example		
Problem	**Convert 8.2 kilometers to decimeters.**	
	km hm dam m dm cm mm ^ ^	Identify locations of kilometers and decimeters.
		Kilometers (km) are larger than decimeters (dm), so you expect there to be more than one dm in a km.
	×10 ×10 ×10 ×10 km hm dam m dm cm mm ^ ^ → → → →	Count the intermediate units, multiplying by 10 as you go.
		Since you are going from a larger unit to a smaller unit, multiply.
	8.2 km • 10 • 10 • 10 • 10 = 82,000 dm	Multiply to find the number of decimeters in 8.2 kilometers.
Answer	8.2 kilometers = 82,000 decimeters	

You can also apply the rules of base 10 to use the “move the decimal” shortcut method in this example. Notice how decimeters (*dm*) is four places to the right of kilometers (*km*); similarly, you move the decimal point four places to the right when converting 8.2 kilometers to decimeters.

km hm dam m dm cm mm

1 2 3 4

$$8.2\ km = 8.2000.\ dm$$
$$1\ 2\ 3\ 4$$
$$8.2\ km = 82{,}000\ dm$$

Example		
Problem	**Convert 0.55 liters to centiliters.**	
	kl hl dal l dl cl ml 1 2	Count two places from liters to centiliters.
	$0.55\ l = 0.55.\ cl$ 1 2	In 0.55 *l*, move the decimal point two places to the right.
	$0.55\ l = 55\ cl$	
Answer	0.55 liters = 55 centiliters	

Self Check A

How many dekaliters are in 0.5 deciliters?

A) 500

B) 5

C) 0.5

D) 0.005

Converting from Smaller to Larger Units

You can use similar processes when converting from smaller to larger units. When converting a larger unit to a smaller one, you multiply; when you convert a smaller unit to a larger one, you divide. Here is an example.

Example		
Problem	**Convert 739 centigrams to grams.**	
	kg | hg | dag | g | dg | cg | mg ^ ^	Identify locations of centigrams and grams. Centigrams (cg) are smaller than grams (g), so you expect there to be less than 739 g in 739 cg.

÷10 ÷10

kg	hg	dag	g	dg	cg	mg

^ ^

← ←

Count the intermediate units, dividing by 10 as you go.

Since you are going from a smaller unit to a larger unit, divide.

739 ÷ 10 ÷ 10 = 7.39 g — Divide to find the number of grams in 739 centigrams.

Answer 739 centigrams = 7.39 grams

Notice that the shortcut method of counting prefixes and moving the decimal the same number of places also works here. Just make sure you are moving the decimal point in the correct direction for the conversion.

Example

Problem **Convert 205.5 milliliters to kiloliters.**

kl hl dal l dl cl ml

6 5 4 3 2 1

Count six places from milliliters to kiloliters.

0.000205.5

6 5 4 3 2 1

Milliliters is smaller than kiloliters, so you expect the number 205.5 to get smaller as you move up the metric chart.

In 205.5 ml, move the decimal point six places to the left.

$205.5\ ml = 0.0002055\ kl$

Answer 205.5 milliliters = 0.0002055 kiloliters

Self Check B

Convert 3,085 milligrams to grams.

A) 3,085,000 grams

B) 308.5 grams

C) 3.085 grams

D) 0.3085 grams

Factor Label Method

There is yet another method that you can use to convert metric measurements—the **factor label method**. You used this method when you were converting measurement units within the U.S. customary system.

The factor label method works the same in the metric system; it relies on the use of unit fractions and the cancelling of intermediate units. The table below shows some of the **unit equivalents** and **unit fractions** for length in the metric system. (You should notice that all of the unit fractions contain a factor of 10. Remember that the metric system is based on the notion that each unit is 10 times larger than the one that came before it.)

Also, notice that two new prefixes have been added here: mega- (which is very big) and micro- (which is very small).

Unit Equivalents	Conversion Factors	
1 meter = 1,000,000 micrometers	$\frac{1\,m}{1{,}000{,}000\,\mu m}$	$\frac{1{,}000{,}000\,\mu m}{1\,m}$
1 meter = 1,000 millimeters	$\frac{1\,m}{1{,}000\,mm}$	$\frac{1{,}000\,mm}{1\,m}$
1 meter = 100 centimeters	$\frac{1\,m}{100\,cm}$	$\frac{100\,cm}{1\,m}$
1 meter = 10 decimeters	$\frac{1\,m}{10\,dm}$	$\frac{10\,dm}{1\,m}$
1 dekameter = 10 meters	$\frac{1\,dam}{10\,m}$	$\frac{10\,m}{1\,dam}$
1 hectometer = 100 meters	$\frac{1\,hm}{100\,m}$	$\frac{100\,m}{1\,hm}$
1 kilometer = 1,000 meters	$\frac{1\,km}{1{,}000\,m}$	$\frac{1{,}000\,m}{1\,km}$
1 megameter = 1,000,000 meters	$\frac{1\,Mm}{1{,}000{,}000\,m}$	$\frac{1{,}000{,}000\,m}{1\,Mm}$

When applying the factor label method in the metric system, be sure to check that you are not skipping over any intermediate units of measurement!

Example		
Problem	**Convert 7,225 centimeters to meters.**	
	7,225 cm = ___ m	Meters is larger than centimeters, so you expect your answer to be less than 7,225.

$$\frac{7{,}225\ cm}{1} \bullet \frac{1\ m}{100\ cm} = \_\_\_\ m$$

Using the factor label method, write 7,225 cm as a fraction and use unit fractions to convert it to m.

$$\frac{7{,}225\ \cancel{cm}}{1} \bullet \frac{1\ m}{100\ \cancel{cm}} = \_\_\_\ m$$

Cancel similar units, multiply, and simplify.

$$\frac{7{,}225}{1} \bullet \frac{1\ m}{100} = \frac{7{,}225}{100} m$$

$$\frac{7{,}225 m}{100} = 72.25 m$$

Answer 7,225 centimeters = 72.25 meters

Self Check C

Using whichever method you prefer, convert 32.5 kilometers to meters.

A) 32,500 m

B) 325 m

C) 0.325 m

D) 0.00325 m

Now that you have seen how to convert among metric measurements in multiple ways, let's revisit the problem posed earlier.

Example

Problem **If you have a prescription for 5,000 mg of medicine, and upon getting it filled, the dosage reads 5 g of medicine, did the pharmacist make a mistake?**

5,000 mg = ___ g? Need to convert mg to g.

$$\frac{5{,}000\ mg}{1} \bullet \frac{1\ g}{1{,}000\ mg} = \_\_\_\ g$$

$$\frac{5{,}000\ \cancel{mg}}{1} \bullet \frac{1\ g}{1{,}000\ \cancel{mg}} = \_\_\_\ g$$

$$\frac{5{,}000 \cdot 1\,g}{1 \cdot 1{,}000} = \frac{5{,}000g}{1{,}000}$$

$$\frac{5{,}000g}{1{,}000} = 5\,g$$

Answer 5 g = 5,000 mg, so the pharmacist did not make a mistake.

Summary

To convert among units in the metric system, identify the unit that you have, the unit that you want to convert to, and then count the number of units between them. If you are going from a larger unit to a smaller unit, you multiply by 10 successively. If you are going from a smaller unit to a larger unit, you divide by 10 successively. The factor label method can also be applied to conversions within the metric system. To use the factor label method, you multiply the original measurement by unit fractions; this allows you to represent the original measurement in a different measurement unit.

1.2.2 Self Check Solutions

Self Check A

How many dekaliters are in 0.5 deciliters?

A) 500

B) 5

C) 0.5

D) 0.005

A) 500
Incorrect. A dekaliter is larger than a deciliter, so you would expect the number of dekaliters in 0.5 deciliters to be smaller than 0.5. The correct answer is 0.005.

B) 5
Incorrect. A dekaliter is larger than a deciliter, so you would expect the number of dekaliters in 0.5 deciliters to be smaller than 0.5. The correct answer is 0.005.

C) 0.5
Incorrect. Deciliters and dekaliters are different units of measurement, so you would not expect 0.5 deciliters to equal 0.5 dekaliters. The correct answer is 0.005.

D) 0.005
Correct. One deciliter is 100 times smaller than a dekaliter, so you move the decimal point two places to the left to convert 0.5 deciliters to 0.005 dekaliters.

Self Check B

Convert 3,085 milligrams to grams.

A) 3,085,000 grams

B) 308.5 grams

C) 3.085 grams

D) 0.3085 grams

A) 3,085,000 grams
Incorrect. Grams are larger than milligrams, so you would expect the number of grams in 3,085 milligrams to be less than 3,085. The correct answer is 3.085 grams.

B) 308.5 grams
Incorrect. One gram is more than 10 times larger than a milligram, so you would expect the number of grams to be less than 308.5. The correct answer is 3.085 grams.

C) 3.085 grams
Correct. One gram is 1,000 times larger than a milligram, so you can move the decimal point in 3,085 three places to the left.

D) 0.3085 grams
Incorrect. This is too small; one gram is 1,000, not 10,000, times larger than a milligram. The correct answer is 3.085 grams.

Self Check C

Using whichever method you prefer, convert 32.5 kilometers to meters.

A) 32,500 m

B) 325 m

C) 0.325 m

D) 0.00325 m

A) 32,500 m
Correct. To find the number of m in 32.5 km, you can set up the following equation: $\frac{32.5\,km}{1} \bullet \frac{1{,}000\,m}{1\,km} = \frac{32{,}500\,m}{1}$. The km units cancel, leaving the answer in m.

B) 325 m
Incorrect. A km is more than 10 times the size of a m; look at the unit fractions and try your calculations again. The correct answer is 32,500 m.

C) 0.325 m
Incorrect. A km is larger than a meter, so you would expect the number of meters in 32.5 km to be more than 32.5. Look at the unit fractions and try your calculations again. The correct answer is 32,500 m.

D) 0.00325 m
Incorrect. A km is larger than a meter, so you would expect the number of meters in 32.5 km to be more than 32.5. Look at the unit fractions and try your calculations again. The correct answer is 32,500 m.

1.2.3 Using Metric Conversions to Solve Problems

Learning Objective(s)
1 Solve application problems involving metric units of length, mass, and volume.

Introduction

Learning how to solve real-world problems using metric conversions is as important as learning how to do the conversions themselves. Mathematicians, scientists, nurses, and even athletes are often confronted with situations where they are presented with information using metric measurements, and must then make informed decisions based on that data.

To solve these problems effectively, you need to understand the context of a problem, perform conversions, and then check the reasonableness of your answer. Do all three of these steps and you will succeed in whatever measurement system you find yourself using.

Understanding Context and Performing Conversions

Objective 1

The first step in solving any real-world problem is to understand its context. This will help you figure out what kinds of solutions are reasonable (and the problem itself may give you clues about what types of conversions are necessary). Here is an example.

Example

Problem **In the Summer Olympic Games, athletes compete in races of the following lengths: 100 meters, 200 meters, 400 meters, 800 meters, 1500 meters, 5000 meters and 10,000 meters. If a runner were to run in all these races, how many kilometers would he run?**

$$\begin{array}{r} 10{,}000 \\ 5{,}000 \\ 1{,}500 \\ 800 \\ 400 \\ 200 \\ +\ \ 100 \\ \hline 18{,}000 \end{array}$$

To figure out how many kilometers he would run, you need to first add all of the lengths of the races together and *then* convert that measurement to kilometers.

$$\frac{18{,}000\ m}{1} \cdot \frac{1\ km}{1{,}000\ m} = \_\_\_\ km$$

Use the factor label method and unit fractions to convert from meters to kilometers.

$$\frac{18{,}000\ \cancel{m}}{1} \bullet \frac{1\,km}{1{,}000\ \cancel{m}} = \_\_\_\ km$$

Cancel, multiply, and solve.

$$\frac{18{,}000}{1} \bullet \frac{1\,km}{1{,}000} = \frac{18{,}000\,km}{1{,}000}$$

$$\frac{18{,}000\,km}{1{,}000} = 18\,km$$

Answer The runner would run 18 kilometers.

This may not be likely to happen (a runner would have to be quite an athlete to compete in all of these races) but it is an interesting question to consider. The problem required you to find the total distance that the runner would run (in kilometers). The example showed how to add the distances, in meters, and then convert that number to kilometers.

An example with a different context, but still requiring conversions, is shown below.

Example

Problem **One bottle holds 295 dl while another one holds 28,000 ml. What is the difference in capacity between the two bottles?**

295 dl = ___ l
28,000 ml = ___ l

The two measurements are in different units. You can convert both units to liters and then compare them.

$$\frac{295\,dl}{1} \bullet \frac{1\,l}{10\,dl} = \_\_\_\ l$$

Convert dl to liters.

$$\frac{295\ \cancel{dl}}{1} \bullet \frac{1\,l}{10\ \cancel{dl}} = \_\_\_\ l$$

Cancel similar units and multiply.

$$\frac{295}{1} \bullet \frac{1\,l}{10} = \frac{295\,l}{10}$$

$$\frac{295\,l}{10} = 29.5\,l$$

295 dl = 29.5 liters.

$$\frac{28{,}000\,ml}{1} \bullet \frac{1\,l}{1{,}000\,ml} = \_\_\_\ l$$

Convert ml to liters.

$$\frac{28{,}000\ \cancel{ml}}{1} \bullet \frac{1\,l}{1{,}000\ \cancel{ml}} = \_\_\_\ l$$

$$\frac{28{,}000}{1} \bullet \frac{1\,l}{1{,}000} = \frac{28{,}000\,l}{1{,}000}$$

$$\frac{28{,}000\,l}{1{,}000} = 28\,l$$ 28,000 ml = 28 liters

29.5 liters – 28 liters = 1.5 liters The question asks for "difference in capacity" between the bottles.

Answer There is a difference in capacity of 1.5 liters between the two bottles.

This problem asked for the difference between two quantities. The easiest way to find this is to convert one quantity so that both quantities are measured in the same unit, and then subtract one from the other.

Self Check A

One boxer weighs in at 85 kg. He is 80 dag heavier than his opponent. How much does his opponent weigh?

A) 5 kg

B) 84.2 kg

C) 84.92 kg

D) 85.8 kg

Checking your Conversions

Sometimes it is a good idea to check your conversions using a second method. This usually helps you catch any errors that you may make, such as using the wrong unit fractions or moving the decimal point the wrong way.

Example

Problem **A two-liter bottle contains 87 centiliters of oil and 4.1 deciliters of water. How much more liquid is needed to fill the bottle?**

87 cl + 4.1 dl + ___ = 2 l You are looking for the amount of liquid needed to fill the bottle. Convert both measurements to liters and then solve the problem.

87 cl = ___ l Convert 87 cl to liters.

$$\frac{87\ cl}{1} \bullet \frac{1\ l}{100\ cl} = \_\_\_\ l$$

$$\frac{87\ \cancel{cl}}{1} \bullet \frac{1\ l}{100\ \cancel{cl}} = \_\_\_\ l$$

$$\frac{87}{1} \bullet \frac{1\ l}{100} = \frac{87\ l}{100}$$

$$\frac{87\ l}{100} = 0.87\ l$$

4.1 dl = ___ l Convert 4.1 dl to liters.

$$\frac{4.1\ dl}{1} \bullet \frac{1\ l}{10\ dl} = \_\_\_\ l$$

$$\frac{4.1\ \cancel{dl}}{1} \bullet \frac{1\ l}{10\ \cancel{dl}} = \_\_\_\ l$$

$$\frac{4.1}{1} \bullet \frac{1\ l}{10} = \frac{4.1\ l}{10}$$

$$\frac{4.1\ l}{10} = 0.41\ l$$

87 cl + 4.1 dl + ___ = 2 l Subtract to find how much more liquid is needed to fill the bottle.

0.87 liter + 0.41 liter + ___ = 2 liters

rs – 0.87 liter – 0.41 liter = 0.72 liter

Answer The amount of liquid needed to fill the bottle is 0.72 liter.

Having come up with the answer, you could also check your conversions using the quicker “move the decimal” method, shown below.

Example

Problem **A two-liter bottle contains 87 centiliters of oil and 4.1 deciliters of water. How much more liquid is needed to fill the bottle?**

87 cl + 4.1 dl + ___ = 2 l	You are looking for the amount of liquid needed to fill the bottle.
87 cl = ___ l	Convert 87 cl to liters.
$kl \quad hl \quad dal \quad l \quad \underset{2}{dl} \quad \underset{1}{cl} \quad ml$	On the chart, l is two places to the left of cl.
$.87.\underset{2\ 1}{cl}$	Move the decimal point two places to the left in 87 cl.
$87\,cl = 0.87\,l$	
4.1 dl = ___ l	Convert 4.1 dl to liters.
$kl \quad hl \quad dal \quad l \quad \underset{1}{dl} \quad cl \quad ml$	On the chart, l is one place to the left of dl.
$.4.1\underset{1}{dl}$	Move the decimal point one place to the left in 4.1 dl.
$4.1dl = 0.41l$	
87 cl + 4.1 dl + ___ = 2 l	Subtract to find how much more liquid is needed to fill the bottle.
0.87 liter + 0.41 liter + ___ = 2 liters	
2 liters – 0.87 liter – 0.41 liter = 0.72 liter	

Answer The amount of liquid needed to fill the bottle is 0.72 liter.

The initial answer checks out—0.72 liter of liquid is needed to fill the bottle. Checking one conversion with another method is a good practice for catching any errors in scale.

Summary

Understanding the context of real-life application problems is important. Look for words within the problem that help you identify what operations are needed, and then apply the correct unit conversions. Checking your final answer by using another conversion method (such as the “move the decimal” method, if you have used the factor label method to solve the problem) can cut down on errors in your calculations.

1.2.3 Self Check Solutions

Self Check A

One boxer weighs in at 85 kg. He is 80 dag heavier than his opponent. How much does his opponent weigh?

A) 5 kg

B) 84.2 kg

C) 84.92 kg

D) 85.8 kg

A) 5 kg
Incorrect. Look at the unit labels—the boxer is 80 *dag* heavier, not 80 *kg* heavier. The correct answer is 84.2 kg.

B) 84.2 kg
Correct. 80 dag = 0.8 kg, and 85 – 0.8 = 84.2.

C) 84.92 kg
Incorrect. This would have been true if the difference in weight was 8 dag, not 80 dag. The correct answer is 84.2 kg.

D) 85.8 kg
Incorrect. The first boxer is 80 dag *heavier*, not *lighter* than his opponent. This question asks for the opponent's weight. The correct answer is 84.2 kg.

1.3.1 Temperature Scales

Learning Objective(s)

1. State the freezing and boiling points of water on the Celsius and Fahrenheit temperature scales.
2. Convert from one temperature scale to the other, using conversion formulas.

Introduction

Turn on the television any morning and you will see meteorologists talking about the day's weather forecast. In addition to telling you what the weather conditions will be like (sunny, cloudy, rainy, muggy), they also tell you the day's forecast for high and low temperatures. A hot summer day may reach 100° in Philadelphia, while a cool spring day may have a low of 40° in Seattle.

If you have been to other countries, though, you may notice that meteorologists measure heat and cold differently outside of the United States. For example, a TV weatherman in San Diego may forecast a high of 89°, but a similar forecaster in Tijuana, Mexico—which is only 20 miles south—may look at the same weather pattern and say that the day's high temperature is going to be 32°. What's going on here?

The difference is that the two countries use different temperature scales. In the United States, temperatures are usually measured using the **Fahrenheit** scale, while most countries that use the metric system use the **Celsius** scale to record temperatures. Learning about the different scales—including how to convert between them—will help you figure out what the weather is going to be like, no matter which country you find yourself in.

Objective **1**

Measuring Temperature on Two Scales

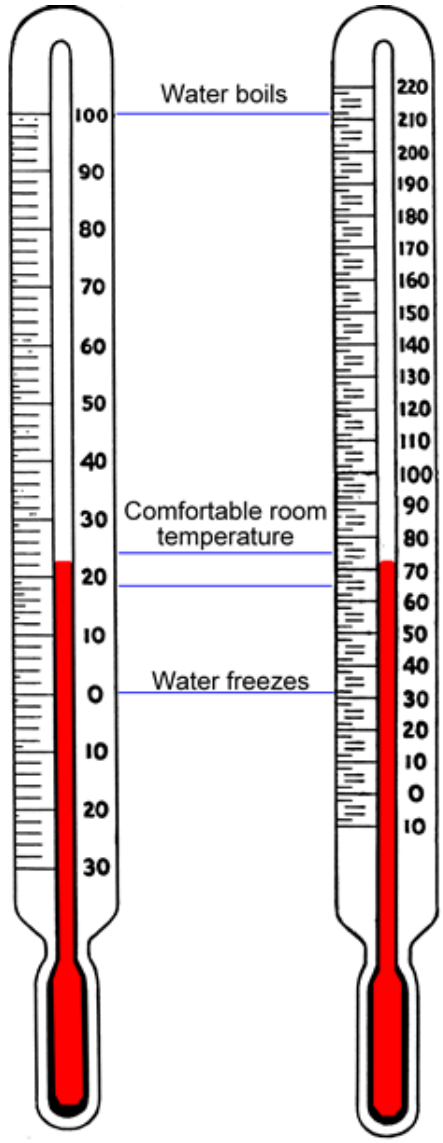
Fahrenheit and Celsius are two different scales for measuring temperature.

A thermometer measuring a temperature of 22° Celsius is shown here.

On the Celsius scale, water freezes at 0° and boils at 100°.

If the United States were to adopt the Celsius scale, forecast temperatures would rarely go below -30° or above 45°. (A temperature of -18° may be forecast for a cold winter day in Michigan, while a temperature of 43° may be predicted for a hot summer day in Arizona.)

Most office buildings maintain an indoor temperature between 18°C and 24°C to keep employees comfortable.



A thermometer measuring a temperature of 72° Fahrenheit is shown here.

On the Fahrenheit scale, water freezes at 32° and boils at 212°.

In the United States, forecast temperatures measured in Fahrenheit rarely go below -20° or above 120°. (A temperature of 0° may be forecast for a cold winter day in Michigan, while a temperature of 110° may be predicted for a hot summer day in Arizona.)

Most office buildings maintain an indoor temperature between 65°F and 75°F to keep employees comfortable.

Self Check A

A cook puts a thermometer into a pot of water to see how hot it is. The thermometer reads 132°, but the water is not boiling yet. Which temperature scale is the thermometer measuring?

A) Celsius

B) Fahrenheit

Converting Between the Scales

Objective **2**

By looking at the two thermometers shown, you can make some general comparisons between the scales. For example, many people tend to be comfortable in outdoor temperatures between 50°F and 80°F (or between 10°C and 25°C). If a meteorologist predicts an average temperature of 0°C (or 32°F), then it is a safe bet that you will need a winter jacket.

Sometimes, it is necessary to convert a Celsius measurement to its exact Fahrenheit measurement or vice versa. For example, what if you want to know the temperature of your child in Fahrenheit, and the only thermometer you have measures temperature in Celsius measurement? Converting temperature between the systems is a straightforward process as long as you use the formulas provided below.

Temperature Conversion Formulas

To convert a Fahrenheit measurement to a Celsius measurement, use this formula.

$$C = \frac{5}{9}(F - 32)$$

To convert a Celsius measurement to a Fahrenheit measurement, use this formula.

$$F = \frac{9}{5}C + 32$$

How were these formulas developed? They came from comparing the two scales. Since the freezing point is 0° in the Celsius scale and 32° on the Fahrenheit scale, we subtract 32 when converting from Fahrenheit to Celsius, and add 32 when converting from Celsius to Fahrenheit.

There is a reason for the fractions $\frac{5}{9}$ and $\frac{9}{5}$, also. There are 100 degrees between the freezing (0°) and boiling points (100°) of water on the Celsius scale and 180 degrees

between the similar points (32° and 212°) on the Fahrenheit scale. Writing these two scales as a ratio, $\frac{F^\circ}{C^\circ}$, gives $\frac{180^\circ}{100^\circ} = \frac{180^\circ \div 20}{100^\circ \div 20} = \frac{9}{5}$. If you flip the ratio to be $\frac{C^\circ}{F^\circ}$, you get $\frac{100^\circ}{180^\circ} = \frac{100^\circ \div 20}{180^\circ \div 20} = \frac{5}{9}$. Notice how these fractions are used in the conversion formulas.

The example below illustrates the conversion of Celsius temperature to Fahrenheit temperature, using the boiling point of water, which is 100° C.

Example

Problem **The boiling point of water is 100°C. What temperature does water boil at in the Fahrenheit scale?**

$F = \frac{9}{5}C + 32$	A Celsius temperature is given. To convert it to the Fahrenheit scale, use the formula at the left.
$F = \frac{9}{5}(100) + 32$	Substitute 100 for *C* and multiply.
$F = \frac{900}{5} + 32$	
$F = \frac{900 \div 5}{5 \div 5} + 32$	Simplify $\frac{900}{5}$ by dividing numerator and denominator by 5.
$F = \frac{180}{1} + 32$	
$F = 212$	Add 180 + 32.

Answer The boiling point of water is 212°F.

Example

Problem **Water freezes at 32°F. On the Celsius scale, what temperature is this?**

$$C = \frac{5}{9}(F - 32)$$ A Fahrenheit temperature is given. To convert it to the Celsius scale, use the formula at the left.

$$C = \frac{5}{9}(32 - 32)$$ Substitute 32 for F and subtract.

$$C = \frac{5}{9}(0)$$ Any number multiplied by 0 is 0.

$$C = 0$$

Answer The freezing point of water is 0°C.

The two previous problems used the conversion formulas to verify some temperature conversions that were discussed earlier—the boiling and freezing points of water. The next example shows how these formulas can be used to solve a real-world problem using different temperature scales.

Example

Problem **Two scientists are doing an experiment designed to identify the boiling point of an unknown liquid. One scientist gets a result of 120°C; the other gets a result of 250°F. Which temperature is higher and by how much?**

What is the difference between 120°C and 250°F? One temperature is given in °C, and the other is given in °F. To find the difference between them, we need to measure them on the same scale.

$$F = \frac{9}{5}C + 32$$ Use the conversion formula to convert 120°C to °F. (You could convert 250°F to °C instead; this is explained in the text after this example.)

$$F = \frac{9}{5}(120) + 32$$ Substitute 120 for C.

$$F = \frac{1080}{5} + 32$$ Multiply.

$F = \frac{1080 \div 5}{5 \div 5} + 32$	Simplify $\frac{1080}{5}$ by dividing numerator and denominator by 5.
$F = \frac{216}{1} + 32$	Add 216 + 32.
$F = 248$	You have found that 120°C = 248°F.
250°F – 248°F = 2°F	To find the difference between 248°*F* and 250°F, subtract.

Answer 250°F is the higher temperature by 2°F.

You could have converted 250°F to °C instead, and then found the difference in the two measurements. (Had you done it this way, you would have found that 250°F = 121.1°C, and that 121.1°C is 1.1°C higher than 120°C.) Whichever way you choose, it is important to compare the temperature measurements within the same scale, and to apply the conversion formulas accurately.

Self Check B

Tatiana is researching vacation destinations, and she sees that the average summer temperature in Barcelona, Spain is around 26°C. What is the average temperature in degrees Fahrenheit?

A) 79°F

B) -3°F

C) 45°F

D) 58°F

Summary

Temperature is often measured in one of two scales: the Celsius scale and the Fahrenheit scale. A Celsius thermometer will measure the boiling point of water at 100° and its freezing point at 0°; a Fahrenheit thermometer will measure the same events at 212° for the boiling point of water and 32° as its freezing point. You can use conversion formulas to convert a measurement made in one scale to the other scale.

1.3.1 Self Check Solutions

Self Check A

A cook puts a thermometer into a pot of water to see how hot it is. The thermometer reads 132°, but the water is not boiling yet. Which temperature scale is the thermometer measuring?

A) Celsius

B) Fahrenheit

A) Celsius
Incorrect. On the Celsius scale, water boils at 100°, so if the water is not boiling and the measurement is over 100°, then it cannot be Celsius. The correct answer is Fahrenheit.

B) Fahrenheit
Correct. Water boils at 212° on the Fahrenheit scale, so a measurement of 132° on a Fahrenheit scale is legitimate for hot (but non-boiling) water.

Self Check B

Tatiana is researching vacation destinations, and she sees that the average summer temperature in Barcelona, Spain is around 26°C. What is the average temperature in degrees Fahrenheit?

A) 79°F

B) -3°F

C) 45°F

D) 58°F

A) 79°F
Correct. Tatiana can find the Fahrenheit equivalent by solving the equation $F = \frac{9}{5}(26) + 32$. The result is 78.8°F, which rounds to 79°F.

B) -3°F
Incorrect. You used the wrong formula. To find the Fahrenheit equivalent, use the formula $F = \frac{9}{5}C + 32$. The correct answer is 79°F.

C) 45°F
Incorrect. You misapplied the formula; try substituting 26 for *C* in the formula $F = \frac{9}{5}C + 32$. The correct answer is 79°F.

D) 58°F
Incorrect. You misapplied the formula; try substituting 26 for *C* in the formula $F = \frac{9}{5}C + 32$. The correct answer is 79°F.

Unit Recap

1.1.1 Length

The four basic units of measurement that are used in the U.S. customary measurement system are: inch, foot, yard, and mile. Typically, people use yards, miles, and sometimes feet to describe long distances. Measurement in inches is common for shorter objects or lengths.

You need to convert from one unit of measure to another if you are solving problems that include measurements involving more than one type of measurement. Each of the units can be converted to one of the other units using the table of equivalents, the conversion factors, and/or the factor label method shown in this topic.

1.1.2 Weight

In the U.S. customary system of measurement, weight is measured in three units: ounces, pounds, and tons. A pound is equivalent to 16 ounces, and a ton is equivalent to 2,000 pounds. While an object's weight can be described using any of these units, it is typical to describe very heavy objects using tons and very light objects using an ounce. Pounds are used to describe the weight of many objects and people. Often, in order to compare the weights of two objects or people or to solve problems involving weight, you must convert from one unit of measurement to another unit of measurement. Using conversion factors with the factor label method is an effective strategy for converting units and solving problems.

1.1.3 Capacity

There are five basic units for measuring capacity in the U.S. customary measurement system. These are the fluid ounce, cup, pint, quart, and gallon. These measurement units are related to one another, and capacity can be described using any of the units. Typically, people use gallons to describe larger quantities and fluid ounces, cups, pints, or quarts to describe smaller quantities. Often, in order to compare or to solve problems involving the amount of liquid in a container, you need to convert from one unit of measurement to another.

1.2.1 The Metric System

The metric system is an alternative system of measurement used in most countries, as well as in the United States. The metric system is based on joining one of a series of prefixes, including kilo-, hecto-, deka-, deci-, centi-, and milli-, with a base unit of measurement, such as meter, liter, or gram. Units in the metric system are all related by a power of 10, which means that each successive unit is 10 times larger than the previous one. This makes converting one metric measurement to another a straightforward process, and is often as simple as moving a decimal point. It is always important, though, to consider the direction of the conversion. If you are converting a smaller unit to a larger unit, then the decimal point has to move to the left (making your number smaller); if you are converting a larger unit to a smaller unit, then the decimal point has to move to the right (making your number larger).

1.2.2 Converting within the Metric System

To convert among units in the metric system, identify the unit that you have, the unit that you want to convert to, and then count the number of units between them. If you are going from a larger unit to a smaller unit, you multiply by 10 successively. If you are going from a smaller unit to a larger unit, you divide by 10 successively. The factor label method can also be applied to conversions within the metric system. To use the factor label method, you multiply the original measurement by unit fractions; this allows you to represent the original measurement in a different measurement unit.

1.2.3 Using Metric Conversion to Solve Problems

Understanding the context of real-life application problems is important. Look for words within the problem that help you identify what operations are needed, and then apply the correct unit conversions. Checking your final answer by using another conversion method (such as the “move the decimal” method, if you have used the factor label method to solve the problem) can cut down on errors in your calculations.

1.3.1 Temperature Scales

Temperature is often measured in one of two scales: the Celsius scale and the Fahrenheit scale. A Celsius thermometer will measure the boiling point of water at 100° and its freezing point at 0°; a Fahrenheit thermometer will measure the same events at 212° for the boiling point of water and 32° as its freezing point. You can use conversion formulas to convert a measurement made in one scale to the other scale.

Glossary

capacity	The amount of liquid (or other pourable substance) that an object can hold when it's full.
Celsius	A measure of temperature commonly used in countries that use the metric system. On the Celsius scale, water freezes at 0° and boils at 100°.
cup	A unit of capacity equal to 8 fluid ounces.
factor label method	One method of converting a measurement from one unit of measurement to another unit of measurement. In this method, you multiply the original measurement by unit fractions containing different units of measurement to obtain the new unit of measurement.
Fahrenheit	A measure of temperature commonly used in the United States. On the Fahrenheit scale, water freezes at 32° F and boils at 212° F.

fluid ounce	A unit of capacity equal to $\frac{1}{8}$ of a cup. One fluid ounce of water at 62°F weighs about one ounce.
foot	A unit for measuring length in the U.S. customary measurement system. 1 foot = 12 inches
gallon	A unit equal to 4 quarts, or 128 fluid ounces.
gram	The base unit of mass in the Metric system.
inch	A unit for measuring length in the U.S. customary measurement system. 1 foot = 12 inches
length	The distance from one end to the other or the distance from one point to another.
liter	The base unit of volume in the Metric system.
measurement	The use of standard units to find out the size or quantity of items such as length, width, height, mass, weight, volume, temperature or time.
meter	The base unit of length in the Metric system.
metric system	A widely used system of measurement that is based on the decimal system and multiples of 10.
mile	A unit for measuring length in the U.S. customary measurement system. 1 mile = 5,280 feet or 1,760 yards.
ounce	A unit for measuring weight in the U.S. customary measurement system. 16 ounces = 1 pound.
pint	A unit of capacity equal to 16 fluid ounces, or 2 cups.
pound	A unit for measuring weight in the U.S. customary measurement system. 16 ounces = 1 pound.
prefix	A short set of letters that denote the size of measurement units in the Metric System. Metric prefixes include centi-, milli-, kilo-, and hecto-.
quart	A unit of capacity equal to 32 fluid ounces, or 4 cups.
ton	A unit for measuring the weight of heavier items in the U.S. customary measurement system. 1 ton = 2,000 pounds.
U.S. customary measurement	The most common system of measurement used in the United States. It is based on English measurement systems of the 18th

system	century.
unit equivalents	Statements of equivalence between measurement units within a system or in comparison to another system of units. For example 1 foot = 12 inches or 1 inch = 2.54 centimeters are both examples of unit equivalents.
unit fractions	A fraction where the numerator and denominator are equal amounts, as in $\frac{1kg}{1000g}$ or $\frac{12inches}{1foot}$. Unit fractions serve to help with conversions in the Factor Label method.
unit of measurement	A standard amount or quantity. For example, an inch is a unit of measurement.
weight	A mathematical description of how heavy an object is.
yard	A unit for measuring length in the U.S. customary measurement system. 1 yard = 3 feet or 36 inches.