

Solving Complex Equations



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.



This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

- 1) **Make sense of problems and persevere in solving them.**
- 2) **Reason abstractly and quantitatively.**
- 3) **Construct viable arguments and critique the reasoning of others.**
- 4) **Model with mathematics.**
- 5) **Use appropriate tools strategically.**
- 6) **Attend to precision.**
- 7) **Look for and make use of structure.**
- 8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Constructing an equation



Veronica has 58¢ and buys 4 jawbreakers.

Tom has \$1 and buys 7 jawbreakers.

They both receive the same amount of change.



If x is the cost of one jawbreaker, what equation could we use to find x ?



$$58 - 4x = 100 - 7x$$

Veronica's change

"is the same as"

Tom's change

I'm thinking of a number. When I multiply the number by 4, I get the same answer as adding 9 to the number.

What number am I thinking of?

Let's call the unknown number n .

We can solve this problem by writing the equation:

$$4n = n + 9$$

The number multiplied by 4 is the same as the number plus 9.



Solving the equation

Let's solve this equation by transforming both sides of the equation in the same way.

$$4n = n + 9$$

$$\begin{array}{c} \downarrow \\ \textcircled{-n} \\ \downarrow \end{array} \quad \begin{array}{c} \downarrow \\ \textcircled{-n} \\ \downarrow \end{array}$$

$$3n = 9$$

$$\begin{array}{c} \downarrow \\ \textcircled{\div 3} \\ \downarrow \end{array} \quad \begin{array}{c} \downarrow \\ \textcircled{\div 3} \\ \downarrow \end{array}$$

$$n = 3$$

Start by writing the equation down.

Subtract n from both sides.

Always line up the equal signs.

Divide both sides by 3.

This is the solution.

We can check the solution by substituting it back into the original equation:

$$4 \times 3 = 3 + 9 \quad \checkmark$$



Juan and Lucinda have the same number of candies.
Juan started with 3 bags and ate 11 candies.
Lucinda started with 2 bags and ate 3 candies.

How many candies are there in a bag?



Let's call the number of candies in a bag, n .

Can you solve this problem by writing an equation?

$$3n - 11 = 2n - 3$$

How many candies does
Juan have, in terms of n ?

"is the
same as"

How many candies does
Lucinda have, in terms of n ?

Solving word problems

MODELING



board
works

Tom took his nephews to the movie theater. Each ticket cost \$5. Tom treated everyone, including himself, to a popcorn combo for \$8. If he spent \$78, how many people went to the theater?

Press the buttons
for help:

Analyze

Plan

Solve

Check

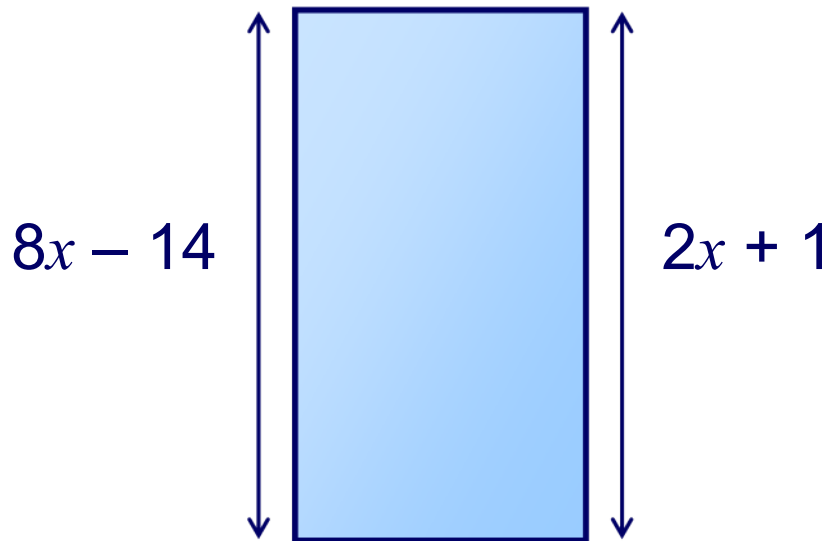


Rectangle problem

MODELING



The area of this rectangle is 27 cm^2 .



Calculate the value of x and use it to find the height of the rectangle.

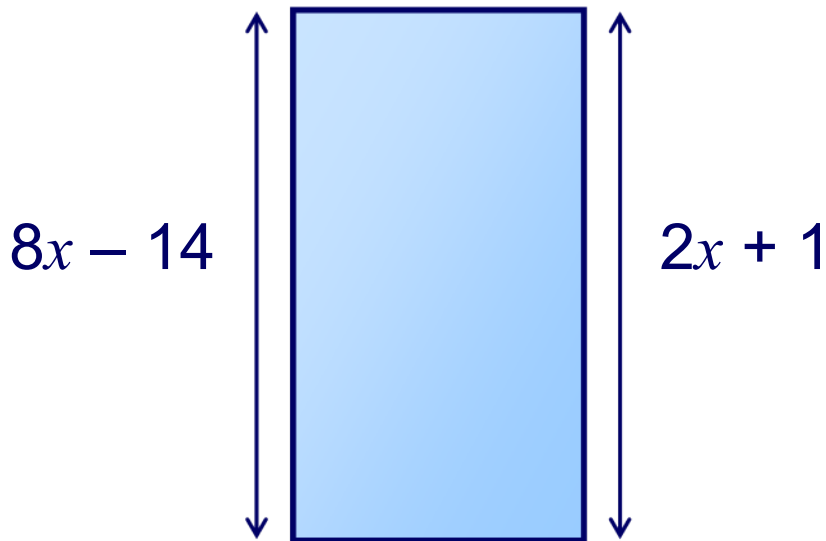
Opposite sides of a rectangle are equal.

We can use this fact to write an equation in terms of x .



Rectangle problem

The area of this rectangle is 27 cm^2 .



$$\begin{array}{r} 8x - 14 = 2x + 1 \\ \downarrow \quad \downarrow \\ \textcircled{-2x} \quad \textcircled{-2x} \\ \downarrow \quad \downarrow \\ 6x - 14 = 1 \\ \downarrow \quad \downarrow \\ \textcircled{+14} \quad \textcircled{+14} \\ \downarrow \quad \downarrow \\ 6x = 15 \\ \downarrow \quad \downarrow \\ \textcircled{\div 6} \quad \textcircled{\div 6} \\ \downarrow \quad \downarrow \\ x = 2.5 \end{array}$$

If $x = 2.5$, we can find the height of the rectangle using substitution:

$$8 \times 2.5 - 14 = 20 - 14 = 6 \text{ cm}$$

Solving word problems

MODELING



board
works

Natalie starts a lawn mowing business. She charges \$25 each time plus a one-time fee of \$20. At the end of summer, she bills one customer \$545. How many times did she mow his lawn?

Press the buttons
for help:

Analyze

Plan

Solve

Check





This equation contains a negative variable.

$$8 - 2n = 4$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ \textcircled{+2n} \quad \textcircled{+2n} \\ \downarrow \quad \downarrow \end{array}$$

$$8 = 4 + 2n$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ \textcircled{-4} \quad \textcircled{-4} \\ \downarrow \quad \downarrow \end{array}$$

$$4 = 2n$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ \textcircled{\div 4} \quad \textcircled{\div 4} \\ \downarrow \quad \downarrow \end{array}$$

$$2 = n$$

$$n = 2$$

Add $2n$ to both sides.

Subtract 4 from both sides.

Divide both sides by 4.

This is the solution.

We always write the variable first.



Equations with parentheses

Equations can contain parentheses. For example:

$$2(3x - 5) = 4x$$

To solve this we can:

Distribute:

$$6x - 10 = 4x$$

Add 10 to both sides:

$$\begin{array}{c} \downarrow \quad \downarrow \\ \textcircled{+10} \quad \textcircled{+10} \\ \downarrow \quad \downarrow \\ 6x = 4x + 10 \end{array}$$

Subtract $4x$ from both sides:

$$\begin{array}{c} \downarrow \quad \downarrow \\ \textcircled{-4x} \quad \textcircled{-4x} \\ \downarrow \quad \downarrow \\ 2x = 10 \end{array}$$

Divide both sides by 2:

$$\begin{array}{c} \downarrow \quad \downarrow \\ \textcircled{\div 2} \quad \textcircled{\div 2} \\ \downarrow \quad \downarrow \\ x = 5 \end{array}$$

Solve these equations by finding x

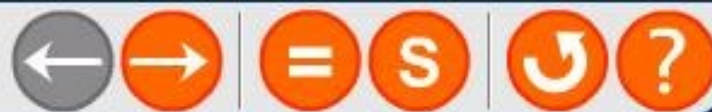
Q1/4 $3x - 11 = 2x - 3$

Press the "=" button to show the work step by step.

8

4

11



Equation dominoes



$$3x+2=17 \quad x=20$$

$$\frac{x}{2}=10 \quad x=-1$$

$$5-3x=8 \quad x=9$$

$$\frac{x}{3}+7=10 \quad x=3$$

$$\frac{12}{x}=4 \quad x=\frac{1}{2}$$

$$6x=3 \quad x=6$$

$$\frac{2x}{3}=10 \quad x=-4$$

$$2x-3=-11 \quad x=12$$

$$23-x=11 \quad x=5$$

$$20+7x=62 \quad x=15$$

Place these dominoes on the grid, matching the equations with their solutions.



Sometimes we can solve equations such as:

$$2(3x - 5) = 4x$$

by first dividing both sides by the number in front of the parentheses:

Divide both sides by 2:

$$3x - 5 = 2x$$

Add 5 to both sides:

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \textcircled{+5} & & \textcircled{+5} \\ \downarrow & & \downarrow \\ 3x = 2x + 5 \end{array}$$

Subtract $2x$ from both sides:

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \textcircled{-2x} & & \textcircled{-2x} \\ \downarrow & & \downarrow \\ x = 5 \end{array}$$

In this example, dividing first means that there are fewer steps.



Solving equations involving division



Linear equations with unknowns on both sides can also involve division.

For example:

$$\frac{5x - 3}{4} = 12 - x$$

In this case we must start by multiplying both sides of the equation by 4.

Distribute:

$$5x - 3 = 4(12 - x)$$

$$5x - 3 = 48 - 4x$$

Add $4x$ to both sides:

$$9x - 3 = 48$$

Subtract 3 from both sides:

$$9x = 45$$

Divide both sides by 9:

$$x = 5$$

Solving equations involving division



Sometimes the expressions on both sides of the equation are divided.

For example:

$$\frac{4}{(x + 3)} = \frac{5}{(3x - 5)}$$

In this example, we can multiply both sides by $(x + 3)$ and $(3x - 5)$ in one step to give:

$$4(3x - 5) = 5(x + 3)$$

Distribute:

$$12x - 20 = 5x + 15$$

Subtract $5x$ from both sides:

$$7x - 20 = 15$$

Add 20 to both sides:

$$7x = 35$$

Divide both sides by 7:

$$x = 5$$