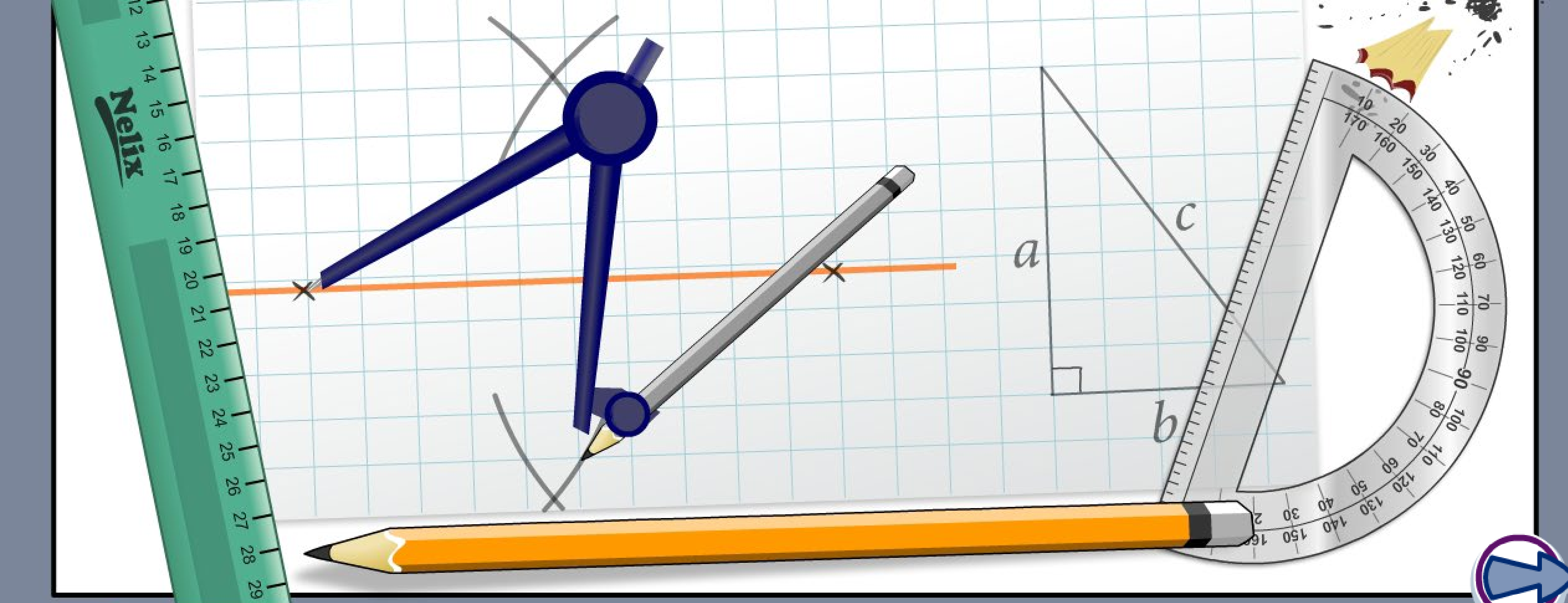
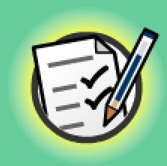
Special Points in Triangles

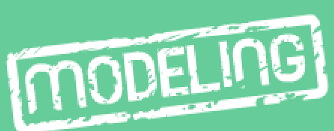


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

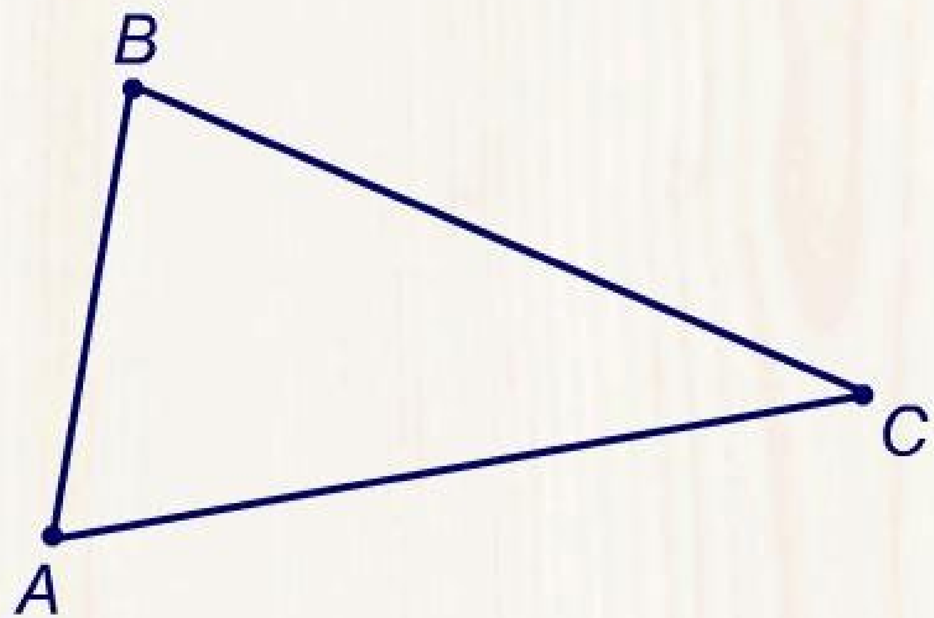
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Constructing the circumcenter of a triangle

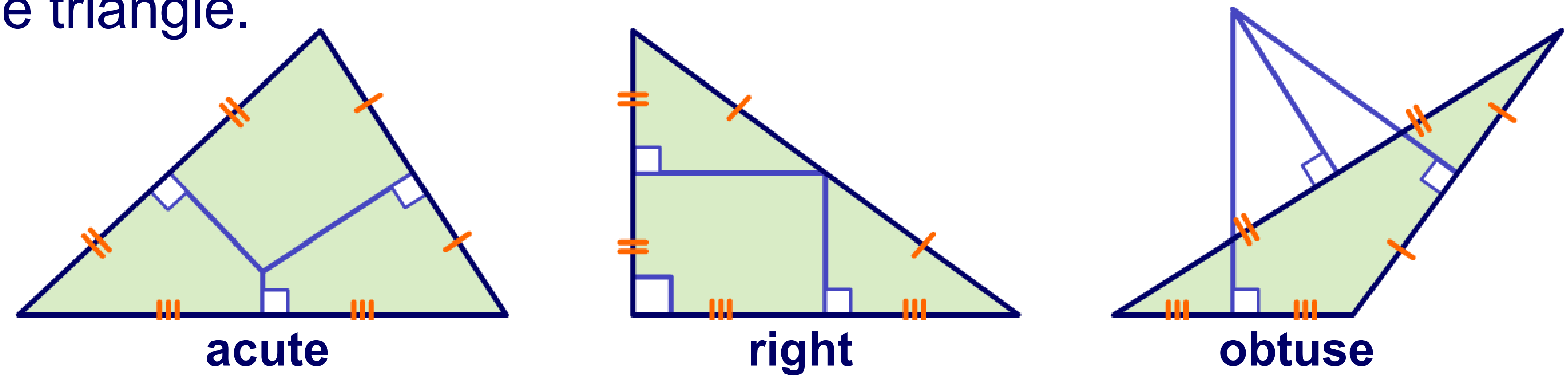
The circumcenter of a triangle is the point where the three perpendicular bisectors of the triangle sides meet.

Press **play** to see how to construct the circumcenter.



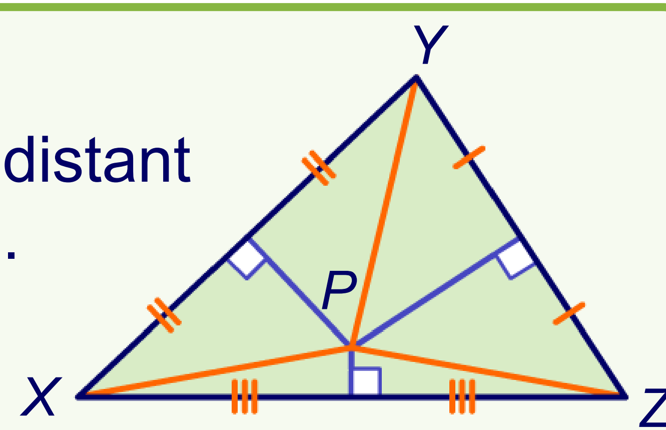
The circumcenter theorem

The **circumcenter of a triangle** is the point where the three perpendicular bisectors meet. It is not always in the interior of the triangle.



Circumcenter theorem:
The circumcenter of a triangle is equidistant from the vertices of the triangle.

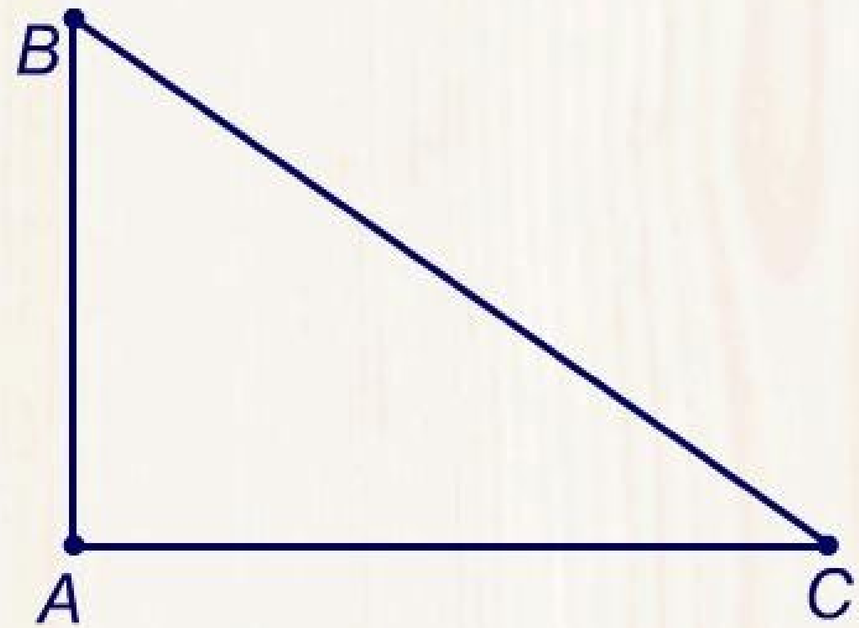
$$PX = PY = PZ$$



Constructing the incenter of a triangle

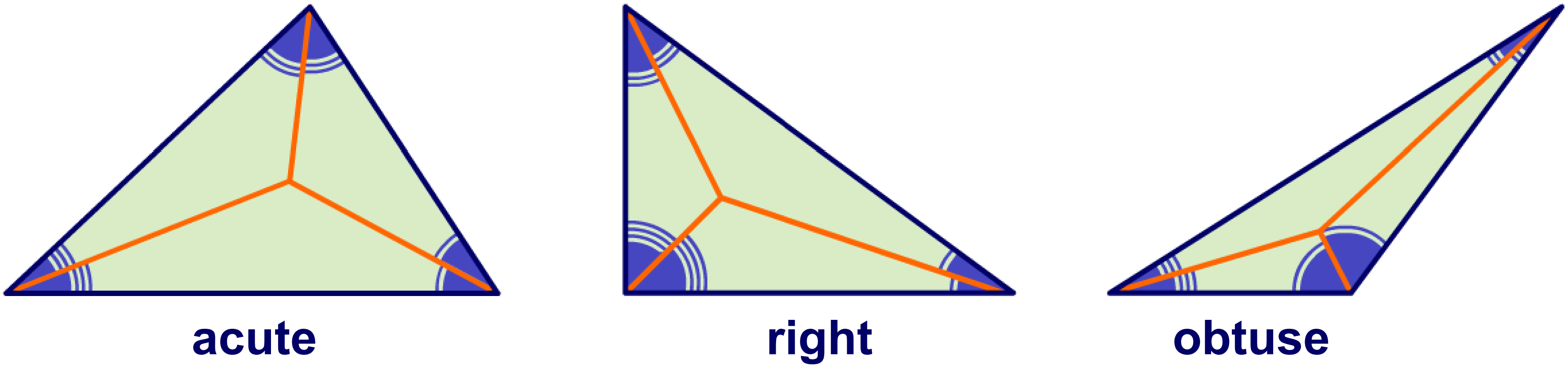
The incenter of a triangle is the point where the three angle bisectors of the triangle meet.

Press **play** to see how to construct the incenter.



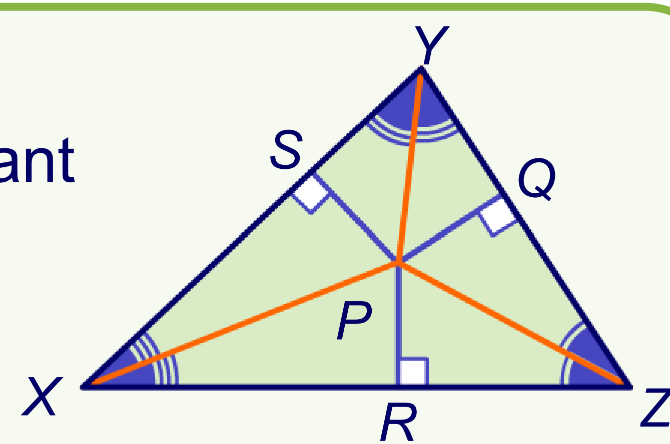
The incenter theorem

The **incenter of a triangle** is the point where the three angle bisectors meet. It is always in the interior of the triangle.



Incenter theorem:
The incenter of a triangle is equidistant from the sides of the triangle.

$$PQ = PR = PS$$

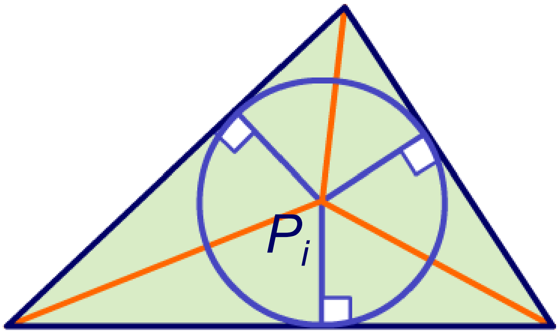


Inscribed and circumscribed shapes

An **inscribed circle** is the largest circle that fits inside the triangle.

Where is the center of the inscribed circle?

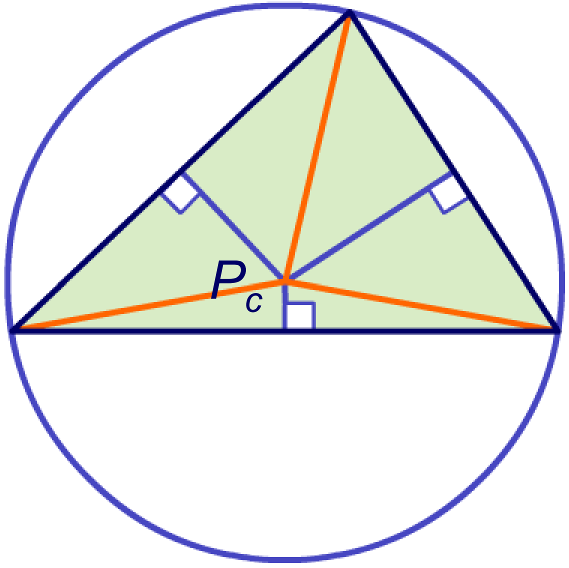
the incenter of the triangle



A **circumscribing circle** is the smallest circle that contains the whole triangle.

Where is the center of the circumscribing circle?

the circumcenter of the triangle

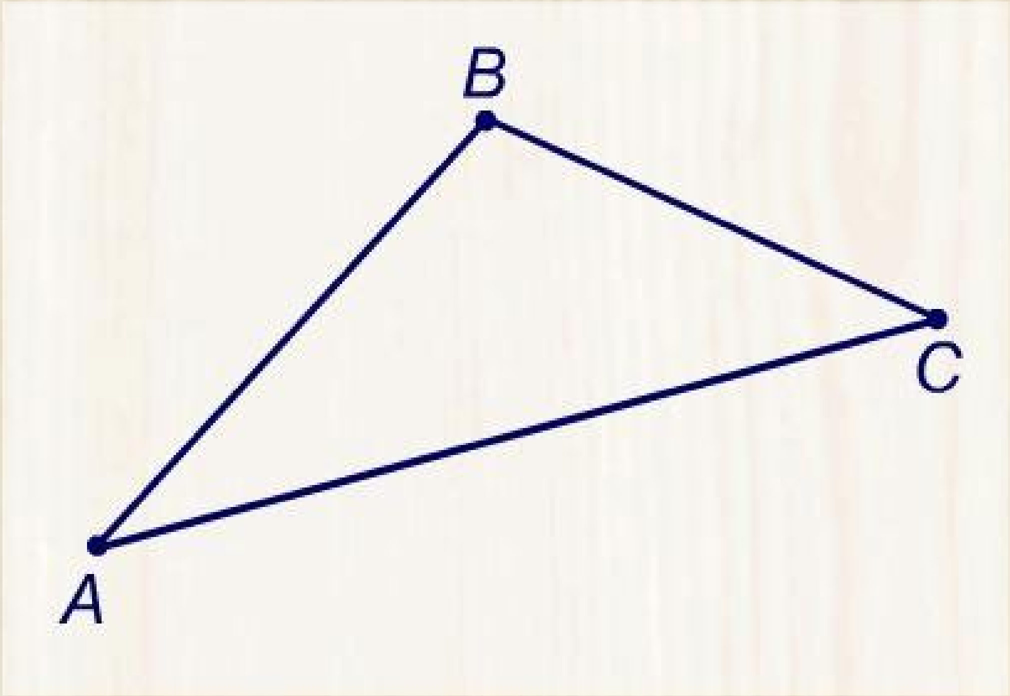


Constructing the centroid

Constructing the centroid of a triangle

The centroid of a triangle is the point where the three medians of the triangle meet.

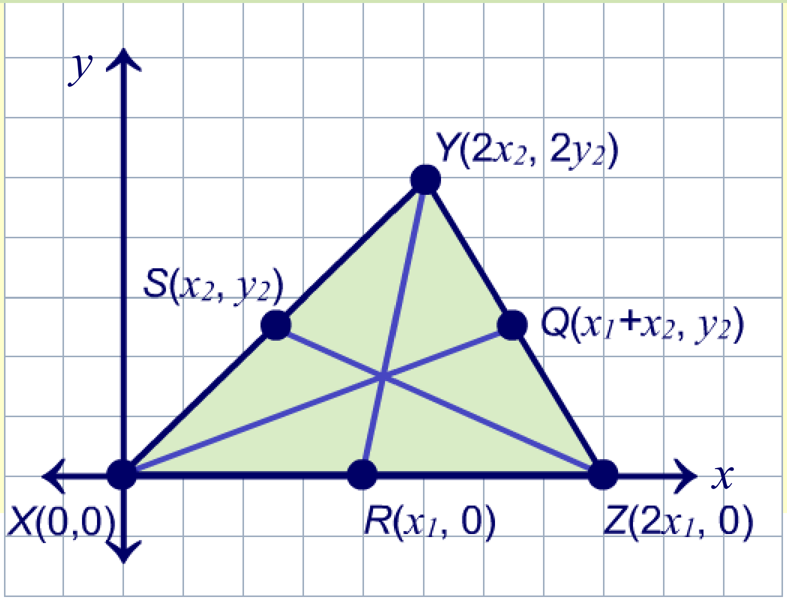
Press **play** to see how to construct the centroid.



The **median of a triangle** is a line segment that joins a vertex to the midpoint of the opposite side.

A triangle has three medians. The point where all three of these meet is called the **centroid**.

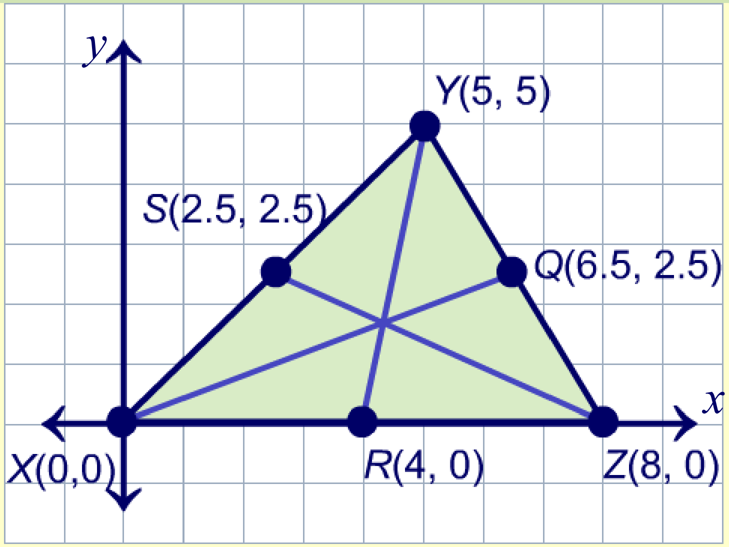
Plan a proof for showing that the three medians of a triangle are concurrent – meaning they meet at a point.



1. find the midpoints of each side
2. find the equations of the medians
3. show algebraically that the medians intersect at the same point.

Centroid example

Find the centroid of the triangle in the figure.



1. find the midpoints of each side:

Q: ((5+8)/2, (5+0)/2) = (6.5, 2.5)

R: ((0+8)/2, (0+0)/2) = (4, 0)

S: ((5+0)/2, (5+0)/2) = (2.5, 2.5)

2. find the equations of the medians:

using point–slope format:

$\overleftrightarrow{XQ}$: $(y–0) = (2.5/6.5)(x–0) \Rightarrow y = 0.38x$

$\overleftrightarrow{RY}$: $(y–0) = (5/1)(x–4) \Rightarrow y = 5x – 20$

$\overleftrightarrow{SZ}$: $(y–0) = (–2.5/5.5)(x–8) \Rightarrow y = –0.45x + 3.6$

3. find the point where two medians intersect:

equate two lines, e.g., $\overleftrightarrow{XQ}$ and $\overleftrightarrow{SZ}$: $0.38x = –0.45x + 3.6$

solve for x: $x = 4.34$

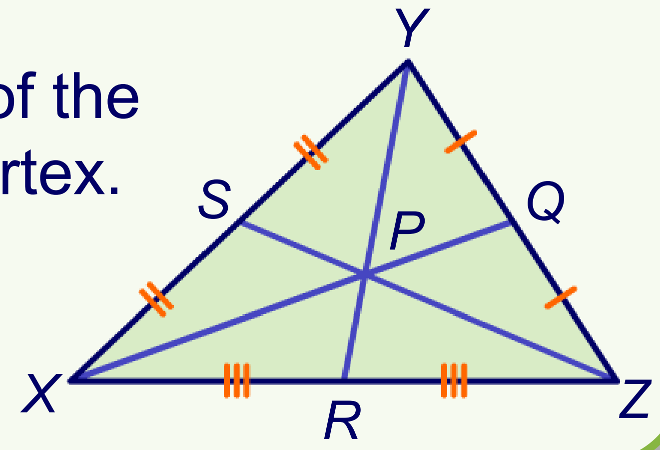
substitute x to find y: $y = 0.38x = 0.38(4.34) = 1.65$

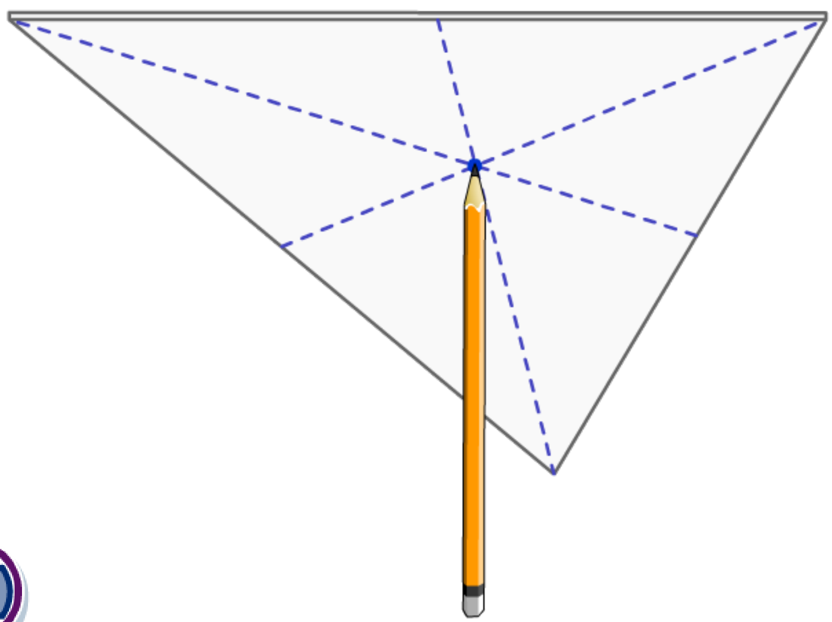
The point of intersection is (4.34, 1.65).

The centroid theorem

Centroid theorem:
The centroid of a triangle is two-thirds of the distance along the median from the vertex.

$XP = \tfrac{2}{3}XQ \quad YP = \tfrac{2}{3}YR \quad ZP = \tfrac{2}{3}ZS$





The **centroid** is the balancing point – or the center of mass – of the triangle.

Applying the centroid theorem

A triangle has a vertex at $X(0,1)$ and a midpoint of the opposite side at $Q(6,4)$. Find the centroid of the triangle using the centroid theorem.

The centroid theorem states that the centroid of a triangle is two-thirds of the distance along a median from the vertex.

First, find x-coordinate that is two-thirds of the way from the vertex.

find the total x-distance:	$6 – 0 = 6$
multiply by ⅔ :	$6 \times ⅔ = 4$
add to x-coordinate of vertex:	$0 + 4 = \mathbf{4}$

Similarly, find y-coordinate that is two-thirds of the way from the vertex.

find the total y-distance:	$4 – 1 = 3$
multiply by ⅔ :	$3 \times ⅔ = 2$
add to y-coordinate of vertex:	$1 + 2 = \mathbf{3}$

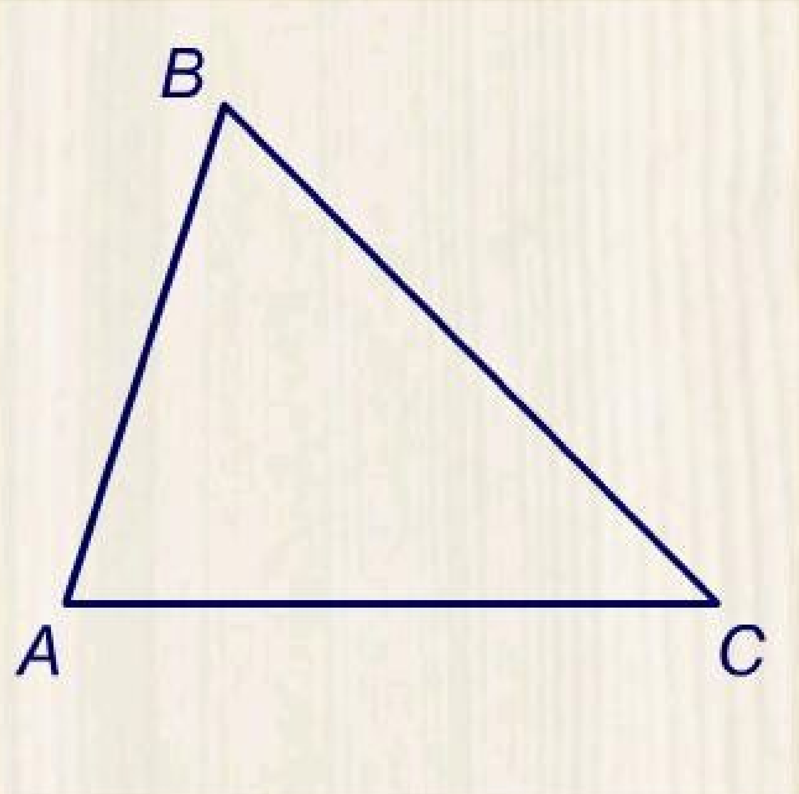
The centroid is at (4, 3).

Constructing the orthocenter of a triangle

The orthocenter of a triangle is the point where the altitudes meet.

First, construct an acute triangle on a piece of tracing paper.

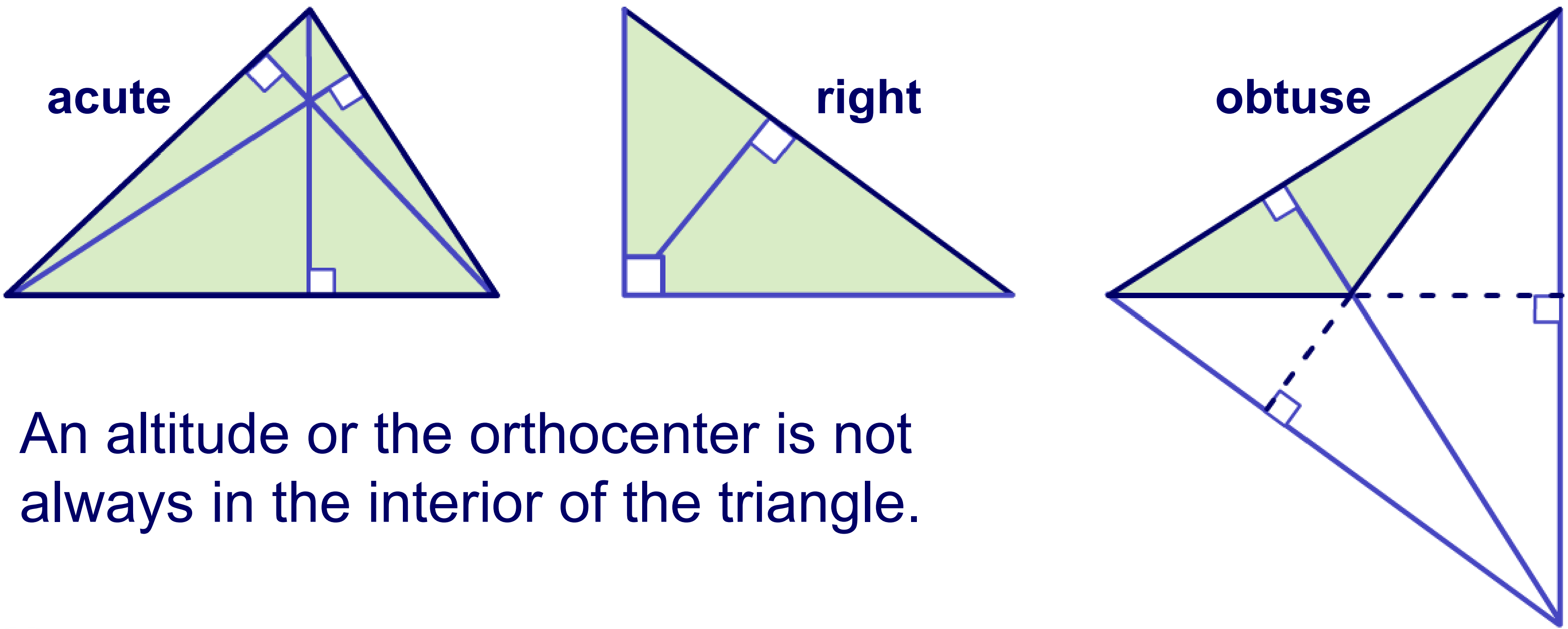
Press **play** to begin the construction.



Altitudes of a triangle

The **altitude of a triangle** is a perpendicular segment that connects a side to the opposite vertex.

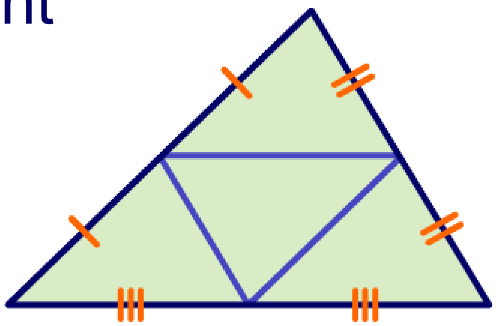
A triangle has three altitudes. The point where all three of these meet is called the **orthocenter**.



An altitude or the orthocenter is not always in the interior of the triangle.

Midsegments of a triangle

The **midsegment of a triangle** is the segment that connects the midpoints of two sides.



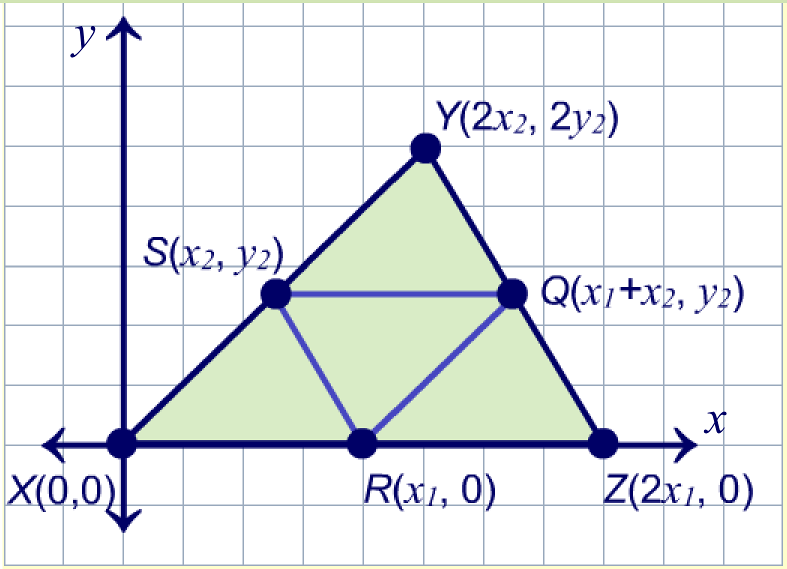
The three midsegments form the midsegment triangle.

***Triangle midsegment theorem*:**
The midsegment of a triangle is parallel to a side of the triangle and half the length of that side.

Describe the midsegment triangle of an equilateral triangle with side lengths 5 cm.

The midsegment triangle would also be an equilateral triangle with side lengths half the length of the triangle sides: 2.5 cm.

Prove the triangle midsegment theorem using the figure.



First, prove that the each midsegment is parallel to a side by finding slopes.

find the midpoints of each side:

(x_2, y_2) $(x_1, 0)$ $(x_1 + x_2, y_2)$

find the slopes of each side:

0 1 $y_2 \div (x_1 - x_2)$

find the slopes of the midsegments:

0 1 $y_2 \div (x_1 - x_2)$

Each midsegment slope matches a side slope, so they are parallel. ✓

Then prove that the midsegment is half the length of its corresponding parallel side using the distance formula.

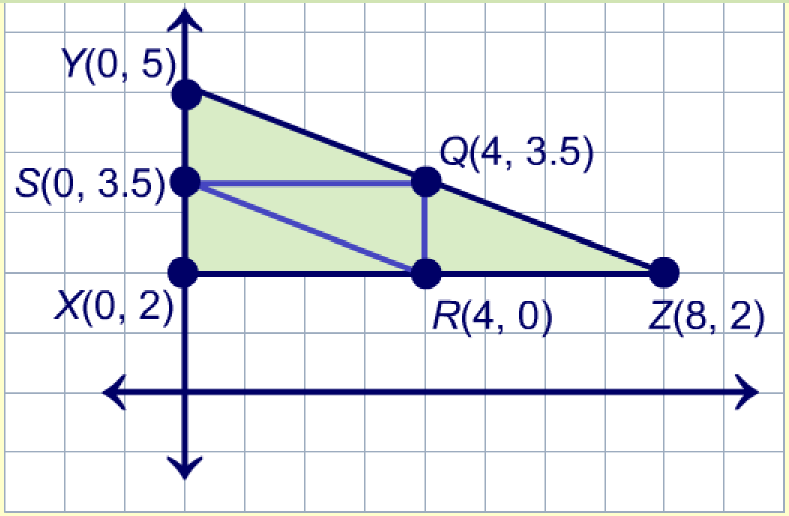
find the lengths of the sides: $2x_1$ $2\sqrt{(x_2-x_1)^2+y_2^2}$ $2\sqrt{x_2^2+y_2^2}$

find the lengths of the midsegments: x_1 $\sqrt{(x_2-x_1)^2+y_2^2}$ $\sqrt{x_2^2+y_2^2}$

Each midsegment is half the length of its corresponding side. ✓

Triangle midsegment theorem

For the triangle shown, find the area of its midsegment triangle. What fraction of the triangle's area is the midsegment triangle's area?



find base and height of original triangle:

XZ = 8–0 = 8 XY = 5–2 = 3

find base and height of midsegment triangle using the midsegment theorem:

QR = ½XY = ½×3 = 1.5

QS = ½XZ = ½×8 = 4

find the area of the midseg. triangle:

A_{ms} = ½ × *base* × *height* = ½ × 4 × 1.5 = 3

find the area of the original triangle:

A = ½ × *base* × *height* = ½ × 8 × 3 = 12

The midsegment triangle has 3 ÷ 12 = ¼ the area of the large triangle.