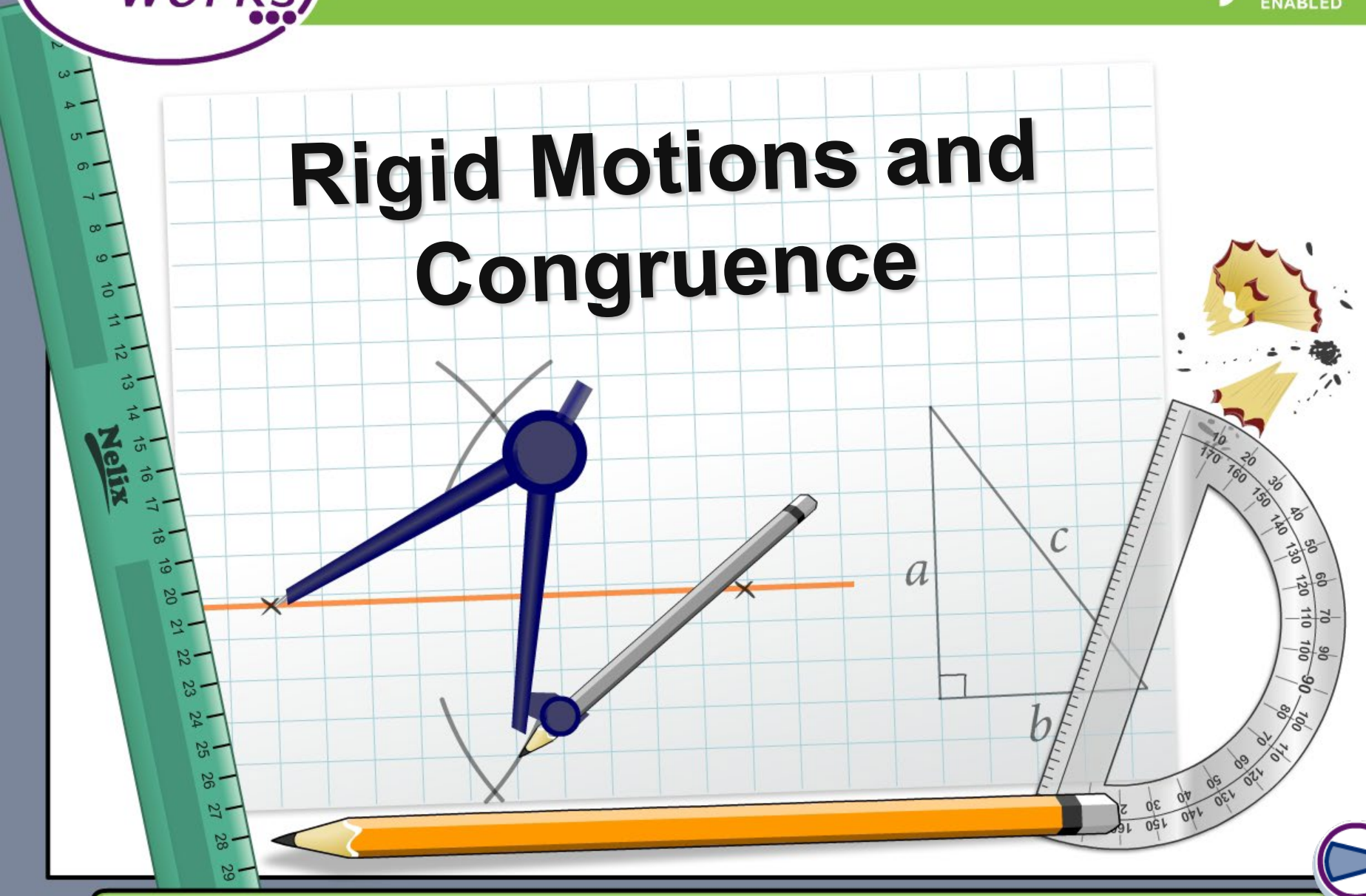
Rigid Motions and Congruence



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

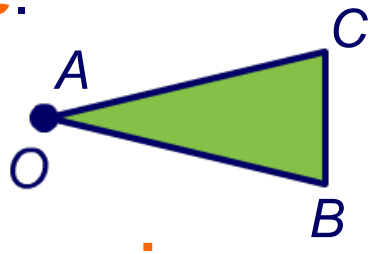
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

What is a transformation?

In geometry, a **transformation** is a mathematical operation performed on a figure that changes its position, size *OR* shape.

The figure before the transformation is called the **object** or **pre-image**. After a transformation is performed, the resulting figure is called the **image**.



pre-image

image

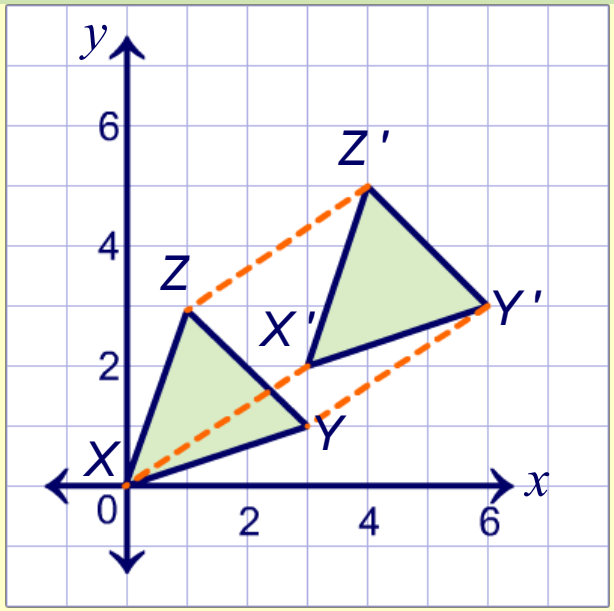
A transformation takes points in the pre-image as inputs and maps them to other points in the image as outputs.

Transformations are represented using a capital letter and an arrow, for example T: $\triangle ABC \rightarrow \triangle A'B'C'$

Translation

A **translation** moves every point in a figure over the same distance and in the same direction.

A triangle has vertices $X(0,0)$, $Y(3,1)$ and $Z(1,3)$.
Its image has vertices $X'(3,2)$, $Y'(6,3)$ and $Z'(4,5)$.
Draw the pre-image and image and specify the translation.



Every point in the shape moves the same distance in the same direction and the side lengths remain the same.

From the figures, the triangle has been moved 3 in the x-direction and 2 in the y-direction, but its shape has not changed.

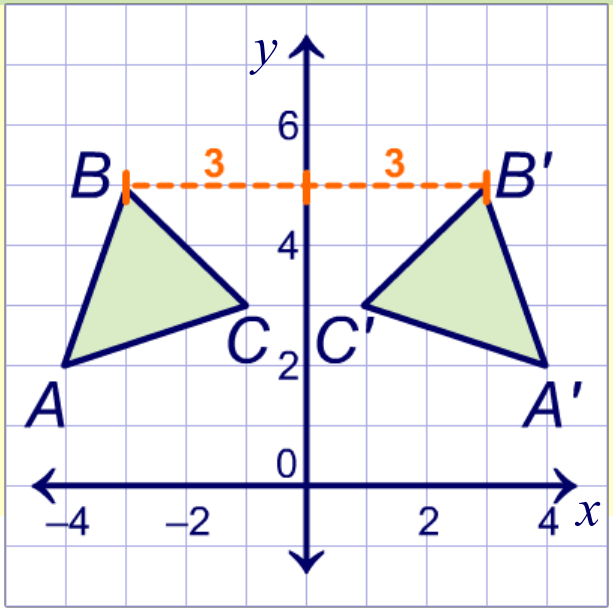
$T: (x, y) \longrightarrow (x + 3, y + 2)$ or $T_{(3,2)}$

Reflection

An object can be **reflected** across a **line of reflection** to produce an image.

Each point in the image is the same distance from the line of reflection as the corresponding point of the pre-image.

$\triangle ABC$ is reflected across the y-axis. What are the coordinates of B'? Write the transformation.



B is at (–3, 5). It is 3 units from the y-axis.

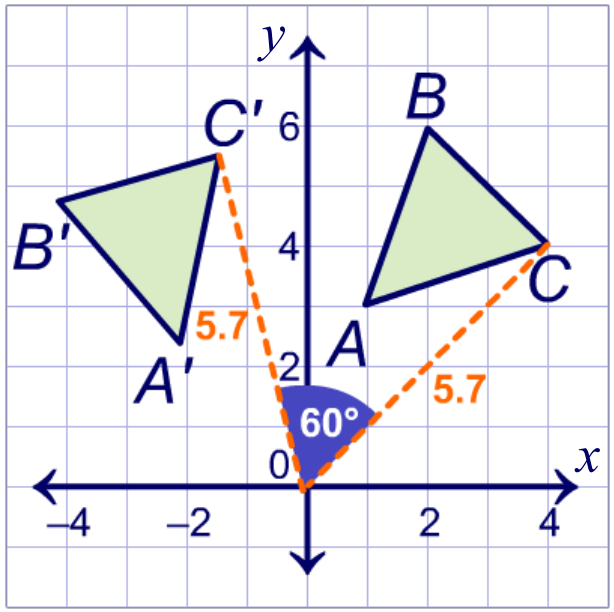
Because the image is a reflection, B' is the same distance from the y-axis as B, which is 3 units. Therefore B' is at (3,5).

r: $(x, y) \longrightarrow (-x, y)$ or $r_{y\text{-axis}}$

Rotation

A rotation moves a figure around the **center of rotation** by a given angle.

A positive angle rotates the figure in the counter-clockwise direction, and a negative angle rotates it clockwise.



$R_{60°}$: $\triangle ABC$ is rotated 60° about the origin.

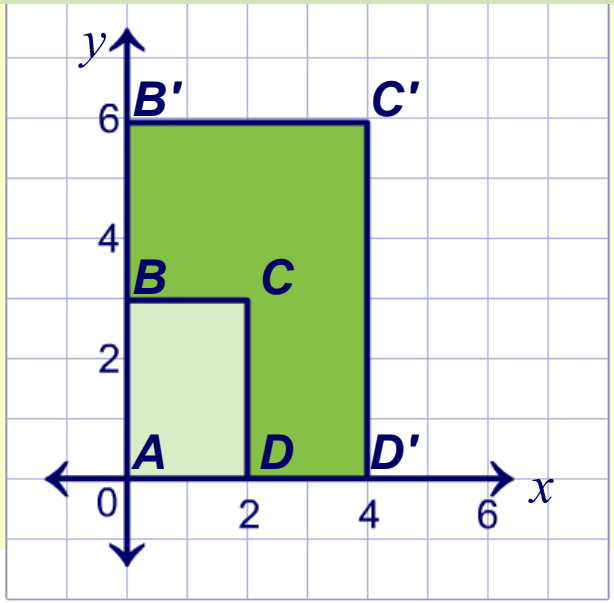
How does the distance from C to the origin compare to the distance from C' to the origin?

The distance between a point on the object and the center of rotation is the same as the distance between the corresponding point on the image and the center.

Dilation

A **dilation** of an object changes the size of the figure, but not its shape.

The dilation D: $(x, y) \rightarrow (2x, 2y)$ (or D_2) is applied to the rectangle shown. Draw the image of the dilation. What is the relationship between length of corresponding sides and angles?



The length of a side on the image is twice the length of the corresponding side of the pre-image.

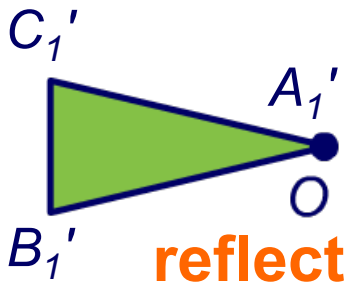
The factor 2 is called the **scale factor**.

Corresponding angles are congruent.

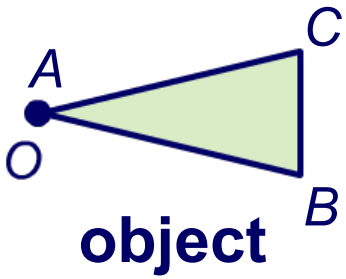
Types of transformation

Can you remember the names of these transformations?
What is different about dilation?

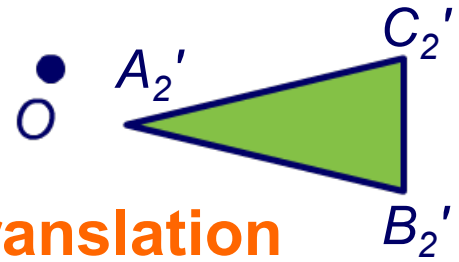
The dilated image is not congruent to the object.
This is because the length of the sides change in the transformation.



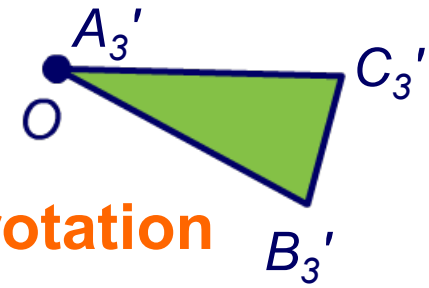
reflection



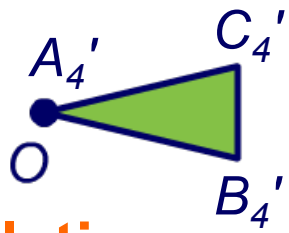
object



translation



rotation



dilation

Rigid motions

Rigid motions are tranformations that
do not change the size or shape of a figure.
Sort the transformations as rigid or non-rigid motions.

rigid motion	non-rigid motion

translation

reflection

dilation

rotation

S C

Rigid motions and congruence

When a **rigid motion** is applied to an object:

- corresponding sides have the same length
- corresponding angles have the same measure.

What can you say about the image of an object that has been translated, rotated and/or reflected?

Remember that if figures are identical in shape and size, they are **congruent**. Applying rigid transformations to a figure does not change the shape or size, so the image and object are congruent.

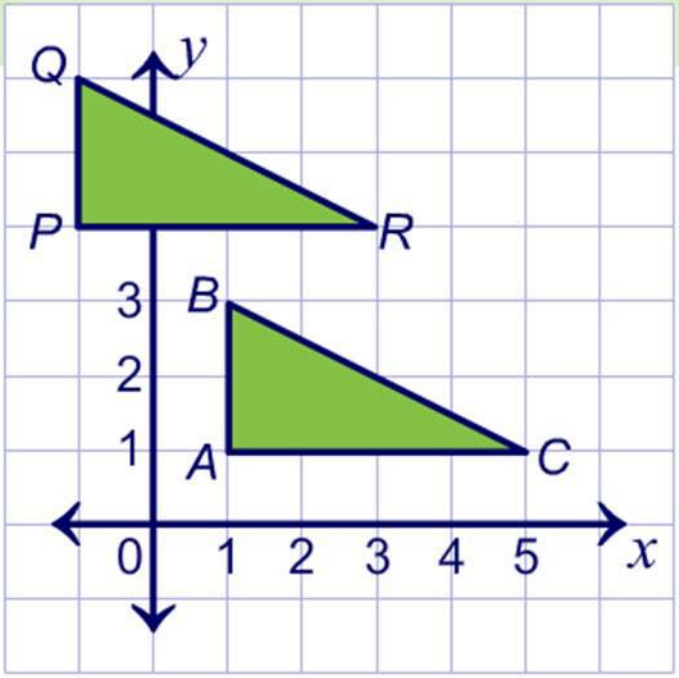
If one object can be mapped to another by rigid motions, the objects are congruent.

Using congruence

Are these figures congruent?

Question: 1/5

Is $\triangle ABC \cong \triangle PQR$? Why or why not?



yes

no

Blueprint for a Ferris wheel

Question: 1/5

How can you describe the relationship between two of the basic triangles used to make up the wheel geometrically?

- rotation
- dilation
- translation
- reflection