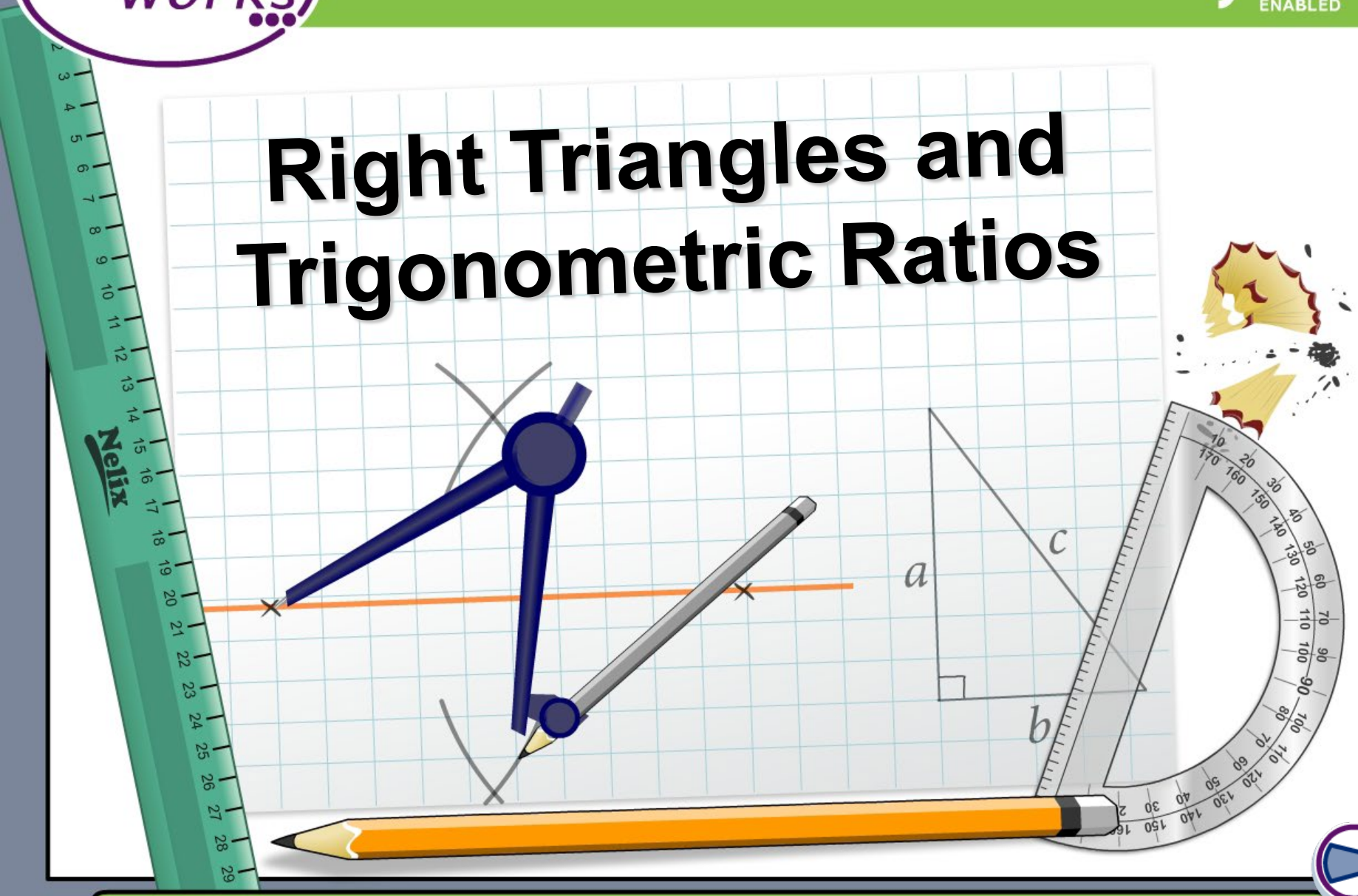
Right Triangles and Trigonometric Ratios



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

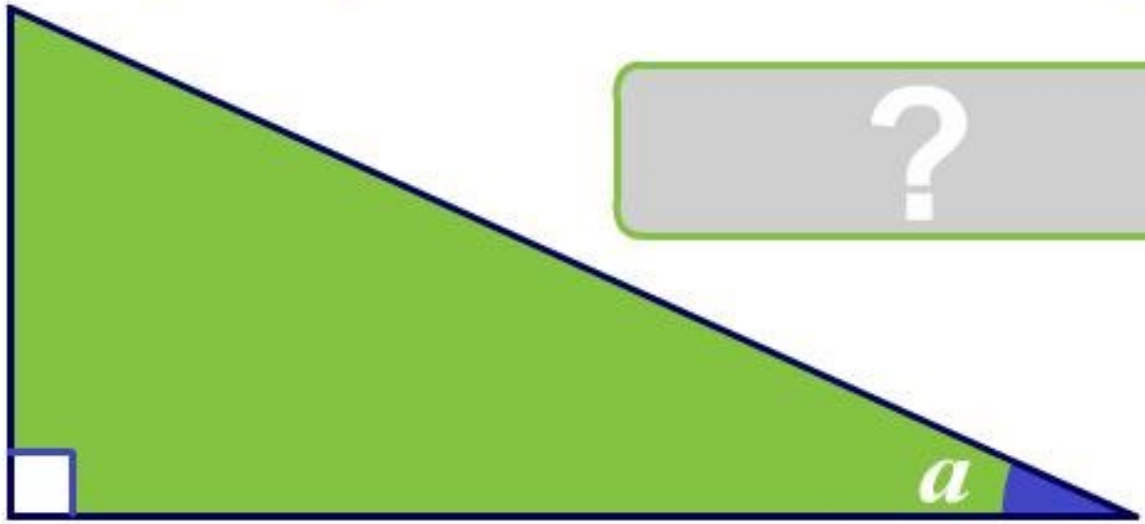
8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

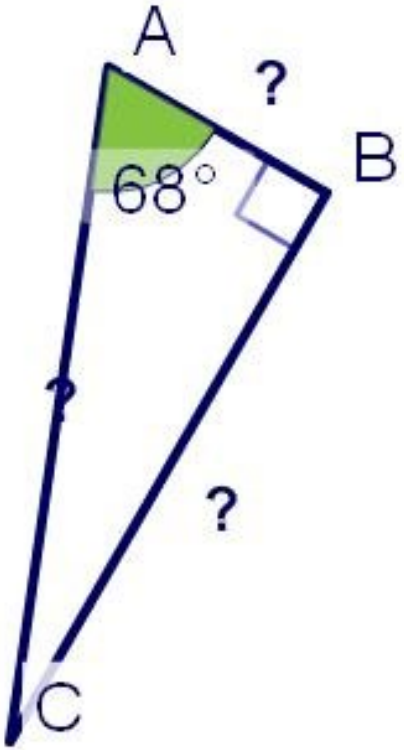
Right triangles

The legs of a right triangle are labeled in relation to the position of the right angle and another marked angle.



Label the sides

Drag the labels onto the sides of the right triangle.



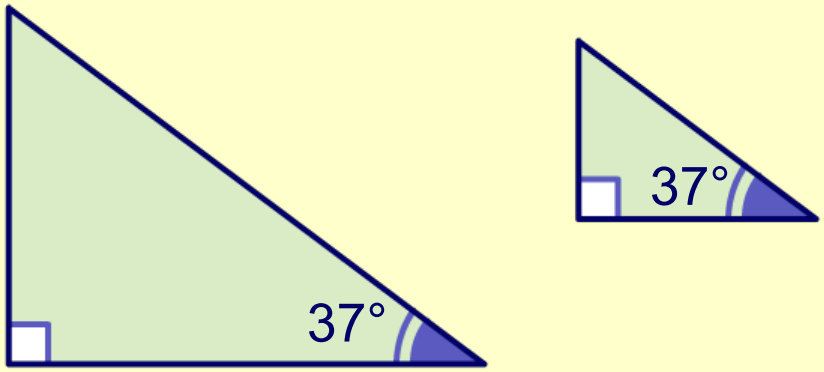
hypotenuse

opposite

adjacent

Similar right triangles

If two right triangles have an acute angle of the same size, what does this show about the triangles?



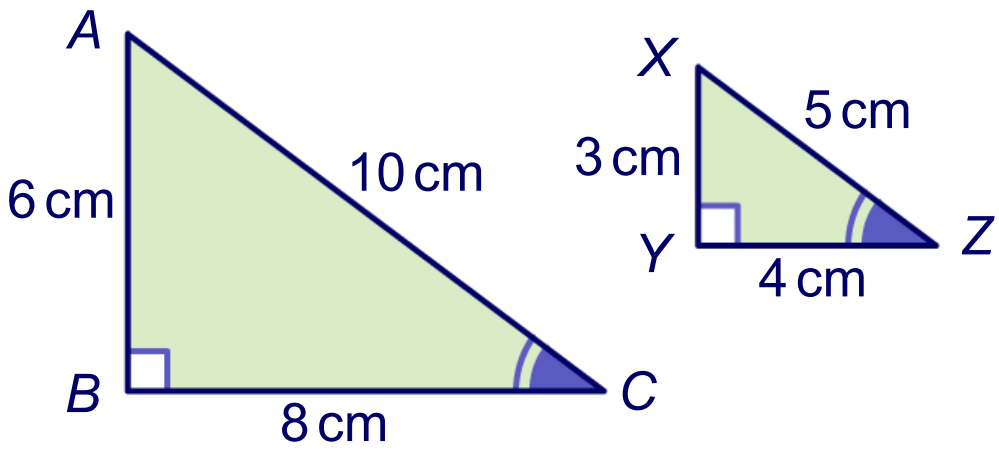
Remember that the ***AA similarity postulate*** states that if a triangle has two angles that are congruent to two angles in another triangle, then the triangles are similar.

These triangles both have a right angle and the same acute angle, so the ***AA similarity postulate*** applies. The two triangles are similar, meaning that one triangle is an enlargement of the other.

All right triangles with an acute angle of the same size are **similar**.

Leg length ratios

Calculate the ratios of the different legs within these similar triangles. What do you notice?



$$\frac{AB}{BC} = \frac{6}{8} = \frac{3}{4} = \frac{XY}{YZ}$$

$$\frac{AB}{AC} = \frac{6}{10} = \frac{3}{5} = \frac{XY}{XZ}$$

$$\frac{BC}{AC} = \frac{8}{10} = \frac{4}{5} = \frac{YZ}{XZ}$$

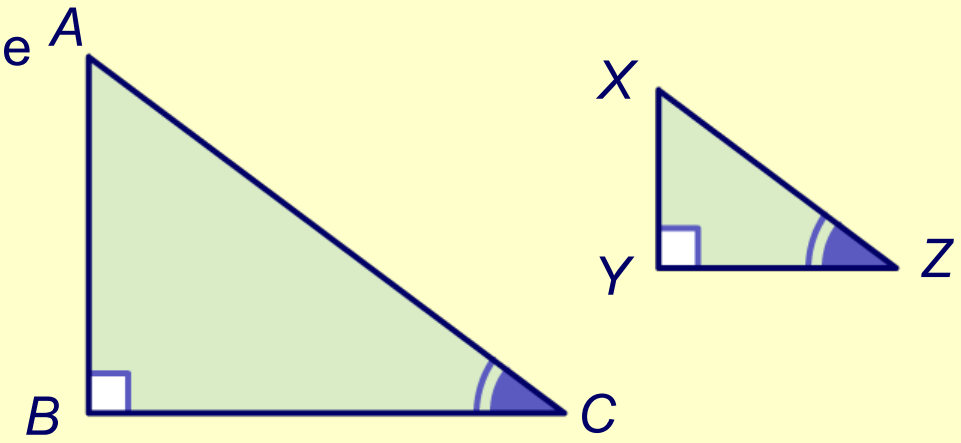
The ratios between different legs of the triangle are the same in both triangles. What do you think you would find if you calculated the leg ratios for another triangle similar to these?

Leg length ratios

Prove that the leg length ratios of a right triangle are the same for any similar triangles, and therefore depend only on the size of the angles.

If two right triangles have the same size acute angle, they are similar and the ratios of corresponding legs are all the same.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$



These can be rearranged to give ratios of legs in one triangle:

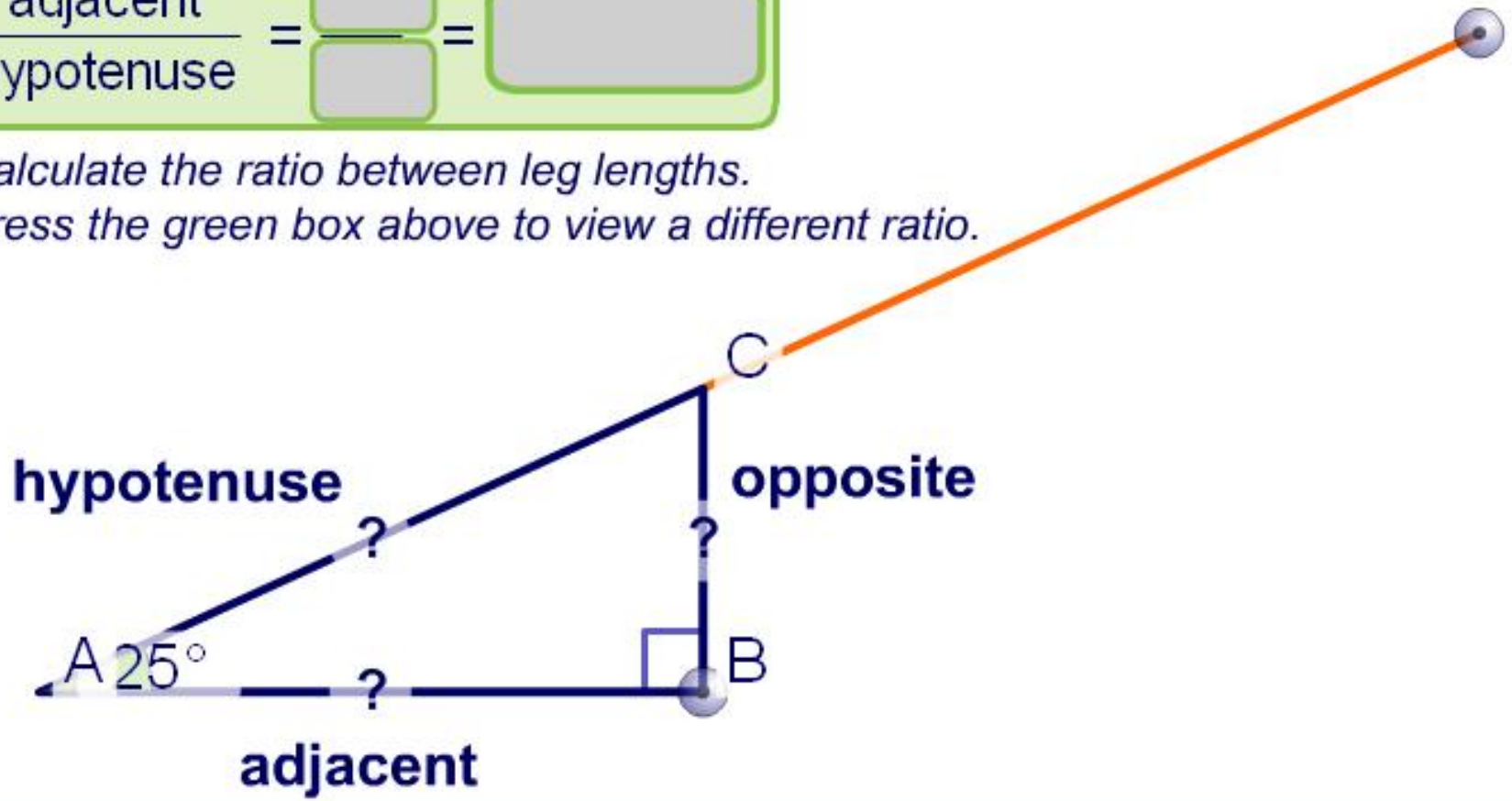
$$\frac{XY}{YZ} = \frac{AB}{BC} \qquad \frac{XY}{XZ} = \frac{AB}{AC} \qquad \frac{YZ}{XZ} = \frac{BC}{AC}$$

These ratios are called **trigonometric ratios**.

Similar right triangles

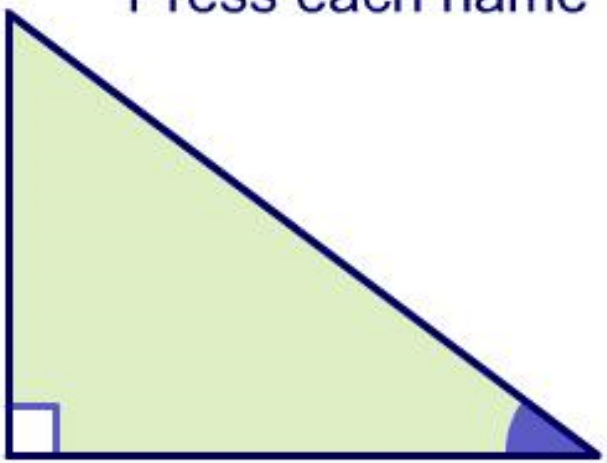
$$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\square}{\square} = \square$$

Calculate the ratio between leg lengths.
Press the green box above to view a different ratio.



Trigonometric ratios

The ratio between the lengths of legs in right triangles depend only on the angles in the triangle. These ratios have special names. Press each name to find out more.



sine

cosine

tangent

The ratio of an angle

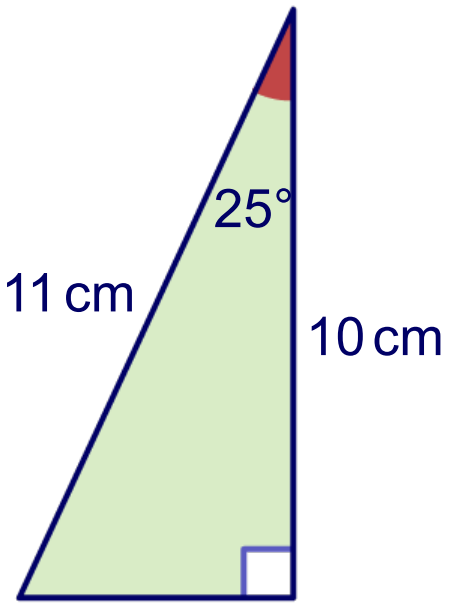
What is the value of cos 25°?

This is the same as asking:

In a right triangle with an angle of 25°, what is the ratio of the adjacent leg to the hypotenuse?

It doesn't matter how big the triangle is because all right triangles with an angle of 25° are similar.

The length of the adjacent leg divided by the length of the hypotenuse will always be the same value, as long as the angle is the same and we have measured accurately.

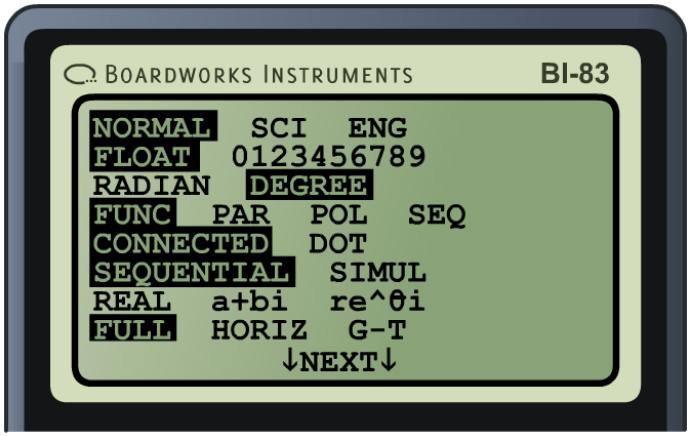


From this triangle, $\cos 25° = \frac{10}{11} \approx 0.91$

Using a calculator

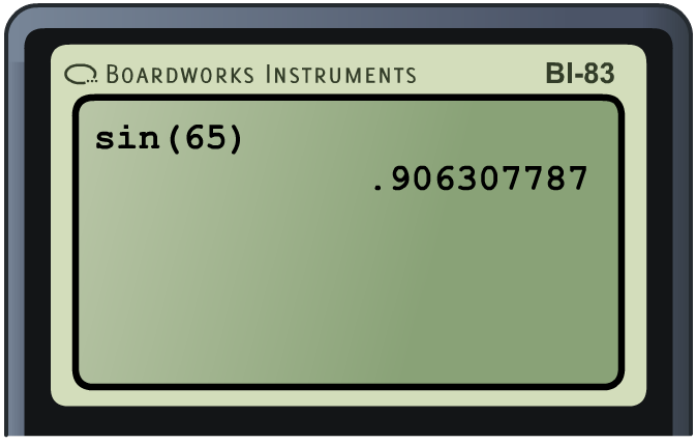
Use a calculator to find the trigonometric ratio for an angle when the leg lengths are unknown.

First make sure that the calculator is set to work in degrees. Press MODE, and use the cursor to select DEGREE.



To evaluate the sine of 65°, press the keys:

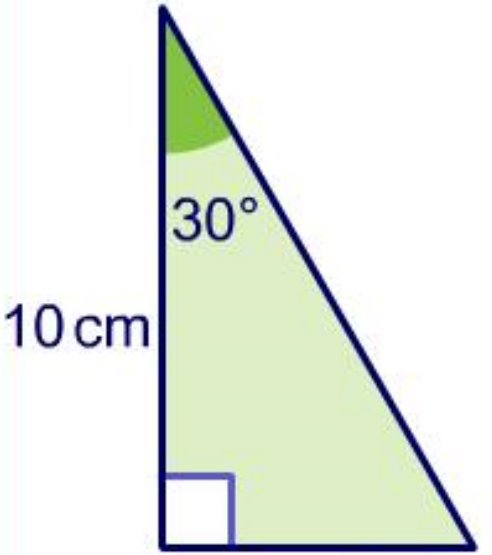
SIN 6 5) ENTER

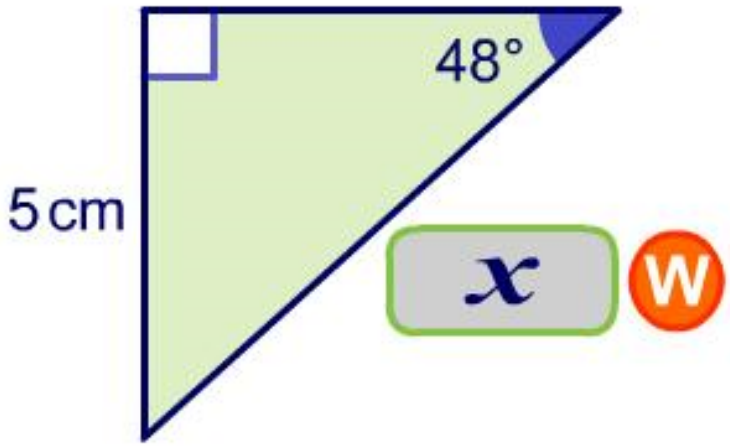


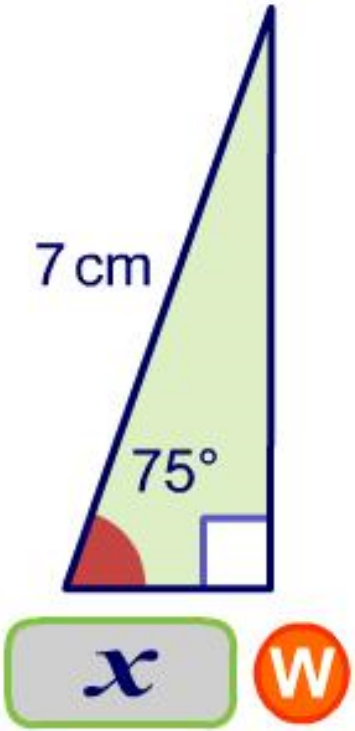
Some calculators require the angle first, or do not need parentheses around the degree. Check what works on your calculator.

Finding leg length

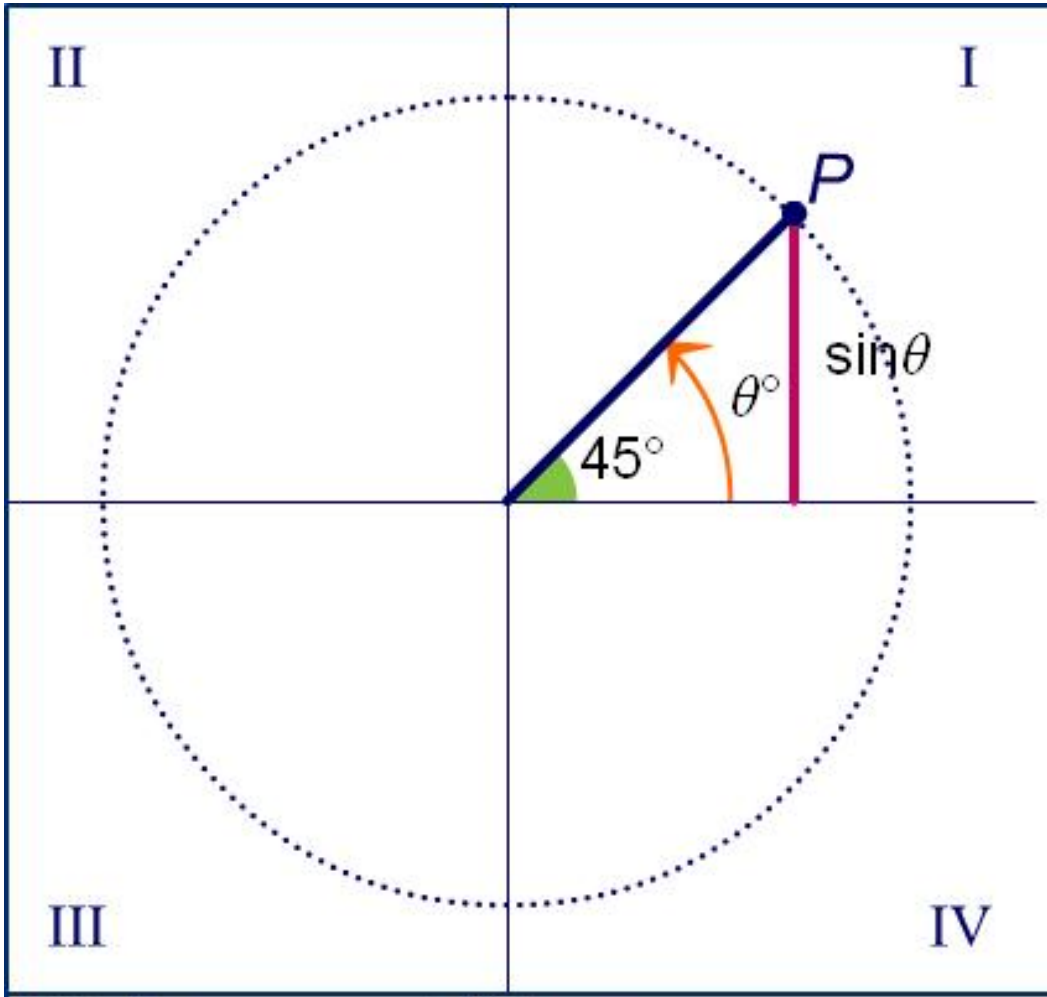
Find the length marked x to the nearest tenth for each of the triangles.
Press the x to reveal its value, or press **W** to see how to find the solution.







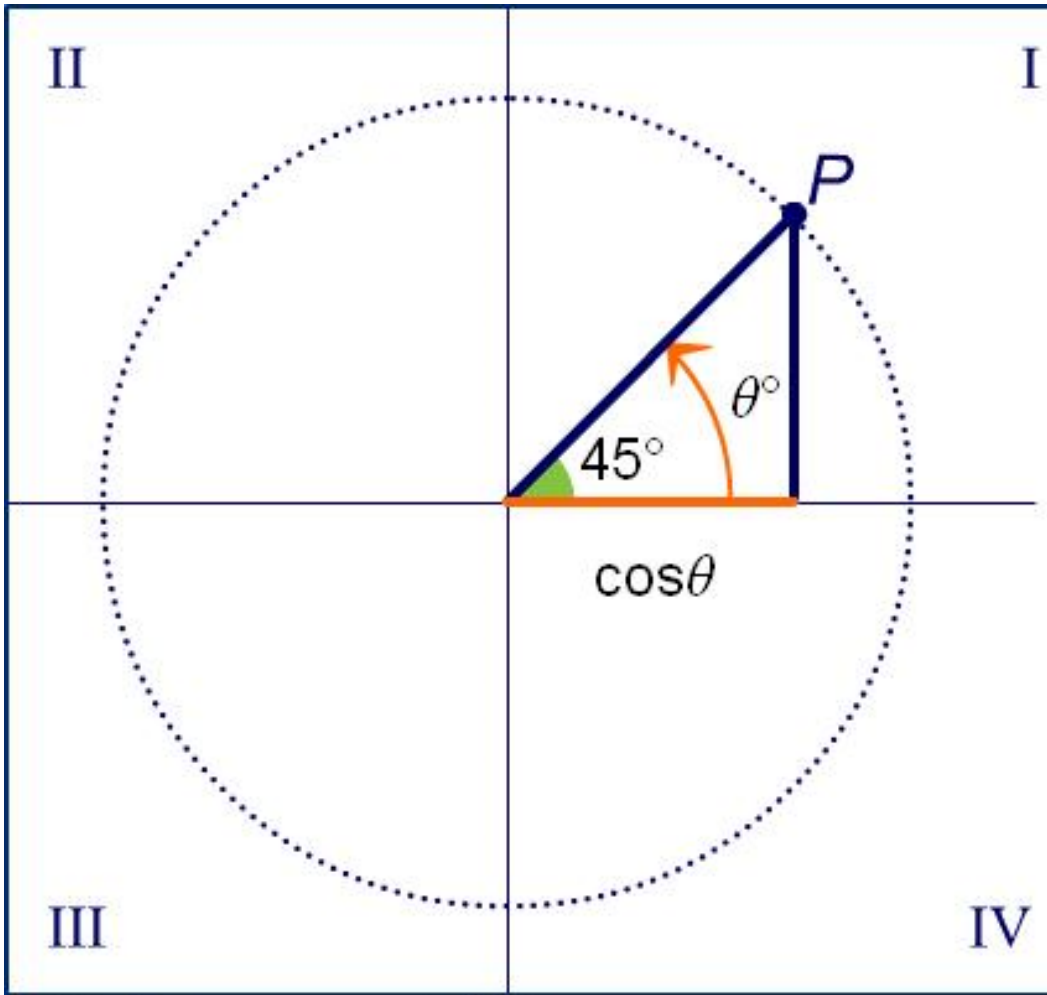
Investigating sine



$\sin\theta°$ is given by the y-coordinate of the point, P.

$\sin 45° = 0.71$

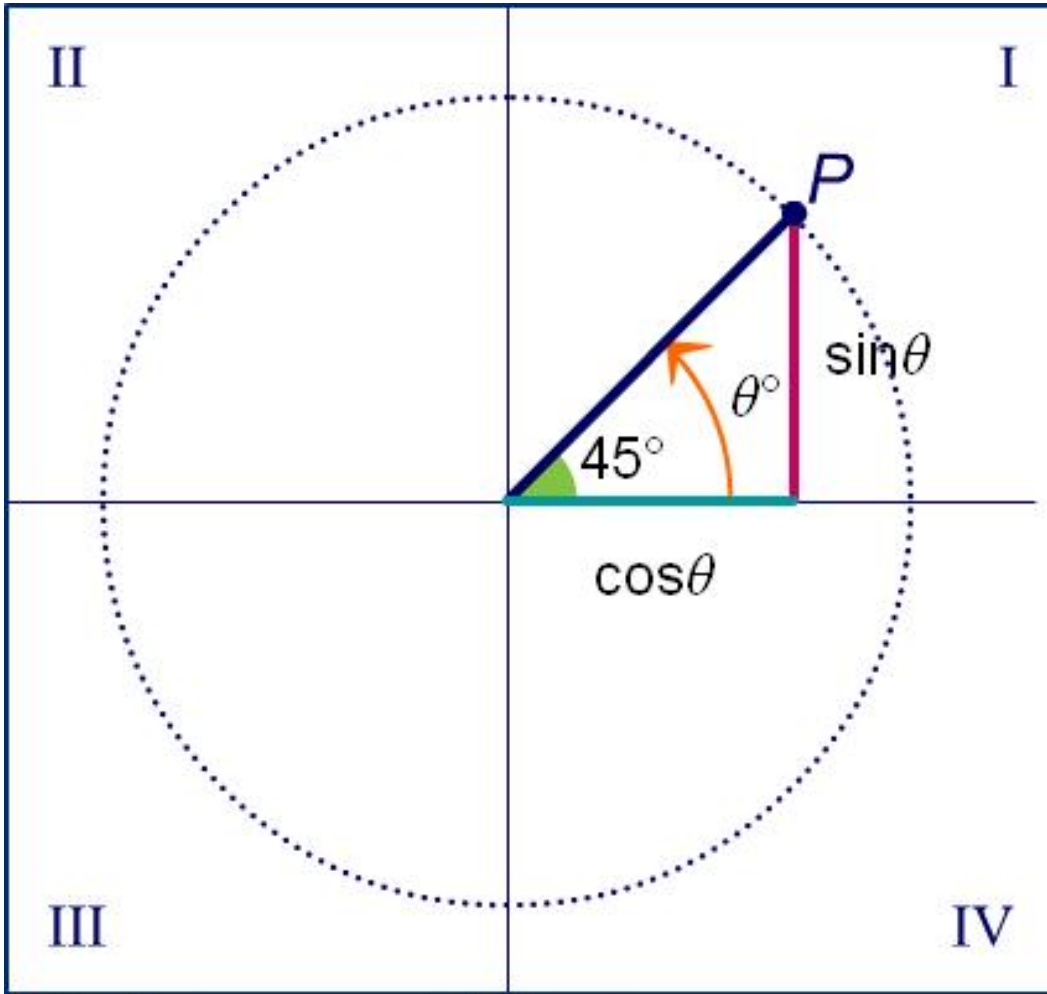
Investigating cosine



cos θ° is given by the x-coordinate of the point, P.

$\cos 45^\circ = 0.70$

Investigating tangent 1

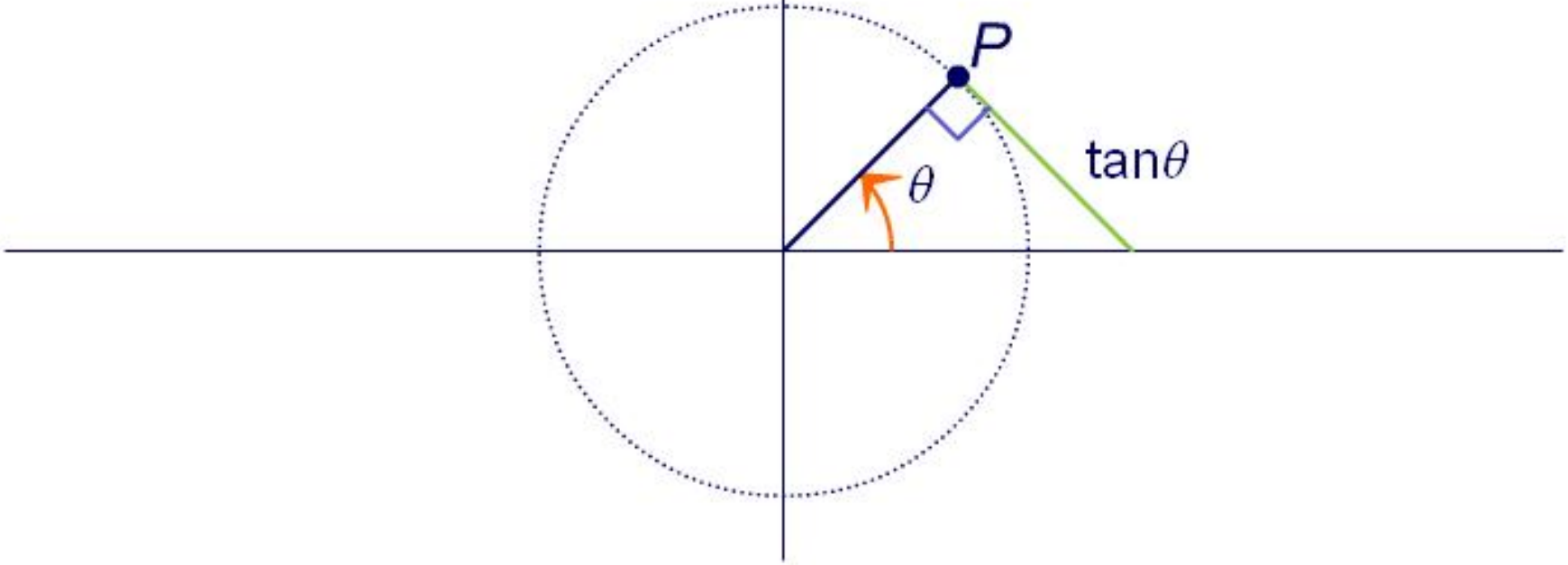


tan θ° is given by the y-coordinate of the point P divided by the x-coordinate.

$$\tan 45^\circ = \frac{0.71}{0.71} = 1$$

Investigating tangent 2

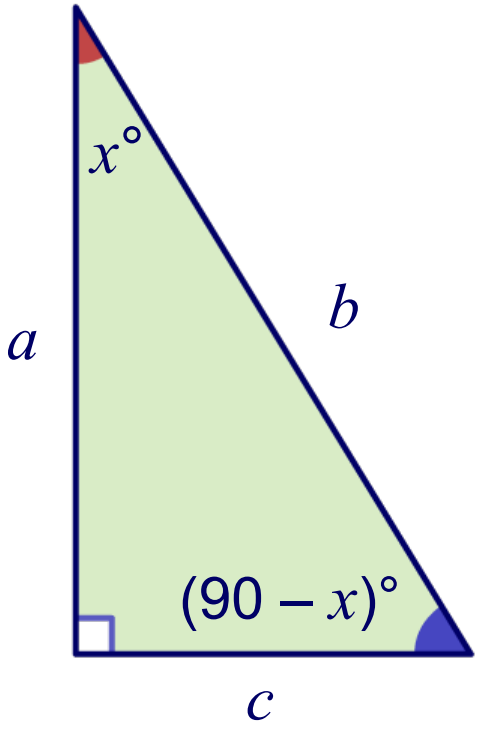
The value of tan θ is given by the length of the tangent from P to the x-axis. Drag the point P around the circle to investigate.



tan45° = 1.00

Relationship between sine and cosine

The acute angles of a right triangle are **complementary**, they sum to 90°.



This means that $\sin x° = \frac{c}{b}$

and $\cos (90 - x)° = \frac{c}{b}$

Therefore $\sin x° = \cos (90 - x)°$

and $\sin (90 - x)° = \cos x°$

The sine of a given angle is equal to the cosine of the complementary angle.

Match the equivalents

Match the equivalents by selecting one of each

sin 79°	0.982
tan 65°	sin 62°
cos 28°	2.14
tan 49°	cos 56°
cos 43°	sin 47°
sin 34°	0.105
tan 6°	1.15

Radio tower problems

A radio tower is supported by cables to keep it upright, at right angles to the ground. The cables form a 60° angle with the ground.

1) Find the trigonometric ratio involving the length of the cable and the height at which it is attached to the tower. ? W

2) What is the length of cable required if it attaches to the tower 20 m above the ground? ? W

3) Find the trigonometric ratio involving the height up the tower and the distance along the ground to where the cable attaches at each end. ? W

4) How far away does the cable attach? ? W