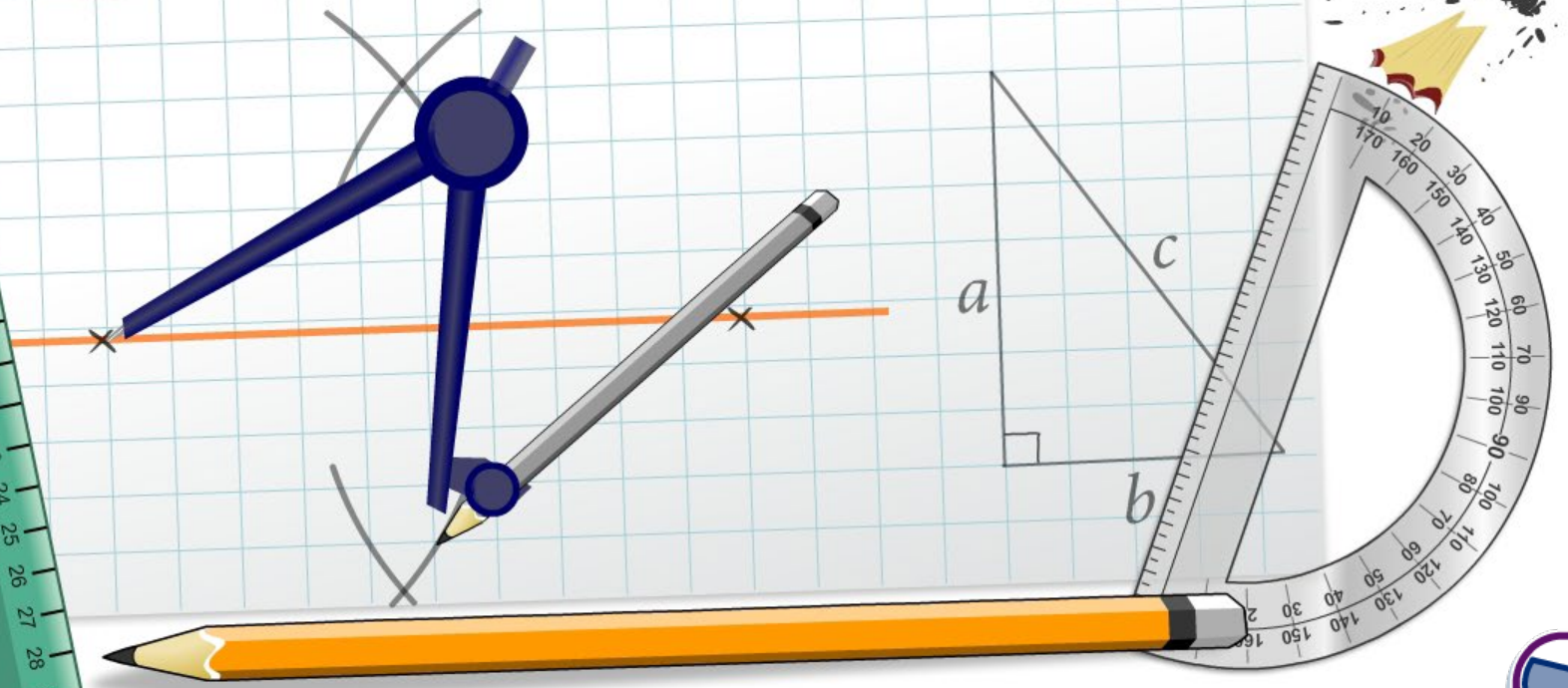
Proving the Pythagorean Theorem



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Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

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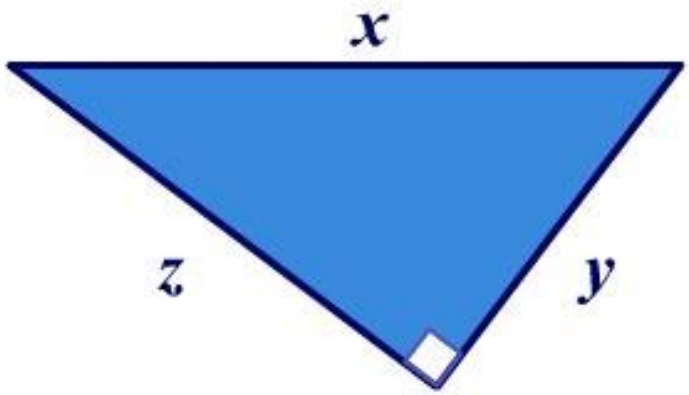
Identify the hypotenuse

Press the letter that matches the hypotenuse.



x y z

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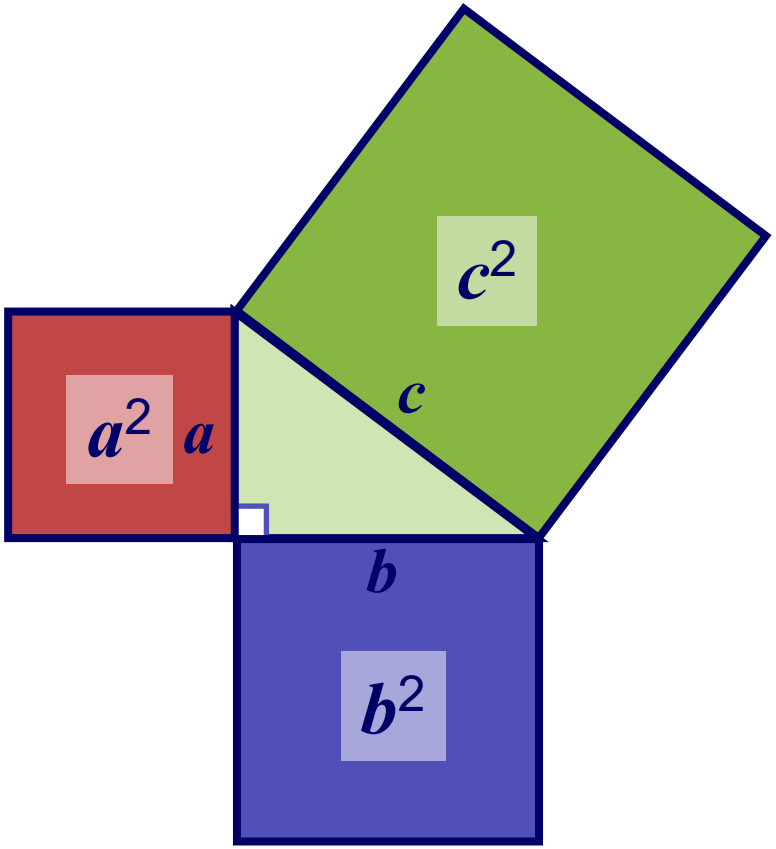
The Pythagorean theorem

The Pythagorean theorem:
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the square of the lengths of the legs.



The area of the largest square is $c \times c$ or c^2.

The areas of the smaller squares are a^2 and b^2.

The Pythagorean theorem can be written as:

$$c^2 = a^2 + b^2$$

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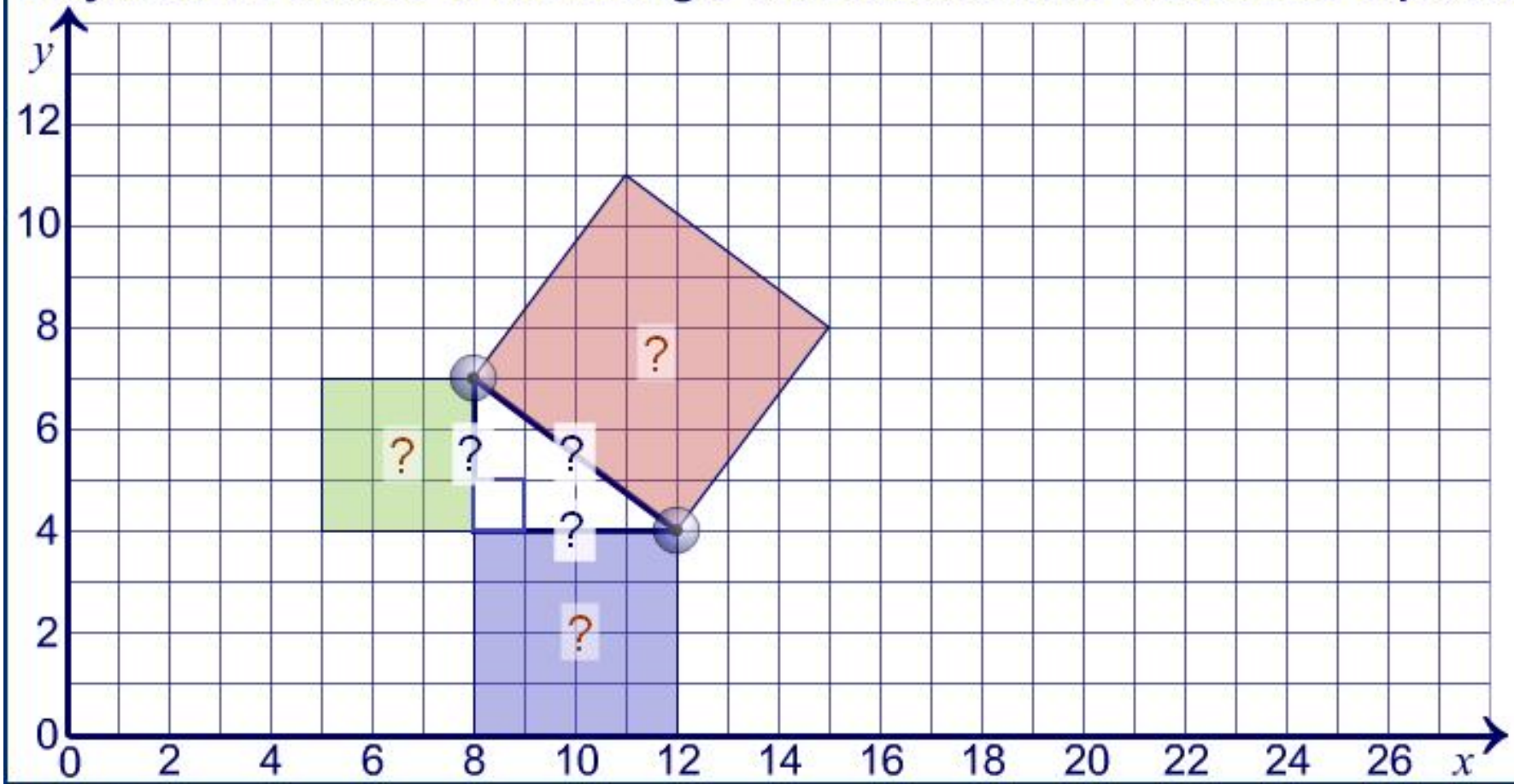
Showing the Pythagorean theorem

Adjust the vertices of the triangle and calculate the area of the squares.



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Right triangles

A **right triangle** contains a right angle.

The sides adjacent to the right angle are called **legs**.

Press **start** to begin.

start

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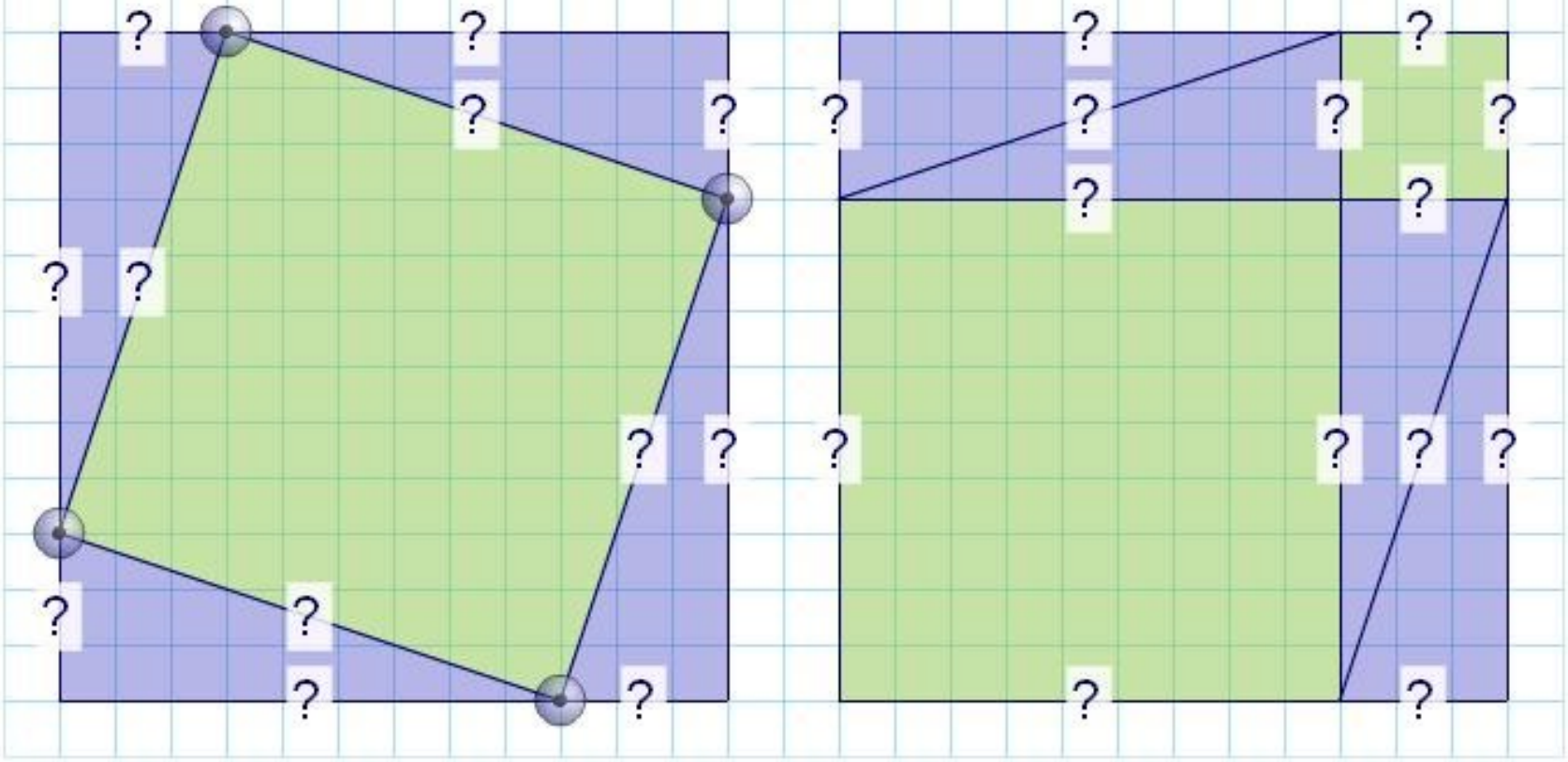
A proof of the Pythagorean theorem

Adjust the points and see how the area of the square with side length c is equal to the sum of the areas of the squares with side lengths a and b.



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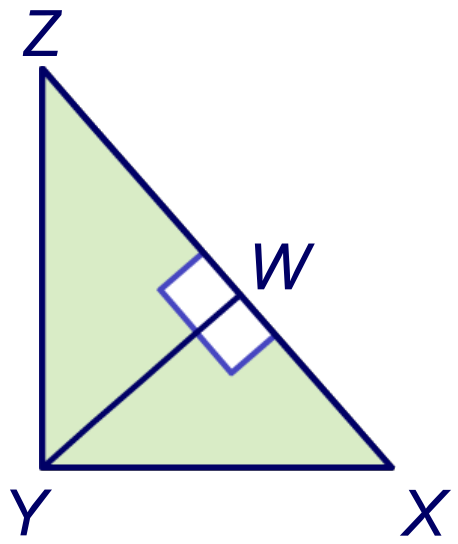
An altitude of a triangle

What is an altitude of a triangle?



An **altitude** of a triangle is a perpendicular line segment from a side to the opposite vertex.

All triangles have three altitudes. This figure shows one altitude of a right triangle.

The Pythagorean theorem can be proved using altitudes and similar triangles.

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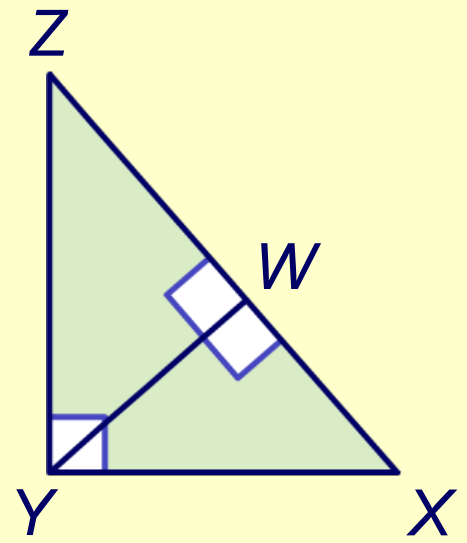
Proof using similarity (1)

Show that the three triangles in this figure are similar.

The altitude to the hypotenuse of a right triangle creates three right triangles: $\triangle XYZ$, $\triangle WXY$ and $\triangle WYZ$.

by definition:	$\angle XYZ$ and $\angle XWY$ are both right angles
by the reflexive property:	$\angle ZXY \cong \angle WXY$
by the AA similarity postulate:	$\triangle XYZ \sim \triangle WXY$ ✓
by definition:	$\angle XYZ$ and $\angle YWZ$ are both right angles
by the reflexive property:	$\angle XZY \cong \angle WZY$
by the AA similarity postulate:	$\triangle XYZ \sim \triangle WYZ$ ✓
by the transitive property of congruence:	$\triangle WXY \sim \triangle WYZ$ ✓



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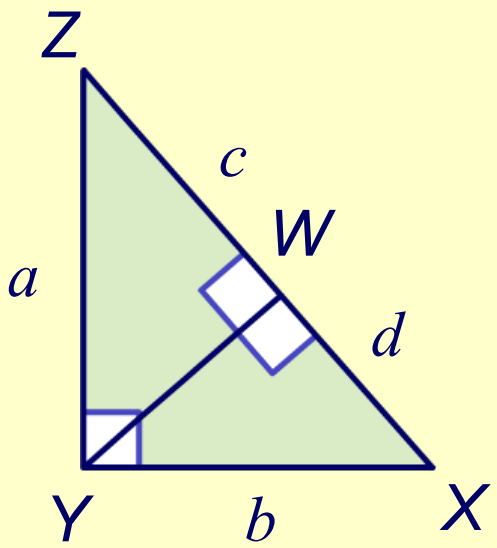
Proof using similarity (2)

Show that $a^2 + b^2 = (c + d)^2$ using similar triangles.

$\triangle XYZ \sim \triangle YWZ \sim \triangle XWY$



$\triangle XYZ$ is similar to $\triangle XWY$:

$$\frac{b}{c+d} = \frac{d}{b} \qquad b^2 = d(c+d)$$

$\triangle XYZ$ is similar to $\triangle YWZ$:

$$\frac{a}{c+d} = \frac{c}{a} \qquad a^2 = c(c+d)$$

adding the two equations:

$$\begin{aligned} a^2 + b^2 &= c(c+d) + d(c+d) \\ &= c^2 + cd + dc + d^2 \\ &= c^2 + 2cd + d^2 \\ &= (c+d)^2 \checkmark \end{aligned}$$

$(c + d)$ is the hypotenuse of $\triangle XYZ$, which proves the Pythagorean theorem.

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