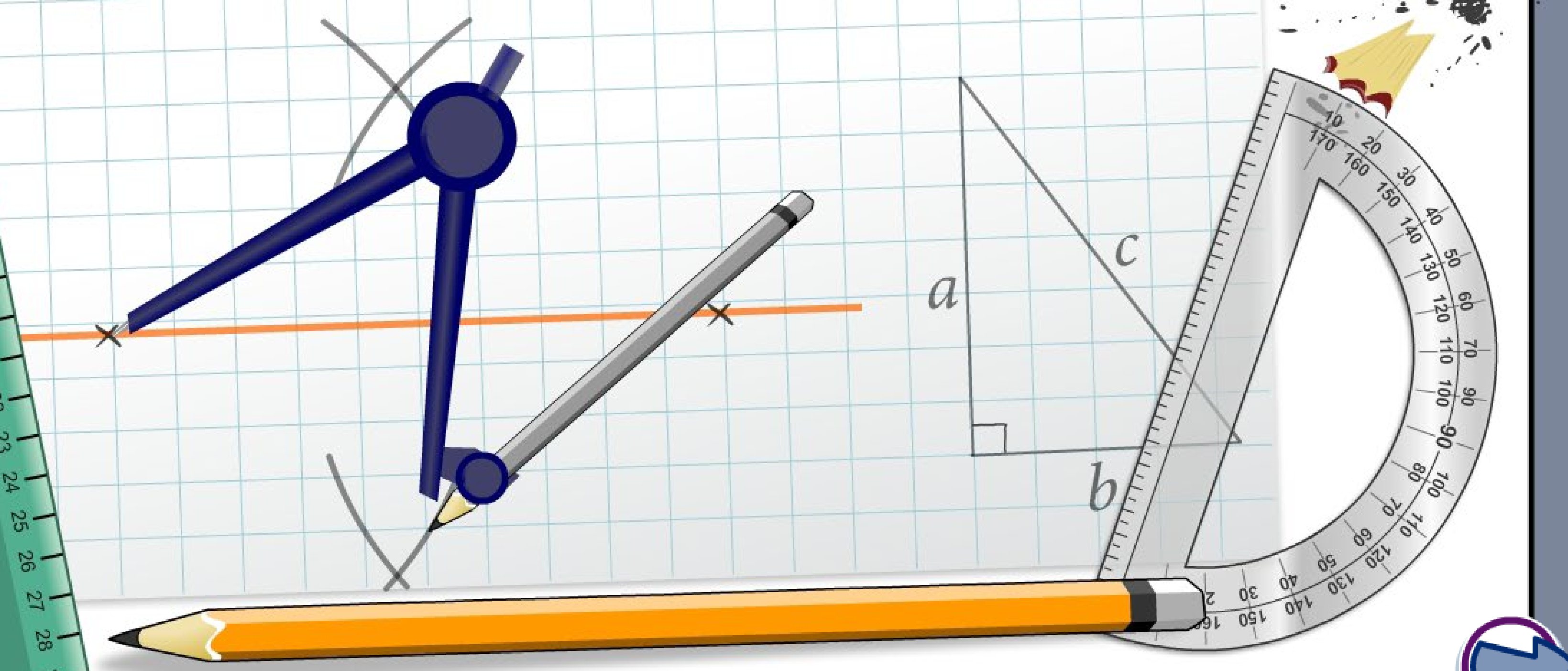
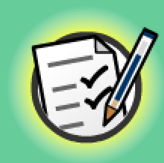
Proofs using Triangle Congruence

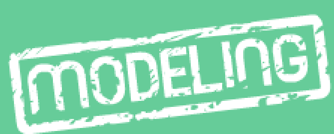


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

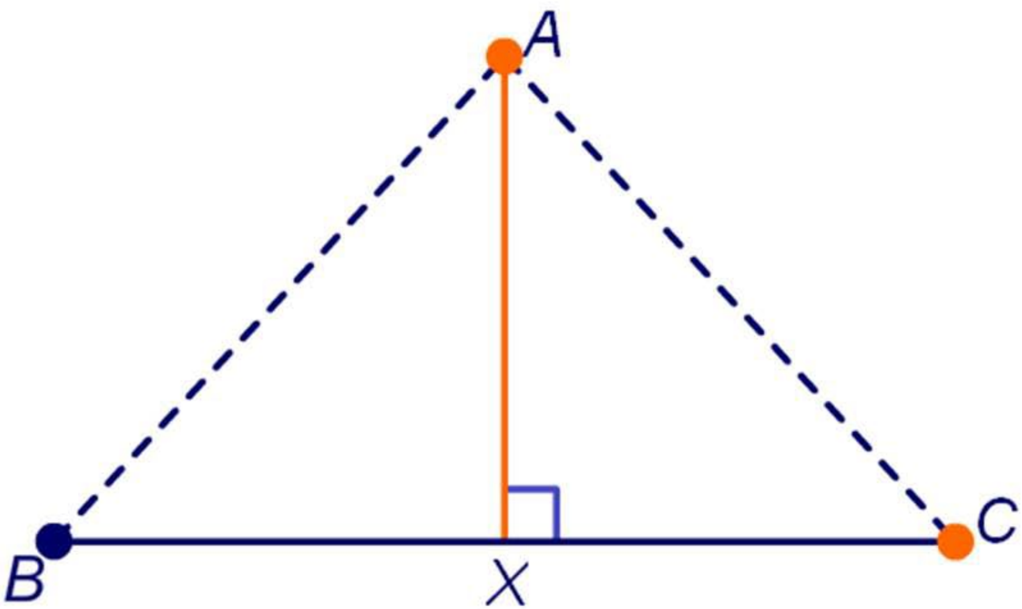
7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Exploring perpendicular bisectors



Drag the orange points to change the length of the line segment and the height of the perpendicular bisector.

show lengths

show angles

Perpendicular bisector theorem: If a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Given that $\overleftrightarrow{AX}$ is the perpendicular bisector of $\overline{BC}$, prove that $AB = AC$.

by definition of perpendicular bisector:

$\overline{BC} \perp \overline{AX} \Rightarrow \angle AXC = \angle AXB = 90^\circ \Rightarrow \angle AXC \cong \angle AXB$

$\overline{BX} \cong \overline{XC}$

by the reflexive property of congruence:

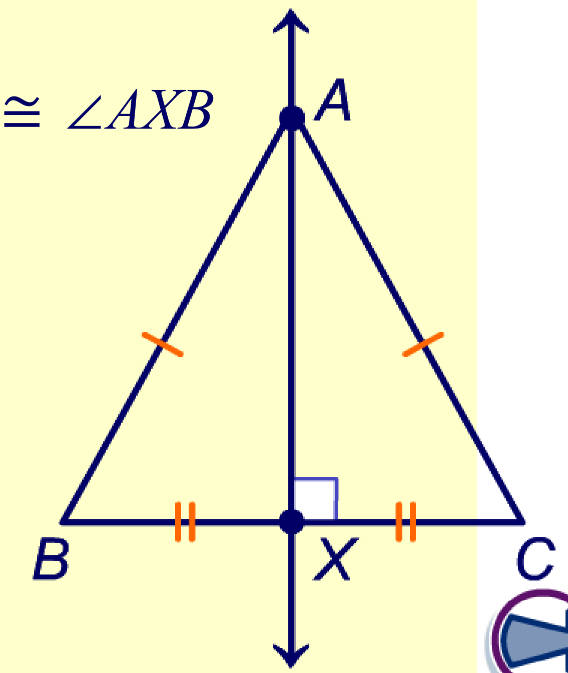
$\overline{AX} \cong \overline{AX}$

by SAS congruence postulate:

$\triangle AXB \cong \triangle AXC$

by CPCTC:

$\overline{AB} \cong \overline{BC} \quad \Rightarrow \quad AB = BC$ ✓



Converse of the perpendicular bisector theorem: If a point is equidistant from endpoints of a segment, then it lies on the perpendicular bisector of the segment.

Set up the proof of the converse of the perpendicular bisector theorem.

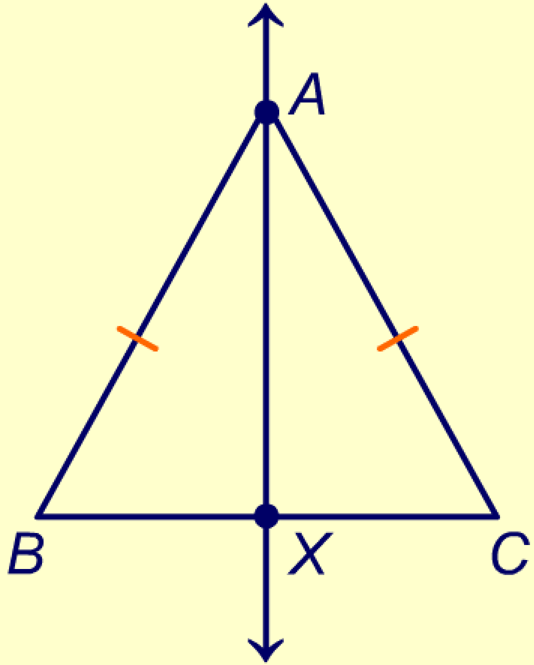
given:

$\overline{AB} \cong \overline{AC}$

need to show:

$\overline{AX} \perp \overline{BC}$

$\overline{BX} \cong \overline{XC}$



Proof of the converse

Converse of the perpendicular bisector theorem: If a point, A, is equidistant from the endpoints of a segment, $\overline{BC}$, then point A lies on the perpendicular bisector of the segment.

A is equidistant from endpoints:	?	(1)
assume X is the midpoint of $\overline{BC}$:	?	(2)
reflexive property of congruence:	?	(3)
by the SSS postulate:	?	(4)

Since $\angle AXB$ and $\angle AXC$ are a linear pair and congruent,

? (5)

Statements (1) and (5) combined means $\overline{AX}$ is the perpendicular bisector of $\overline{BC}$.

$\overline{AX} \cong \overline{AX}$	$\angle AXB = \angle AXC = 90°$	$AB = AC \Rightarrow \overline{AB} \cong \overline{AC}$
$\overline{BX} \cong \overline{XC}$	$\angle AXB + \angle AXC = 180°$	$\triangle AXB \cong \triangle AXC$

S C ?

Angles in isosceles triangles

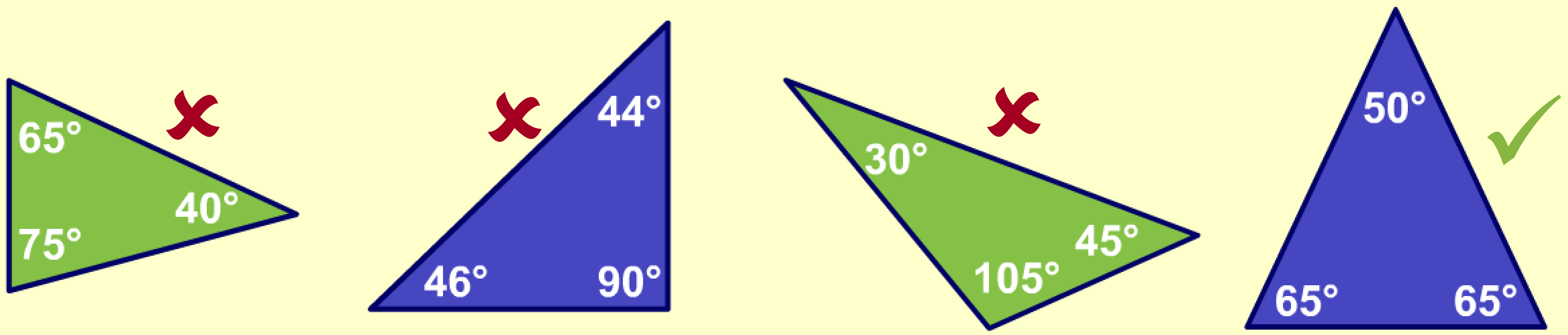
Which of the following choices could be the measures of three angles in an isosceles triangle?

A) 75°, 40°, 65°

B) 44°, 90°, 46°

C) 30°, 45°, 105°

D) 50°, 65°, 65°

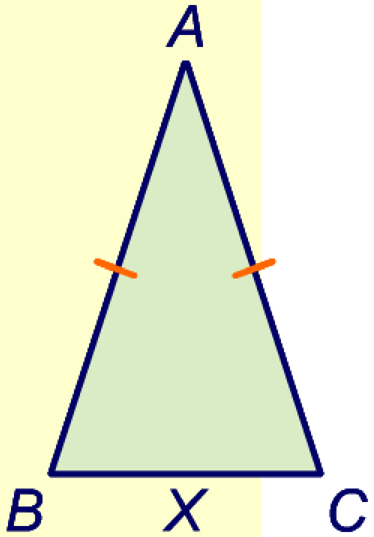


What do you notice about the angles? Is this true in general of isosceles triangles?

Base angle theorem: If two sides in a triangle are congruent, then the angles opposite these sides are also congruent.

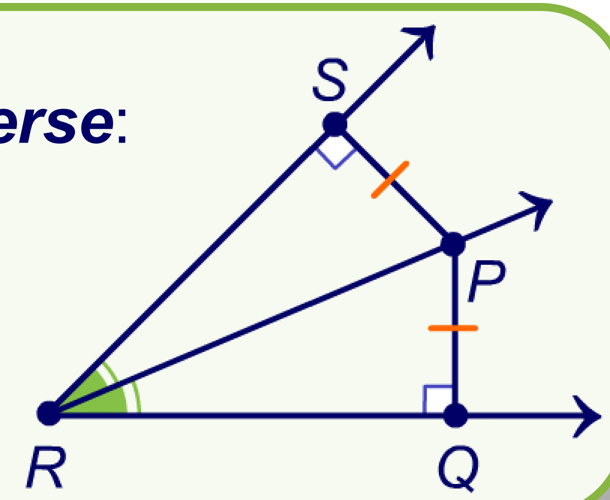
Prove that the angles opposite the congruent sides in an isosceles triangle are congruent.

1. Since A is equidistant from the endpoints of $\overline{BC}$, it lies on the perpendicular bisector of $\overline{BC}$ by the ***converse of the perpendicular bisector theorem***.
2. It is given that $\overline{AB}$ and $\overline{AC}$ are congruent. X bisects $\overline{BC}$, so $\overline{BX}$ and $\overline{CX}$ are congruent. With this, and since $\triangle ABX$ and $\triangle ACX$ share the side $\overline{AX}$, $\triangle ABX$ and $\triangle ACX$ are congruent by ***SSS***.
3. Corresponding parts of congruent triangles are congruent, therefore $\angle B$ is congruent to $\angle C$. ✓



Angle bisector theorem and its converse:

A point lies on the bisector of an angle *if and only if* it is equidistant from the sides of the angle.



State the angle bisector theorem. As a challenge, prove it.

"If a point lies on the bisector of an angle, then it is equidistant from the sides of the angle." proof hint: Use ***AAS.***

State the converse of the theorem. As a challenge, prove it.

"If a point is equidistant from the sides of the angle, then it lies on the bisector of the angle." proof hint: Use ***SSS.***