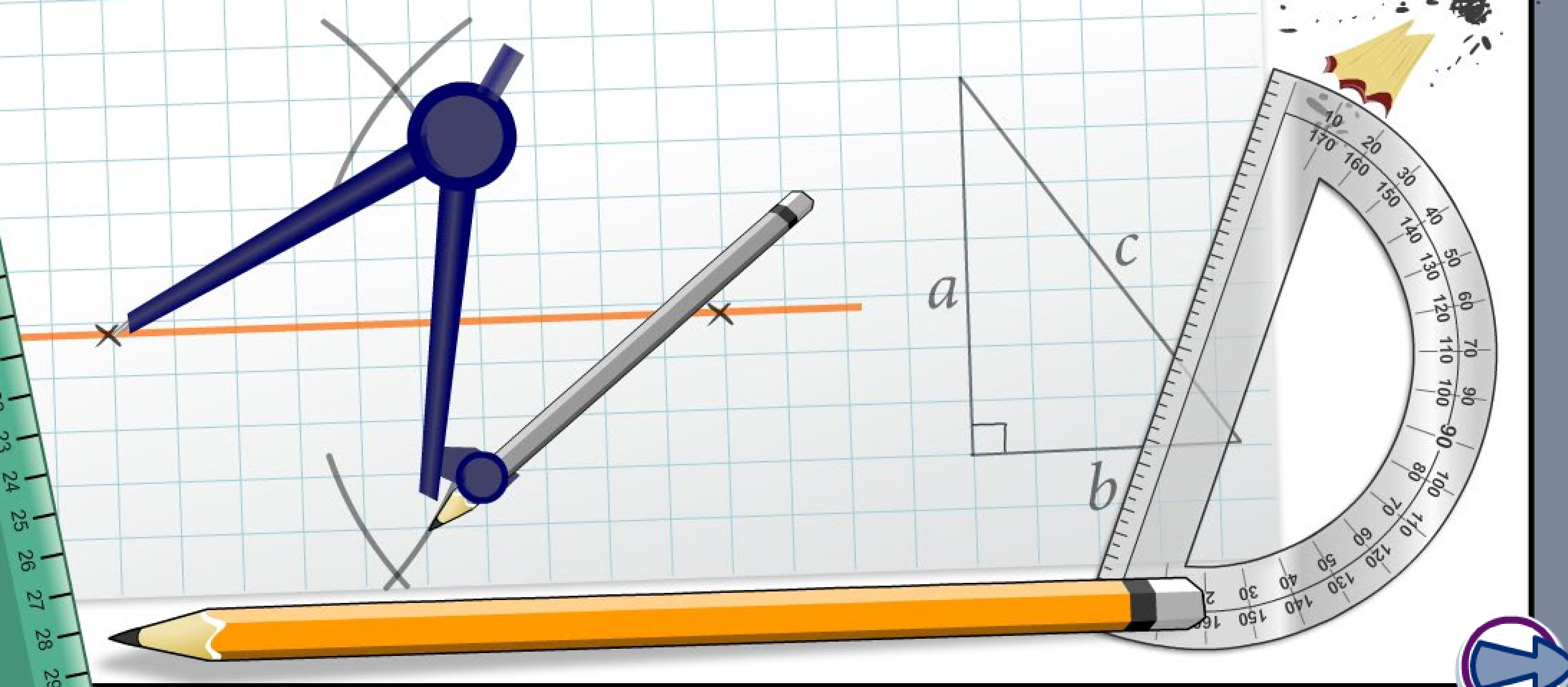
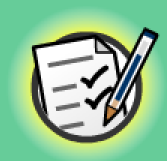
Proofs using Similar Triangles

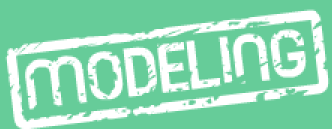


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

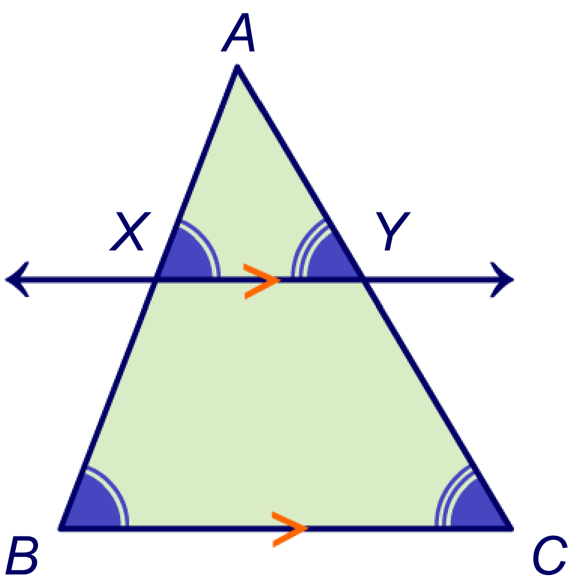
8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Triangle proportionality theorem

Triangle proportionality theorem and converse:
A line is parallel to the side of a triangle and intersects the two other sides *if and only if* it divides the sides proportionally.



State the *triangle proportionality theorem*.

If $\overleftrightarrow{XY}$ is parallel to $\overline{BC}$...

... then $AX/XB = AY/YC$.

State the *converse of the triangle proportionality theorem*.

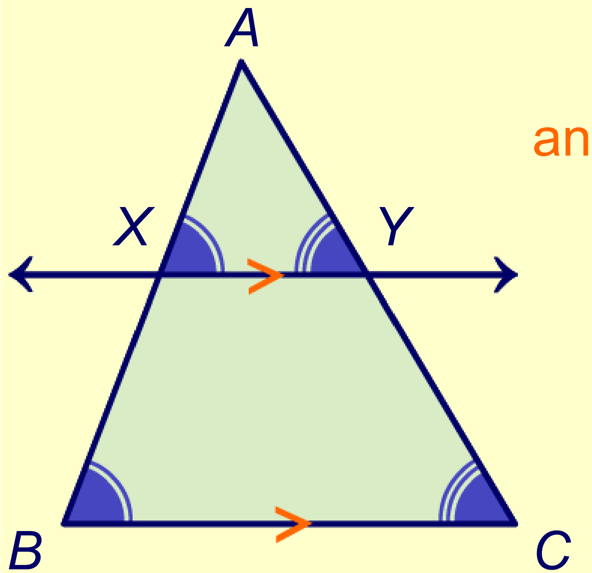
If $AX/XB = AY/YC$...

... then $\overleftrightarrow{XY}$ is parallel to $\overline{BC}$.

Triangle proportionality theorem:

If a line is parallel to the side of a triangle and intersects the two other sides, then it divides the sides proportionally.

Prove the *triangle proportionality theorem.*



given: $\overline{XY} \parallel \overline{BC}$

corresponding angles postulate: $\angle AXY \cong \angle B,\ \angle AYX \cong \angle C$

AA similarity postulate: $\triangle ABC \sim \triangle AXY$

$\Rightarrow AX/(AX+XB) = AY/(AY+YC)$

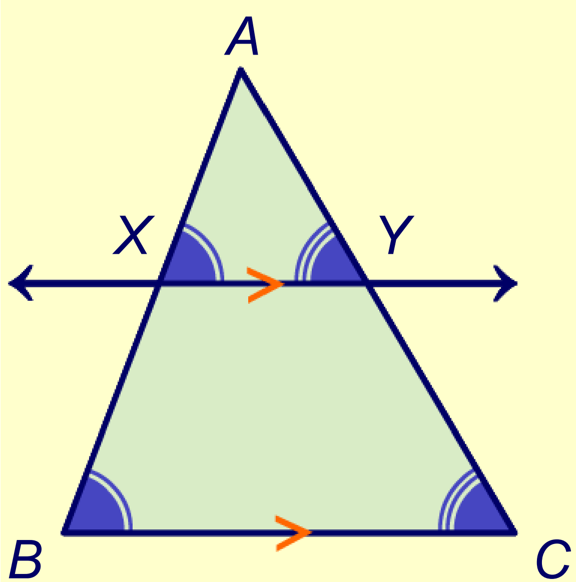
cross-multiply: $AX \cdot AY + AX \cdot YC = AY \cdot AX + AY \cdot XB$

simplify: $AX \cdot YC = AY \cdot XB$

divide by $YC \cdot XB$: $AX/XB = AY/YC$ ✓

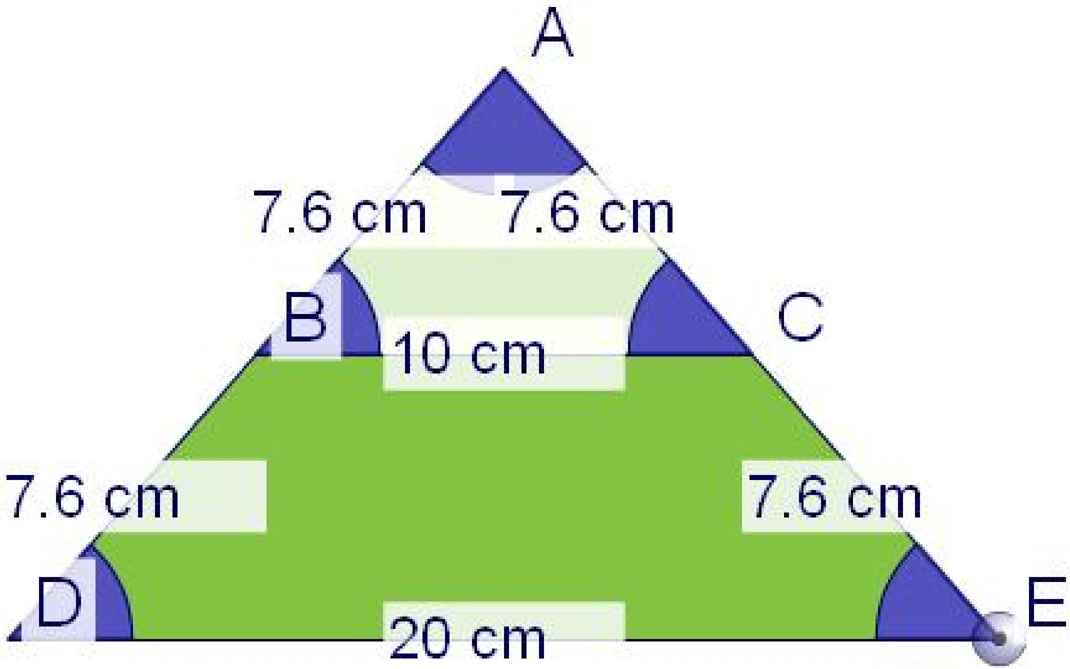
Converse of the triangle proportionality theorem:
If a line divides two sides of a triangle proportionally, then it is parallel to the other side.

Prove the *converse of the triangle proportionality theorem*.



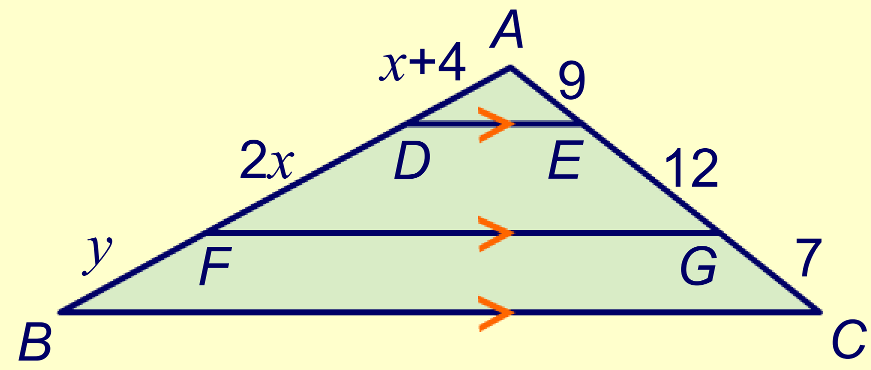
given:	$AX/XB = AY/YC$
cross-multiply :	$AX \cdot YC = AY \cdot XB$
add $AX \cdot AY$:	$AX \cdot YC + AX \cdot AY = AY \cdot XB + AX \cdot AY$
factor:	$AX(AY + YC) = AY(AX + XB)$
substitute:	$AX(AC) = AY(AB)$
rearrange:	$AX/AB = AY/AC$
reflex. prop.:	$\angle A \cong \angle A$
SAS similarity postulate:	$\triangle ABC \sim \triangle AXY$ $\Rightarrow \angle AXY \cong \angle B, \angle AYX \cong \angle C$
conv. corr. ang. theorem:	$\overline{XY} \parallel \overline{BC}$ ✓

Similar triangles



$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} = 2$$

In the triangle shown, find the values of x and y.



Since $DE \parallel FG$ and $BC \parallel FG$, use the ***triangle proportionality theorem***.

First look at $\triangle AFG$.

by the tri. prop. theorem: $AD/DF = AE/EG$

substitute known lengths: $(x + 4)/2x = 9/12$

$12x + 48 = 18x$

solving for x: $x =$ **8**

Then look at $\triangle ABC$.

by the tri. prop. theorem: $AF/FB = AG/GC$

substitute known lengths and x: $(x + 4 + 2x)/y = (9 + 12)/7$

$28/y = 21/7$

solving for x: $y = 28 \times 7 \div 21 =$ **8.05**