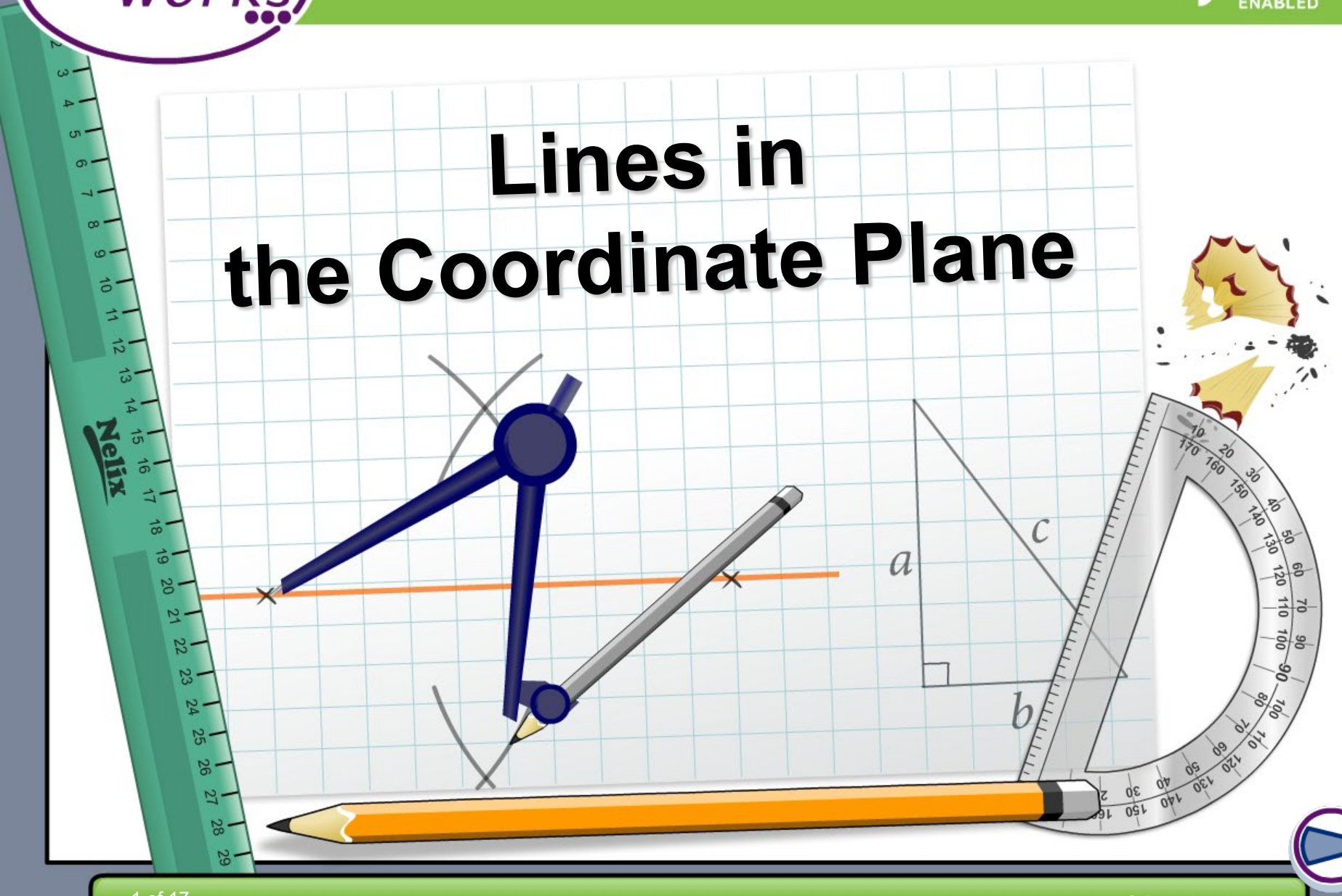
Lines in the Coordinate Plane



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

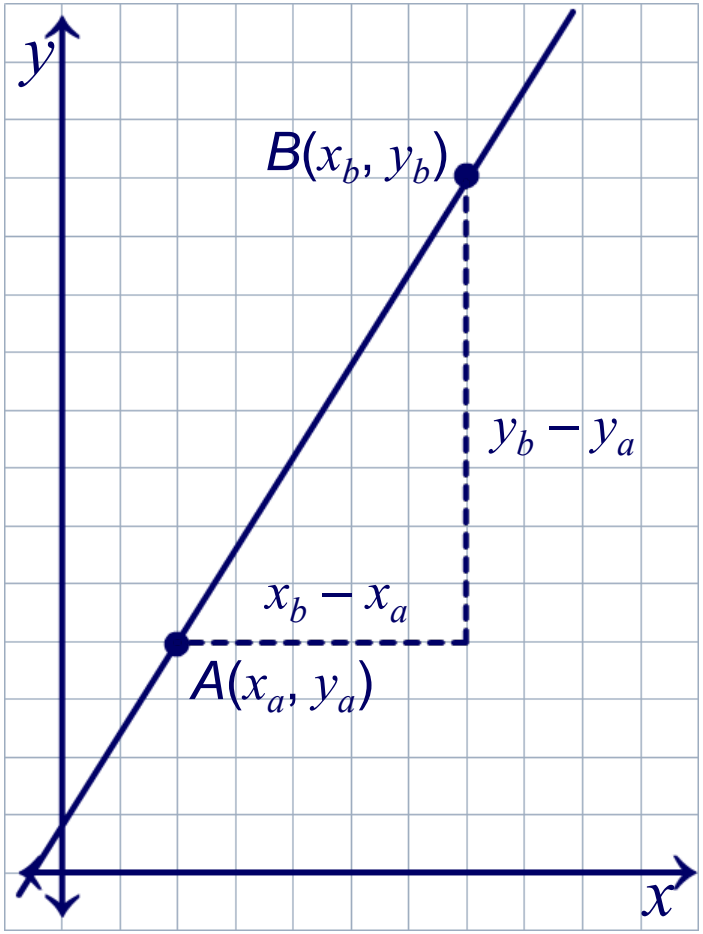
This icon indicates teacher's notes in the Notes field.

Slope

The **slope** of a line through any two points $A(x_a, y_a)$ and $B(x_b, y_b)$ is a measure of steepness.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\textbf{rise}}{\textbf{run}}$$

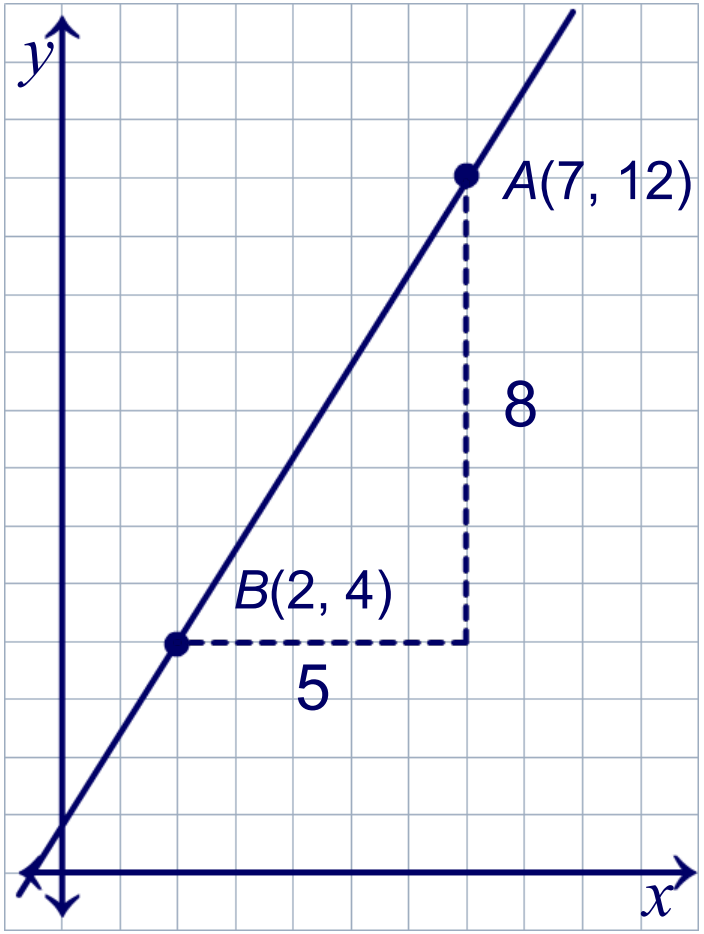
To find this more easily, draw a right triangle with the line segment $\overline{AB}$ as the hypotenuse.



$$\text{slope} = \frac{y_b - y_a}{x_b - x_a}$$

Finding slope

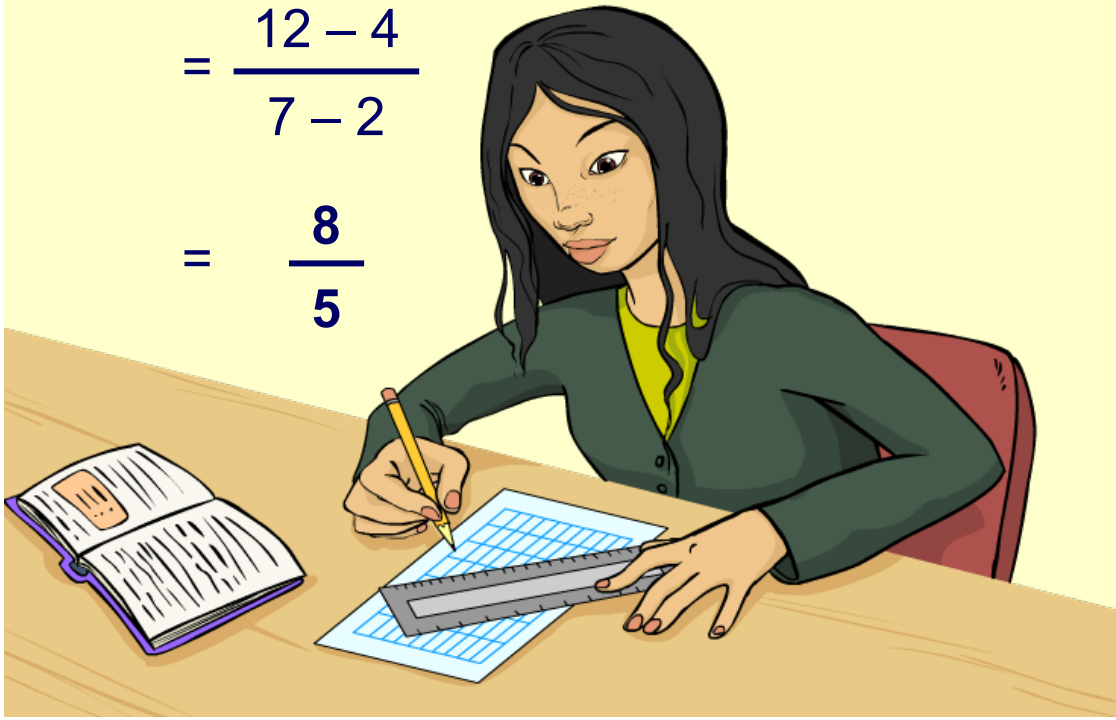
What is the slope of the line through $A(7, 12)$ and $B(2, 4)$?



$$\text{slope} = \frac{y_b - y_a}{x_b - x_a}$$

$$= \frac{12 - 4}{7 - 2}$$

$$= \frac{8}{5}$$



Finding slope modeling

The horizontal distance from Franklin to Greenville is 30 miles. Greenville is 3168 feet above sea level and Franklin is 1056 feet above sea level. There are 5280 feet in a mile.

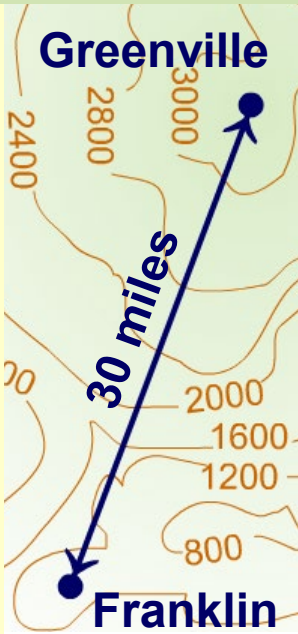
What is the average slope of the road from Franklin to Greenville?

convert feet to miles: $\frac{1056}{5280} = 0.2$ miles $\quad \frac{3168}{5280} = 0.6$ miles

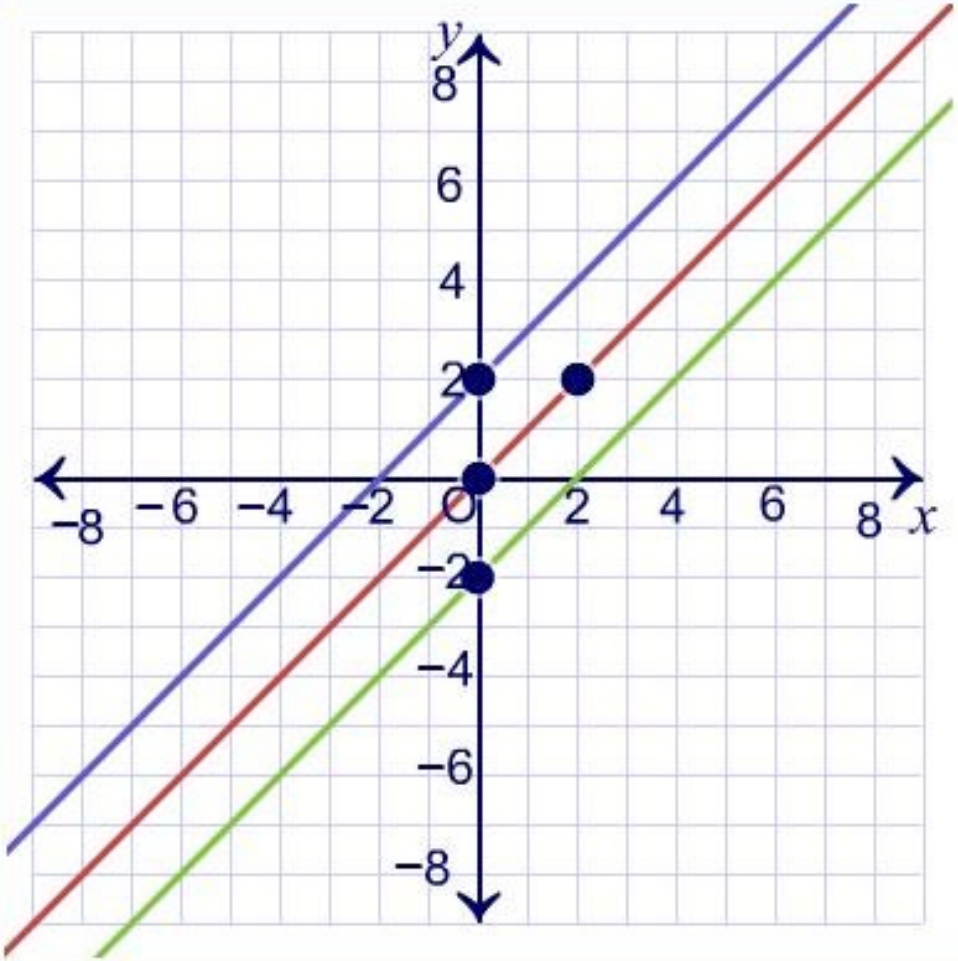
Represent Franklin and Greenville on a coordinate grid. Let the y axis represent the height above sea level and the x axis represent horizontal distance.

Franklin is $F(0, 0.2)$. Greenville is $G(30, 0.6)$.

find the slope: $\frac{\text{rise}}{\text{run}} = \frac{0.6 - 0.2}{30 - 0} = \frac{0.4}{30} = \mathbf{\frac{1}{75}}$



Investigating parallel lines



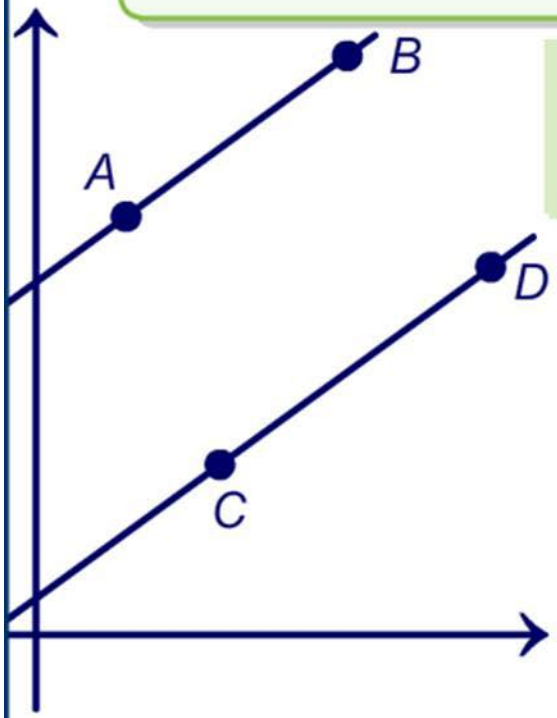
$y = x$

$y = x - 2$

$y = x + 2$

Proof of the parallel lines theorem

The parallel lines theorem: two lines are parallel if and only if they have the same slope.

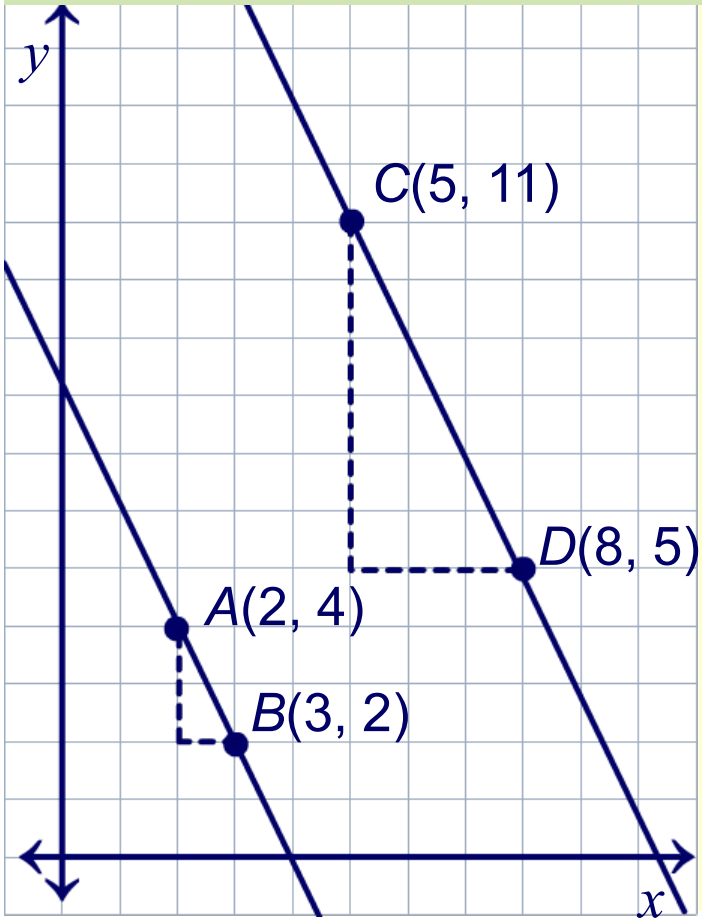


Prove that if $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, they have the same slope.

Press next to see the proof step-by-step.

Parallel lines practice

Show that the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel for $A(2, 4)$, $B(3, 2)$, $C(5, 11)$ and $D(8, 5)$.



Draw the points on a coordinate grid and graph the lines.

find the slope of $\overleftrightarrow{AB}$:

rise = 2 – 4 = –2 run = 3 – 2 = 1

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$$

find the slope of $\overleftrightarrow{CD}$:

rise = 5 – 11 = –6 run = 8 – 5 = 3

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-6}{3} = -2$$

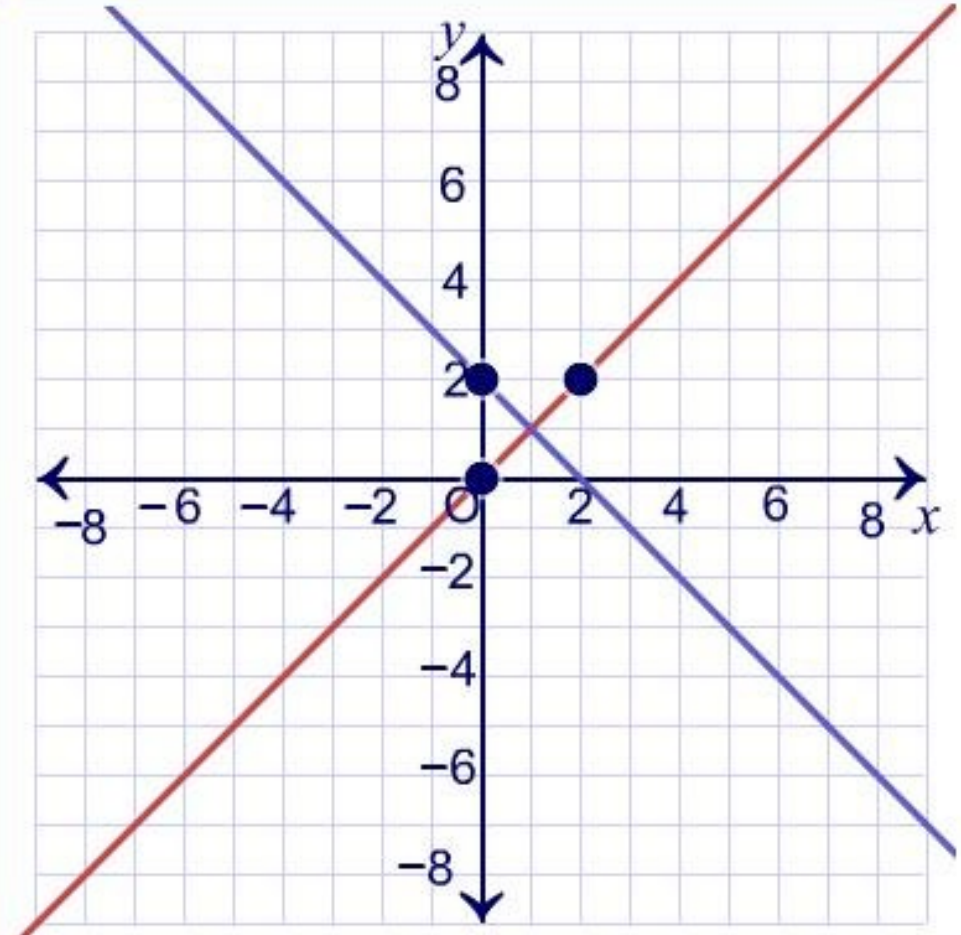
The slopes are the same so by the parallel lines theorem $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel.

Matching parallel lines

$y = \frac{-3}{4}x - 9$	$y = \frac{-3}{4}x - 4$
$y = \frac{-1}{5}x - 9$	$y = \frac{-1}{5}x + 5$
$y = \frac{-1}{4}x - 3$	$y = \frac{-1}{4}x + 1$
$y = \frac{-2}{3}x + 8$	$y = \frac{-2}{3}x + 5$
$y = \frac{-7}{2}x - 10$	$y = \frac{-7}{2}x + 8$

1 2

Investigating perpendicular lines

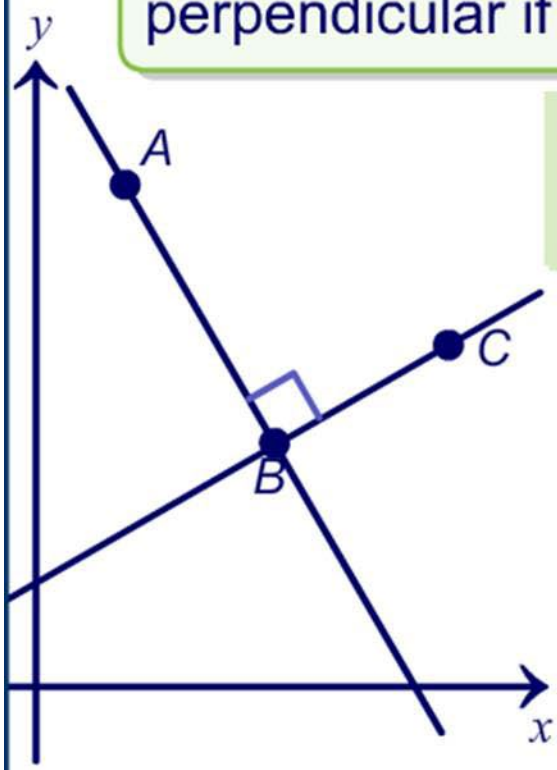


$y = x$

$y = -x + 2$

Proof of the perpendicular lines theorem

The perpendicular lines theorem: two lines are perpendicular if and only if the product of their slopes is –1.

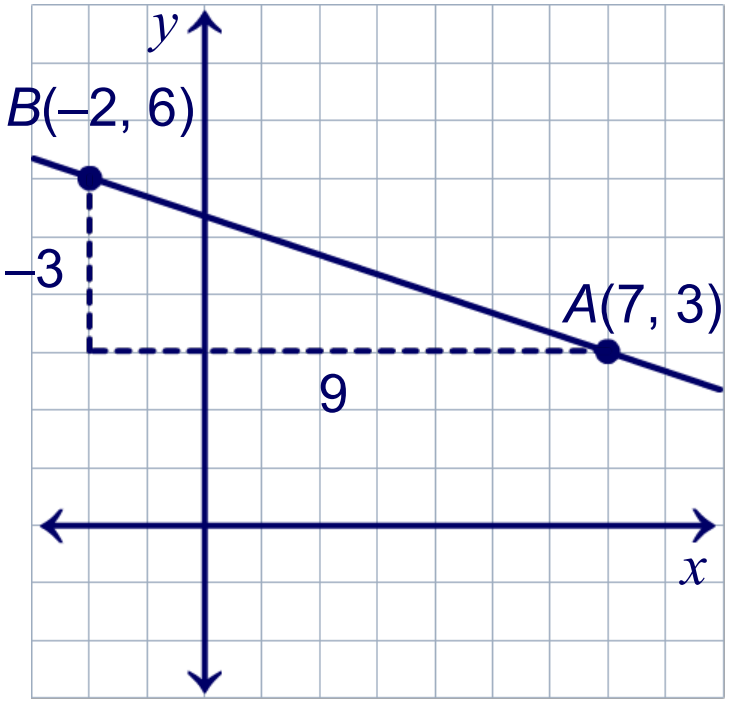


Prove that if $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are perpendicular, the product of their slopes is –1.

Press next to see the proof step-by-step.

Perpendicular lines

What is the slope of a line perpendicular to $\overleftrightarrow{AB}$ for $A(7, 3)$ and $B(–2, 6)$?



Draw the points A and B on a coordinate grid and graph the line.

find the slope of AB:

$$\text{rise} = 3 - 6 = -3 \qquad \text{run} = 7 - (-2) = 9$$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-3}{9} = -\frac{1}{3}$$

find the perpendicular slope:

$$-\frac{1}{3} \times m = -1$$

So $\boldsymbol{m = 3}$

Matching perpendicular lines

$y = \frac{-1}{4}x - 1$	$y = 4x + 3$
$y = \frac{-1}{2}x - 7$	$y = 2x - 8$
$y = -3x + 5$	$y = \frac{1}{3}x - 8$
$y = \frac{10}{3}x - 5$	$y = \frac{-3}{10}x - 10$
$y = \frac{2}{5}x - 4$	$y = \frac{-5}{2}x - 7$

Points partitioning lines

The coordinates of the midpoint of a line segment are found using the mean of the coordinates of the endpoints. How can the coordinates of other points on the line segment be found?

For example, what are the coordinates of the point that is ⅔ of the way from $A(-1, 1)$ to $B(5, 4)$?

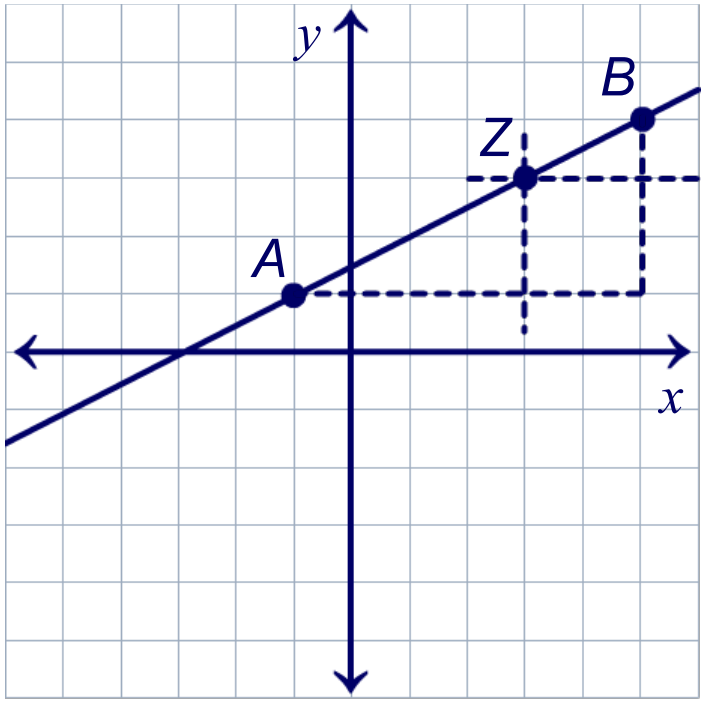
The line segment AB has a **direction** between its endpoints, so it is called a **directed line segment**.

We are asked to find a **partition** of a directed line segment that divides it into two segments in a given ratio, ⅔ or 2:1.

First, find the **rise** and **run** between the two points and then calculate ⅔ of this distance.

Directed line segments

Find the point $Z(x,y)$ in a directed line segment from $A(-1,1)$ to $B(5,4)$ that partitions it in a 2:1 ratio.



find rise from A to B:

$y_b - y_a = 4 - 1$

$= 3$

find run from A to B:

$x_b - x_a = 5 - (-1)$

$= 6$

2:1 ratio means that $AZ/ZB = 2/1$, so $AZ = 2ZB$.

This means that Z is ⅔ of the way along $\overline{AB}$.

⅔ of rise $= ⅔ \times 3$

$= 2$

⅔ of run $= ⅔ \times 6$

$= 4$

Z is the coordinates of A plus the fraction of the run and rise:

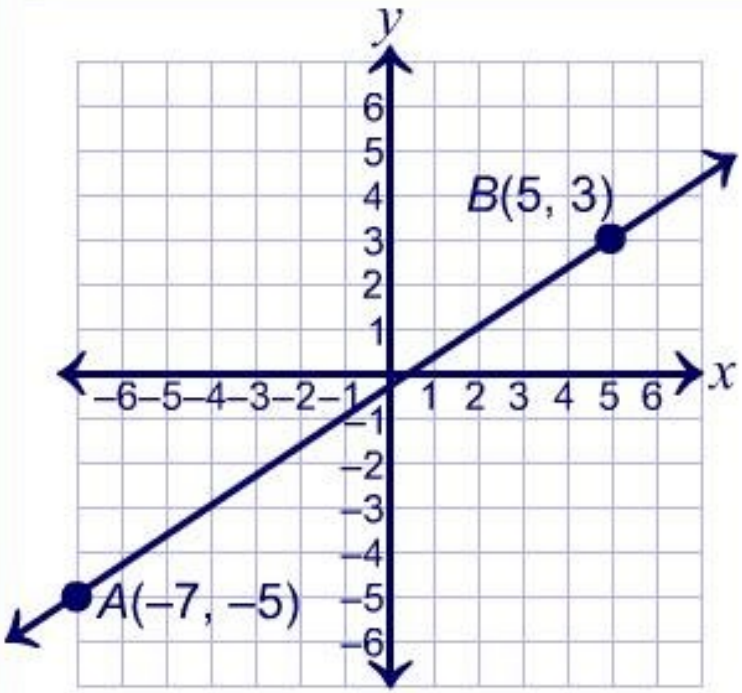
$$Z(x,y) = Z(-1 + 4, 1 + 2)$$

$$= \mathbf{Z(3,3)}$$

Line segment practice

Question: 1/8

What is the slope of line $\overleftrightarrow{AB}$?



5/3	3/7
3/2	2/3

Press the "=" button to show the calculations step-by-step.

Road map problems

The town of Clinton is 40 miles east of Salem, and 20 miles north. Washington is 35 miles north and 15 miles west of Salem, and Fairview is 10 miles south and 50 miles east of Salem.

A straight highway passes through Fairview and Clinton and another through Salem and Washington.

Do these highways ever cross?

*Press the **W** button to see the solution.*