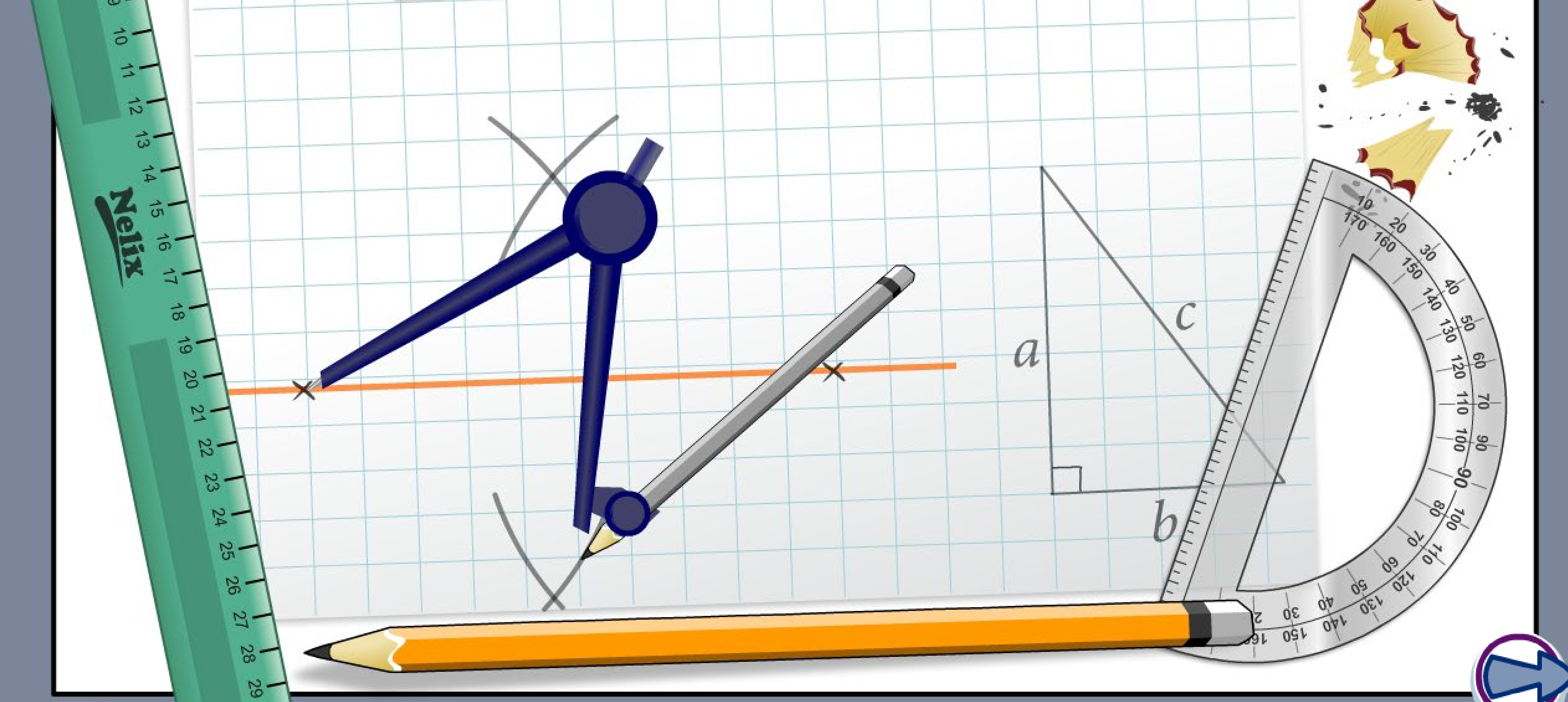
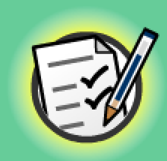
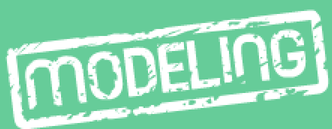
Law of Cosines



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

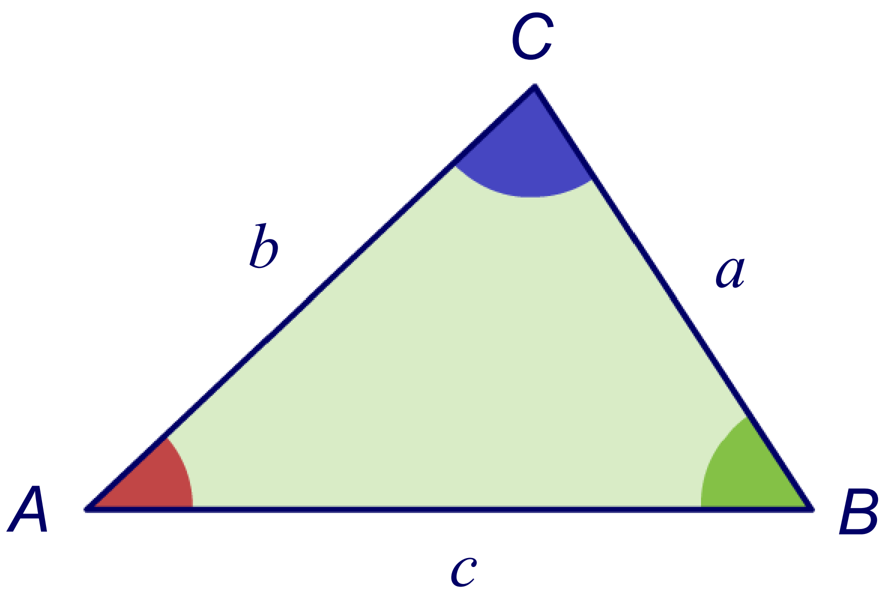
7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Here is a triangle ABC.



a is the length of the side opposite angle A.

b is the length of the side opposite angle B.

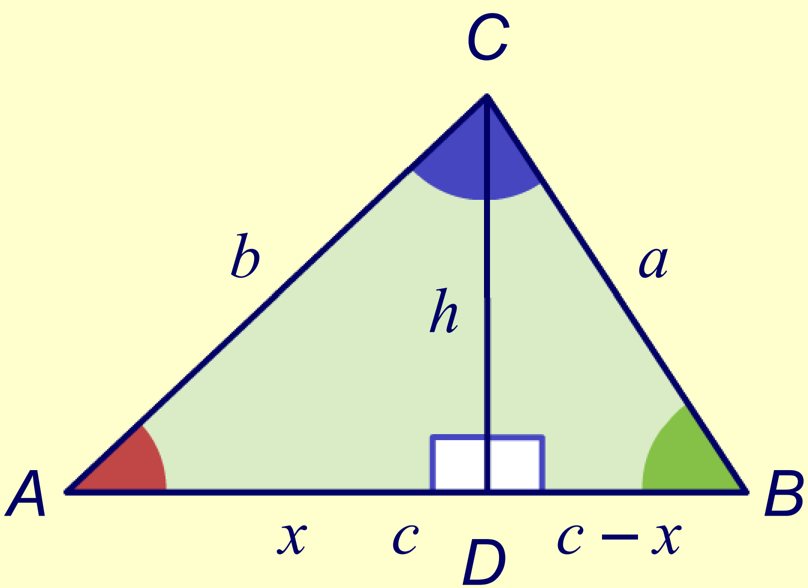
c is the length of the side opposite angle C.

The law of cosines:

$a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Deriving the law of cosines (1)

Use the Pythagorean Theorem to derive the law of cosines.

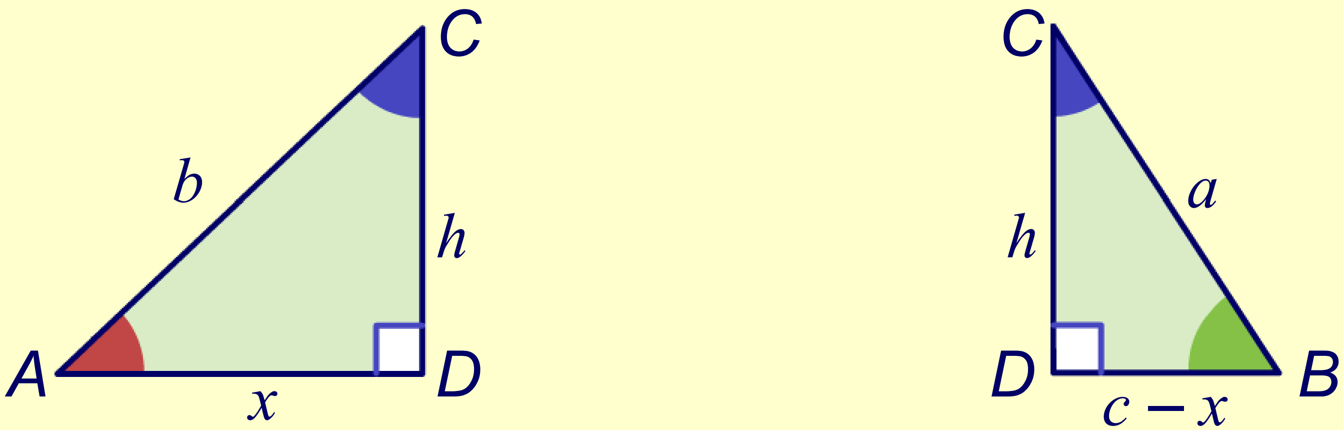


If a line segment, h, is drawn perpendicular from $\overline{AB}$ to C, the triangle can be divided into two right triangles, $\triangle ACD$ and $\triangle BDC$.

By definition, the length AB is equal to c.

If the length AD is called x, then the length BD is equal to $c - x$.

Considering the two triangles, $\triangle ACD$ and $\triangle BDC$, separately:



apply the pythagorean theorem to both triangles:

$b^2 = x^2 + h^2$

$a^2 = (c - x)^2 + h^2$

distribute: $a^2 = c^2 - 2cx + x^2 + h^2$

substitute $x^2 + h^2$ for b^2: $a^2 = c^2 - 2cx + b^2$

substitute x for $b \cos A$ (because $\cos A = \frac{x}{b}$): $a^2 = c^2 - 2cb \cos A + b^2$

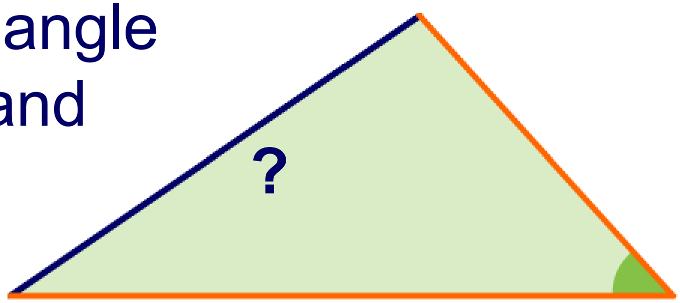
rearrange: $a^2 = b^2 + c^2 - 2bc \cos A$ ✓

When to use the law of cosines

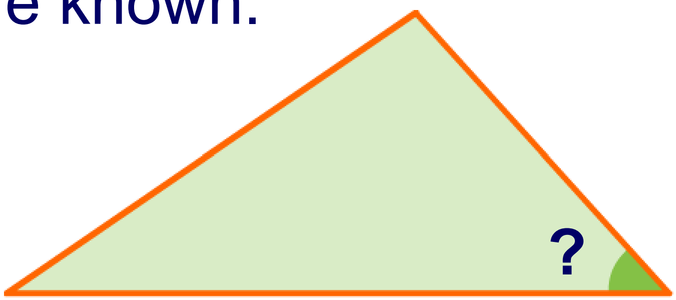
When should the law of cosines be used?

The law of cosines is particularly useful for **non-right triangles**. It can be used:

- to find the length of one side in a triangle if the length of the other two sides and the measure of the angle between them is known

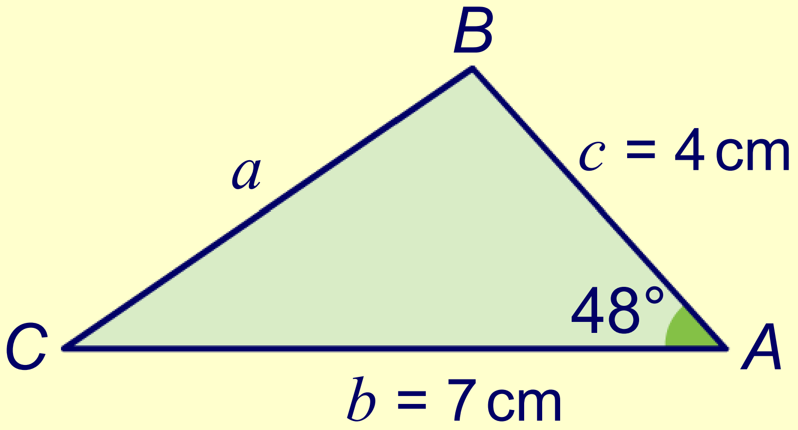


- to find the measure of any one of the angles in the triangle if the lengths of all three sides are known.



If we do not have this information, we must use a different method or a different law.

Find the length of side a.



write the law of cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

substitute for the given values:

$$a^2 = 7^2 + 4^2 - 2(7 \times 4)\cos 48°$$

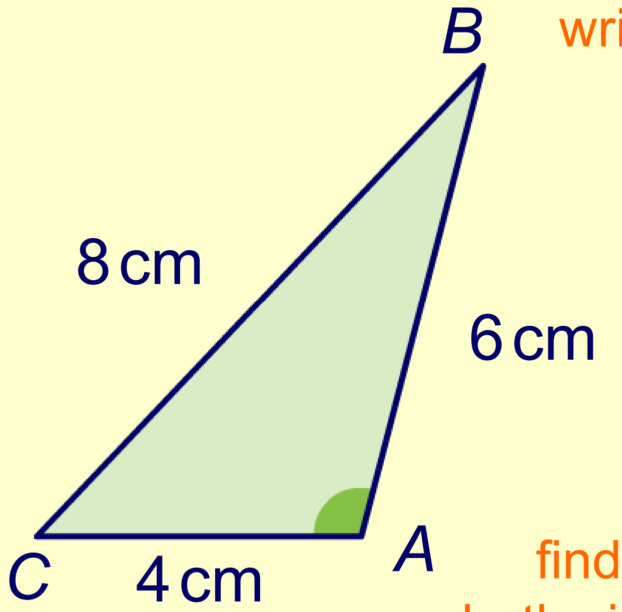
evaluate:

$a^2 = 27.53$ (to the nearest hundredth)

take the square root of both sides of the equal sign:

$\boldsymbol{a}$ **= 5.25 cm** (to the nearest hundredth)

Find the measure of the angle at A.



write the law of cosines: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

substitute for the given values: $\cos A = \dfrac{4^2 + 6^2 - 8^2}{2(4 \times 6)}$

evaluate: $\cos A = -0.25$

find the inverse cosine of both sides of the equal sign: $A = \cos^{-1}(-0.25)$

evaluate: $A =$ **104.48°** (nearest hundredth)

Press the question marks and angle markers to reveal the values of the side lengths and angles. Choose an angle or side and calculate it using the law of cosines. Drag the points to change the triangle.