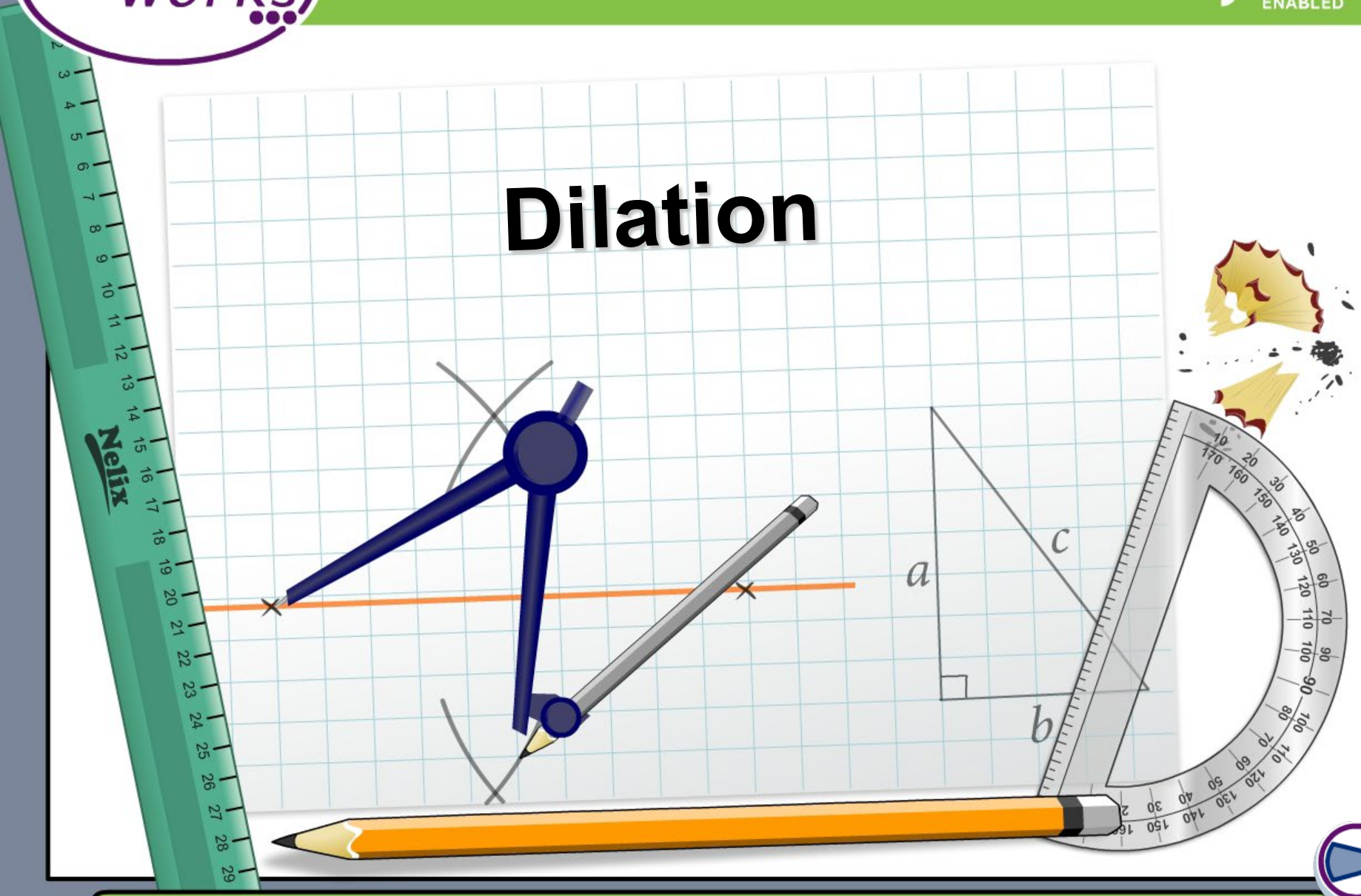
Dilation



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

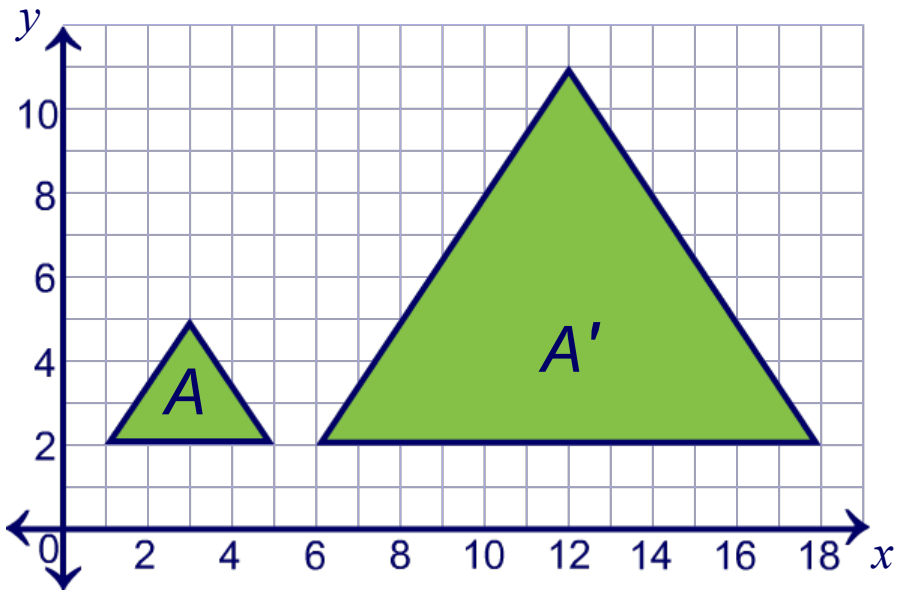
This icon indicates teacher's notes in the Notes field.

Dilation

Shape A' is a **dilation** of shape A.

The length of each side in shape A' is 3 times the length of each side in shape A.

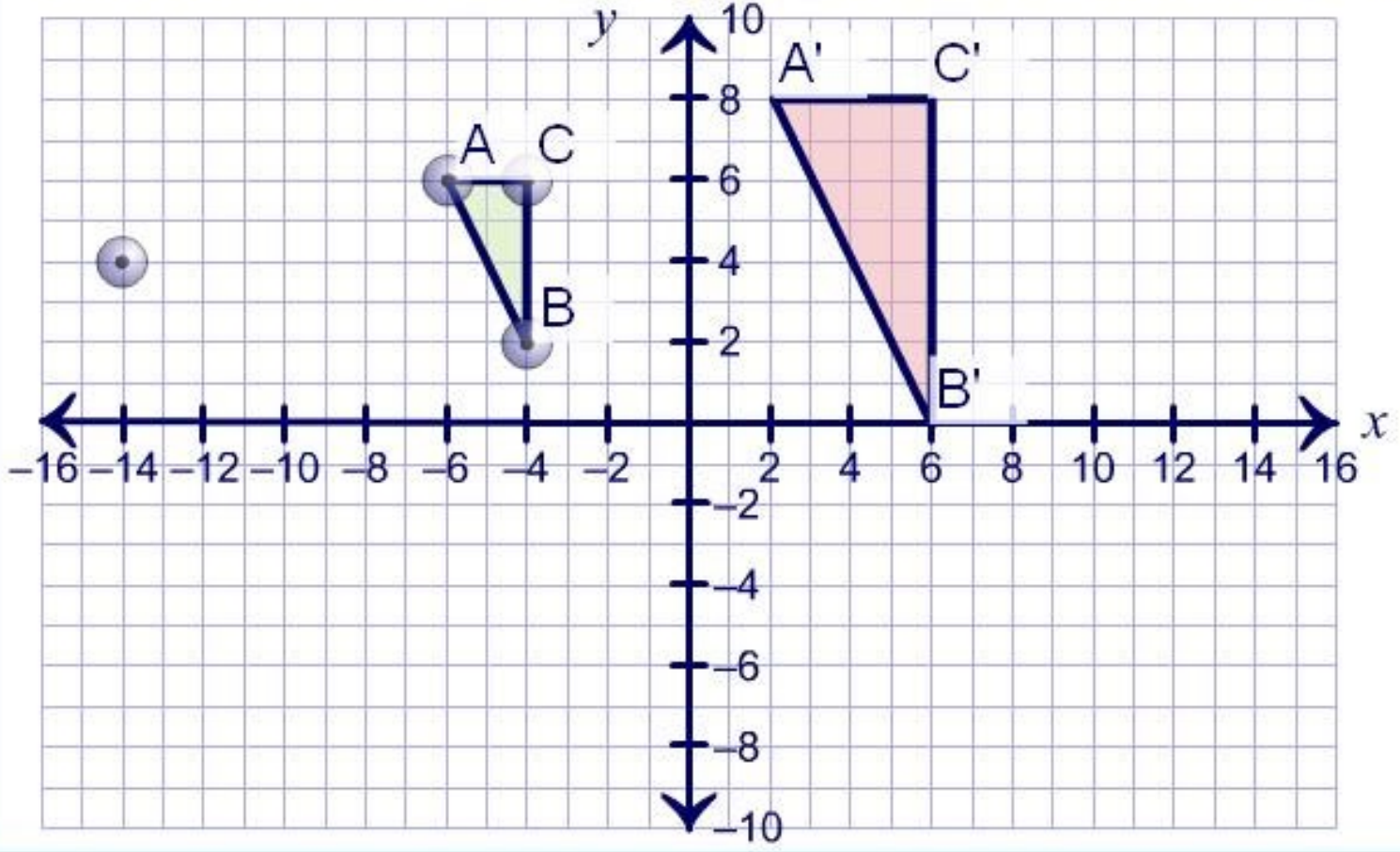
Shape A has been dilated by a **scale factor** of 3.



Every side of shape A is parallel to its corresponding side of the image A'.

Dilation on a coordinate grid

Choose a scale factor, then drag the points to modify the dilation

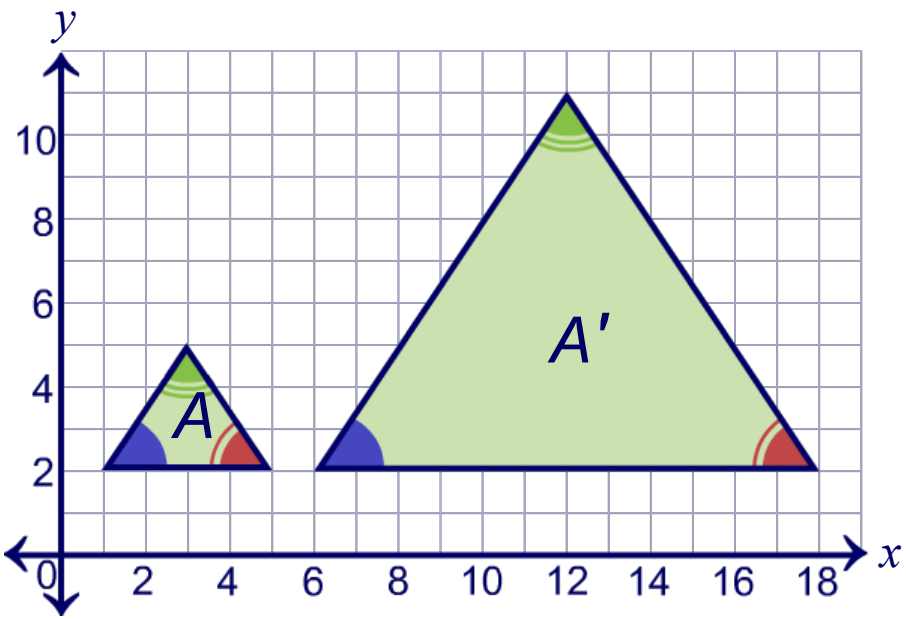


2

Is a dilation congruent to the original object?

The image of a dilation and its object are **similar**. This is different than reflections, rotations and translations, which produce images that are **congruent** to the original shape.

If two shapes are congruent, they are the same shape and size.

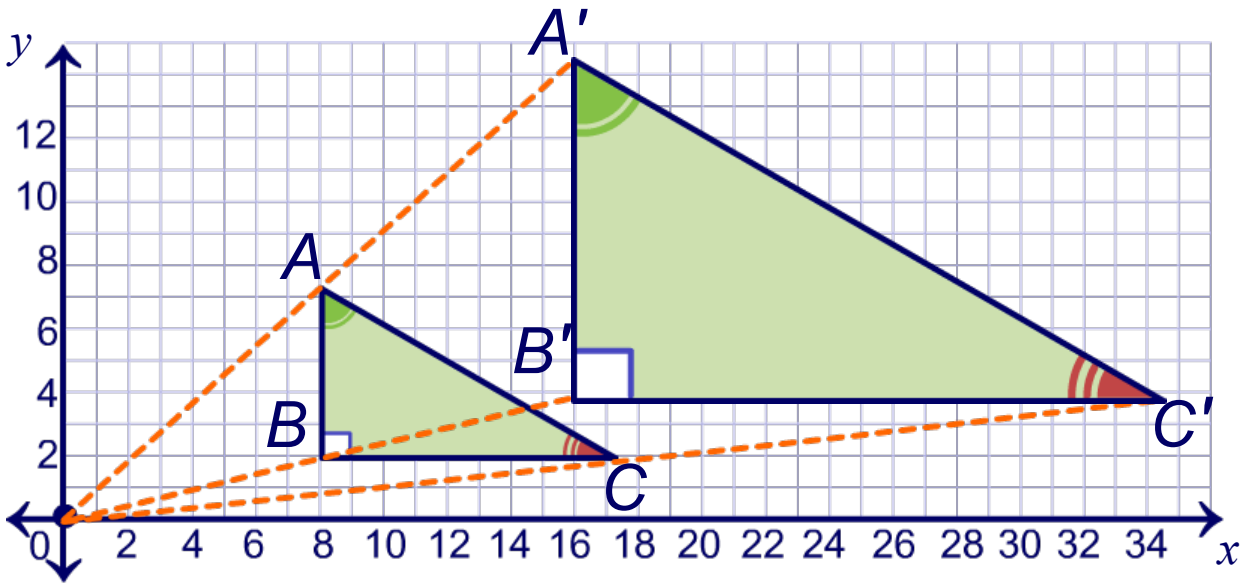


In a dilated shape, the corresponding angles are congruent, but the lengths are different.

Defining a dilation

To define a dilation, the **scale factor** and **center of dilation** must be given.

For example, dilate $\triangle ABC$ by a scale factor of 2 with the center of dilation at the origin.

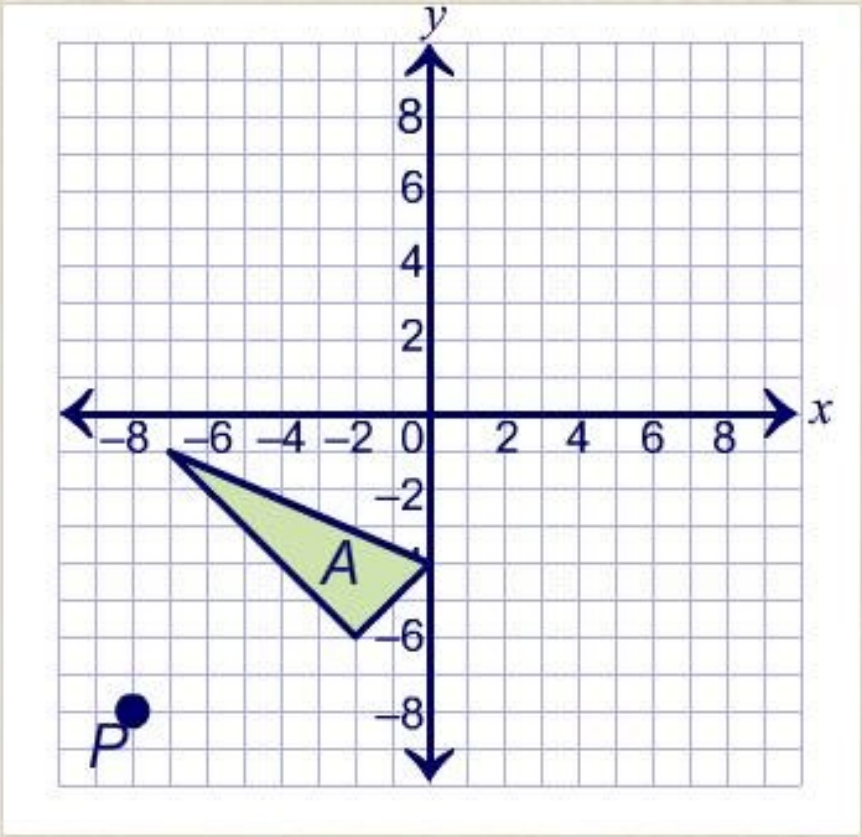


The length of each side has increased by two, but the measures of the angles are the same.

Constructing a dilation

To define a dilation we must be given a **scale factor** and a **center of dilation.**

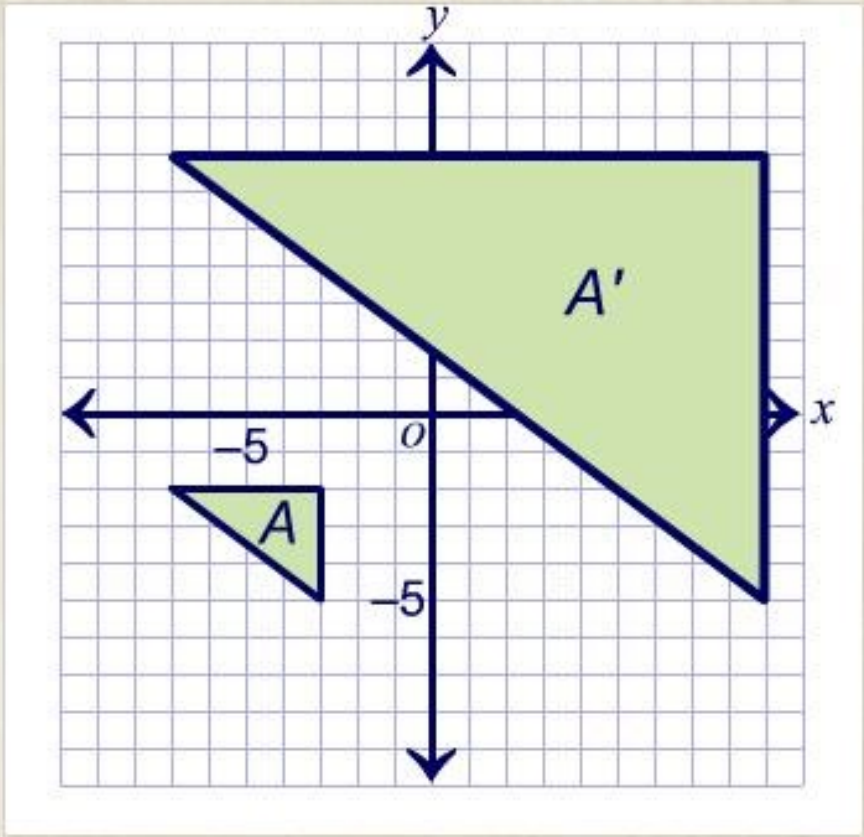
Press **"play"** to see shape A dilated by a scale factor of 2 from the center of dilation P.



Finding the center of dilation

Given an object and its dilation, it is easy to find the center of the dilation.

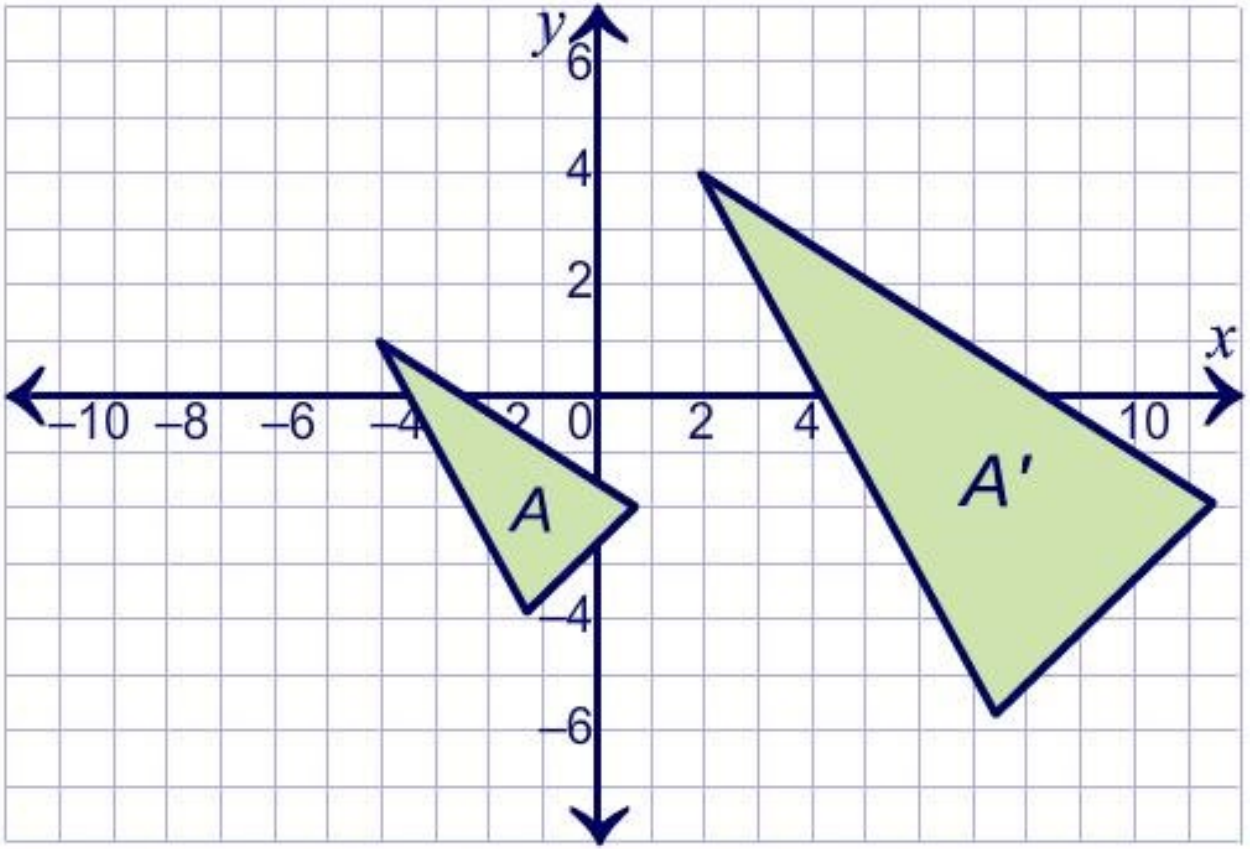
Press **"play"** to see how.



Finding the center of dilation

Center of dilation

Shape A has been dilated by a scale factor of 2 to create the image A'. Drag the point P to the center of dilation on the graph.



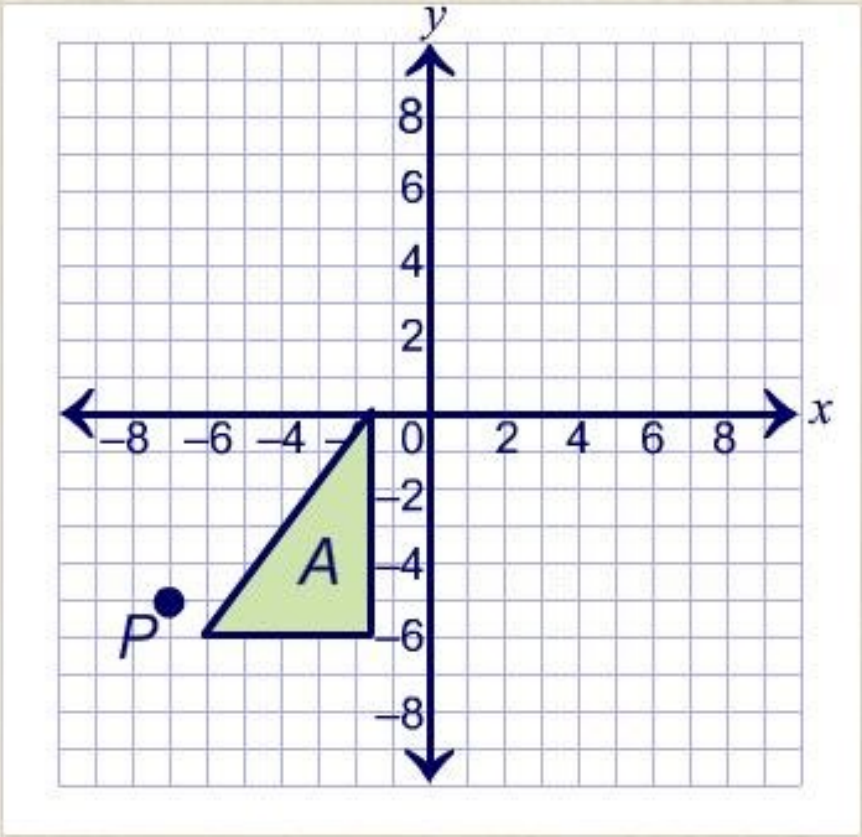
P

Inverse dilations

Inverse dilation

An **inverse dilation** maps the image that has been dilated back onto the original object.

Press **"play"** to see how.



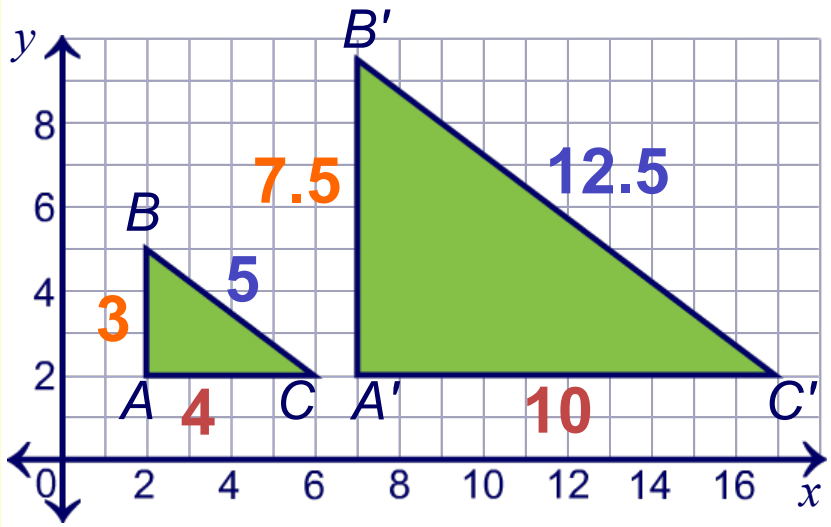
Finding the scale factor of a dilation

The scale factor for a dilation is found by looking at the ratio between any two corresponding lengths.

Find the scale factor of the dilation from A to A'.

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = \text{scale factor}$$

$$\frac{7.5}{3} = \frac{12.5}{5} = \frac{10}{4} = 2.5$$



scale factor = $\dfrac{\text{length of dilation}}{\text{corresponding length of original}}$

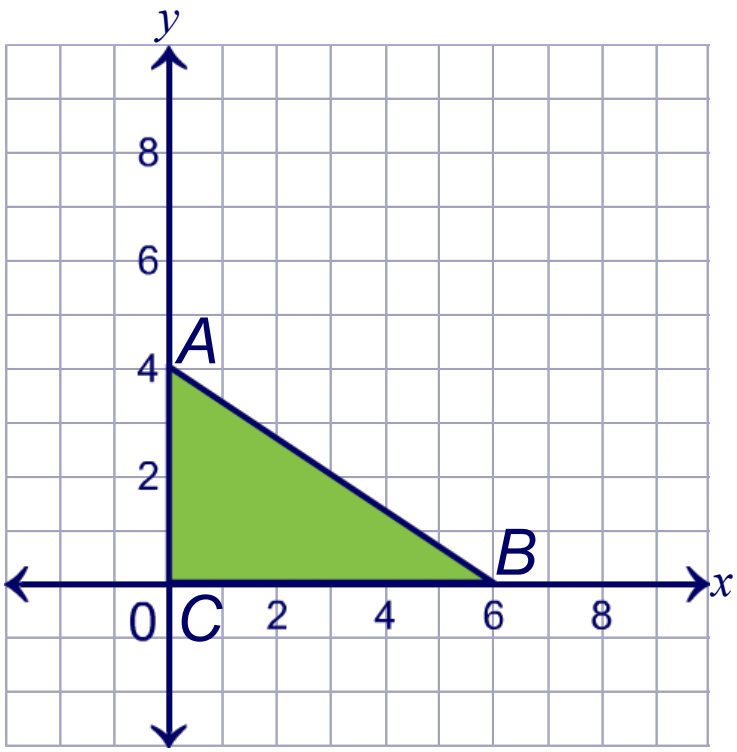
Applying the scale factor

The object is dilated by a scale factor of 2.5 with the center at the origin. What are the new coordinates?

notation for dilation

$(x, y) \rightarrow (kx, ky)$

where k is the scale factor.



A': $(0, 4) \rightarrow (2.5 \times 0, 2.5 \times 4) \rightarrow (0, 10)$

B': $(6, 0) \rightarrow (2.5 \times 6, 2.5 \times 0) \rightarrow (15, 0)$

C': $(0, 0) \rightarrow (0, 0)$

Dilations in the coordinate plane

The triangle ABC has been dilated around the origin. Below are the coordinates of the object and the image.

A	B	C
(3, 4)	(7, 4)	(6, 9)

A'	B'	C'
(20.25, 27)	(47.25, 27)	(40.5, 60.75)

What is the scale factor?

pick a point and write in notation for dilation: $A\,(3, 4) \rightarrow A'\,(k3, k4)$

set equal to coordinates of A': $k3 = 20.25$ and $k4 = 27$

divide by 3: $k = 6.75$ divide by 4: $k = 6.75$

scale factor = 6.75

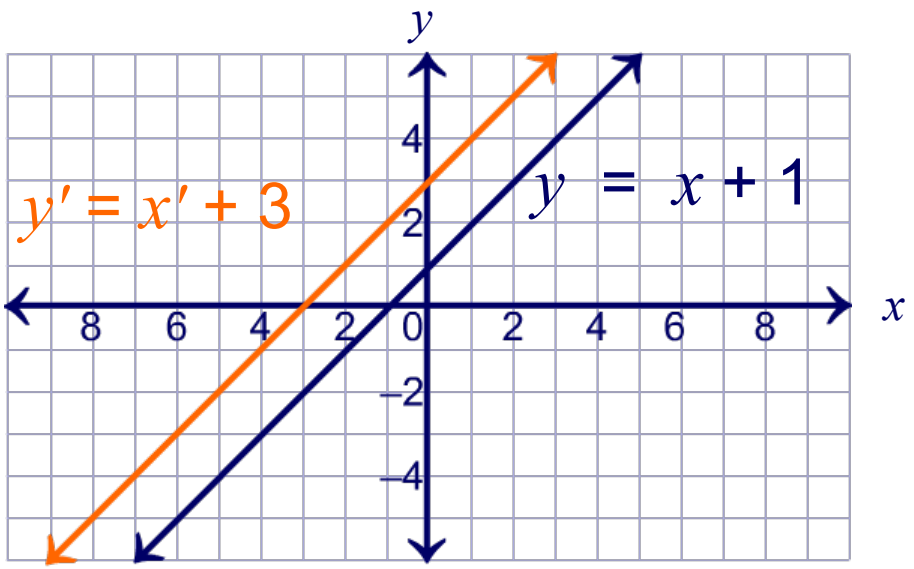
Check the answer using the other points.

Dilations in the coordinate plane

The graph of $y = x + 1$ is dilated by a scale factor of 3 with center at the origin. Using points from the original function and dilation notation, find the corresponding points on the dilation.

x	y
–2	–1
–1	0
0	1
1	2
2	3

x'	y'
–6	–3
–3	0
0	0
3	6
6	9



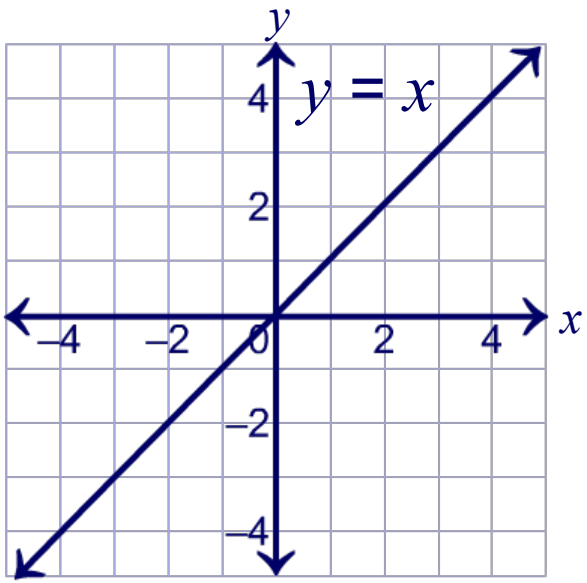
What is the equation of the dilated function?

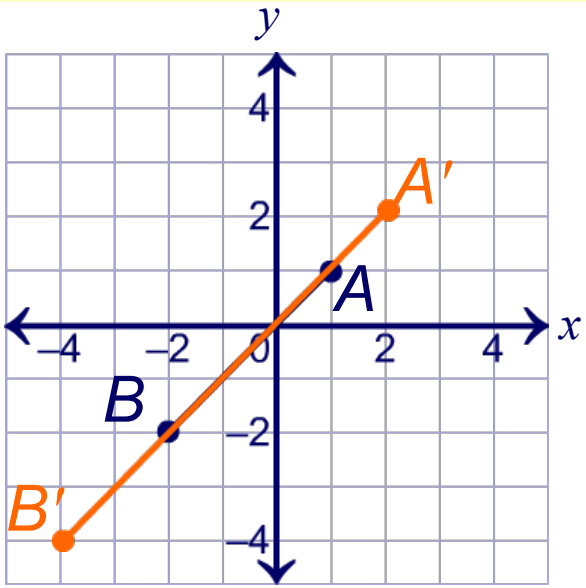
$y = x + 3$

Dilating lines

Describe the dilation of the graph $y = x$ by a scale factor of 3.

The dilation by any scale factor of a line that passes through the center of dilation will leave the line unchanged.

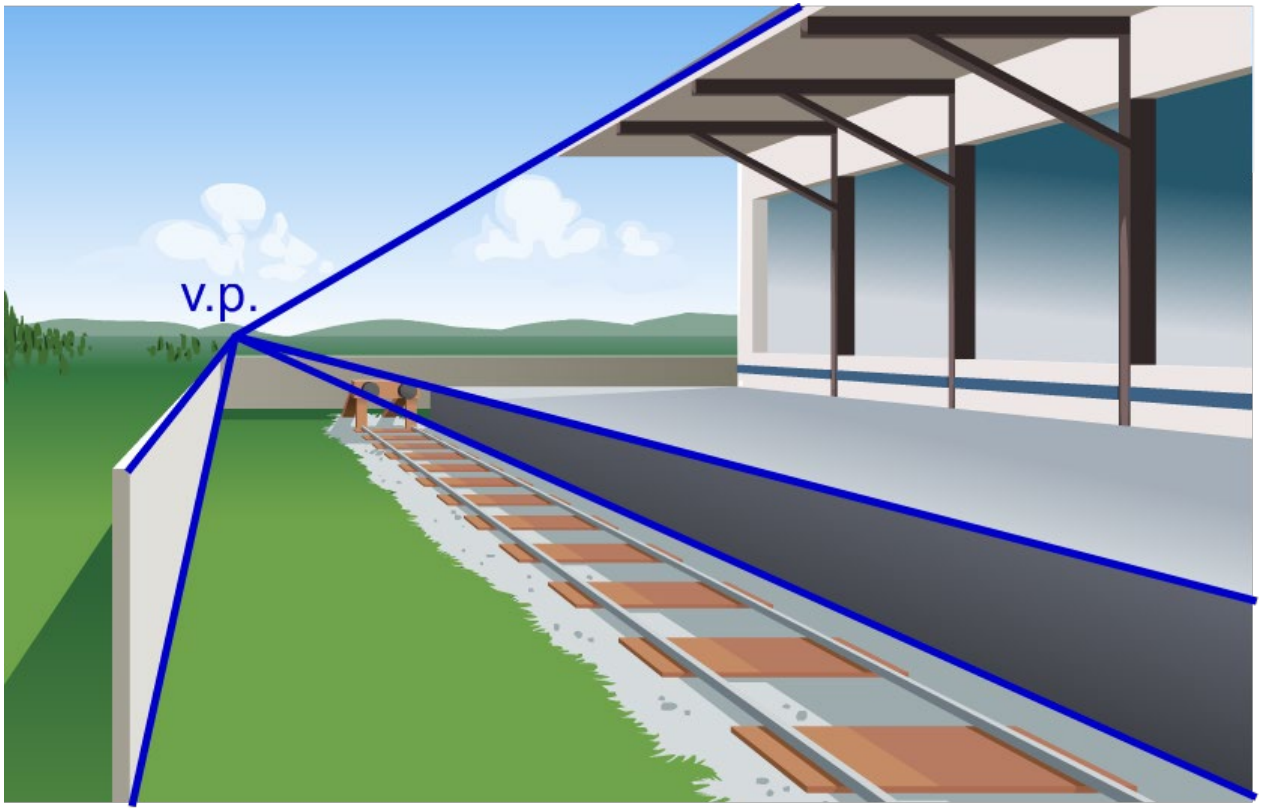




The dilation of a line segment passing through the center of dilation will increase or decrease in length by the scale factor.

Finding the center of dilation

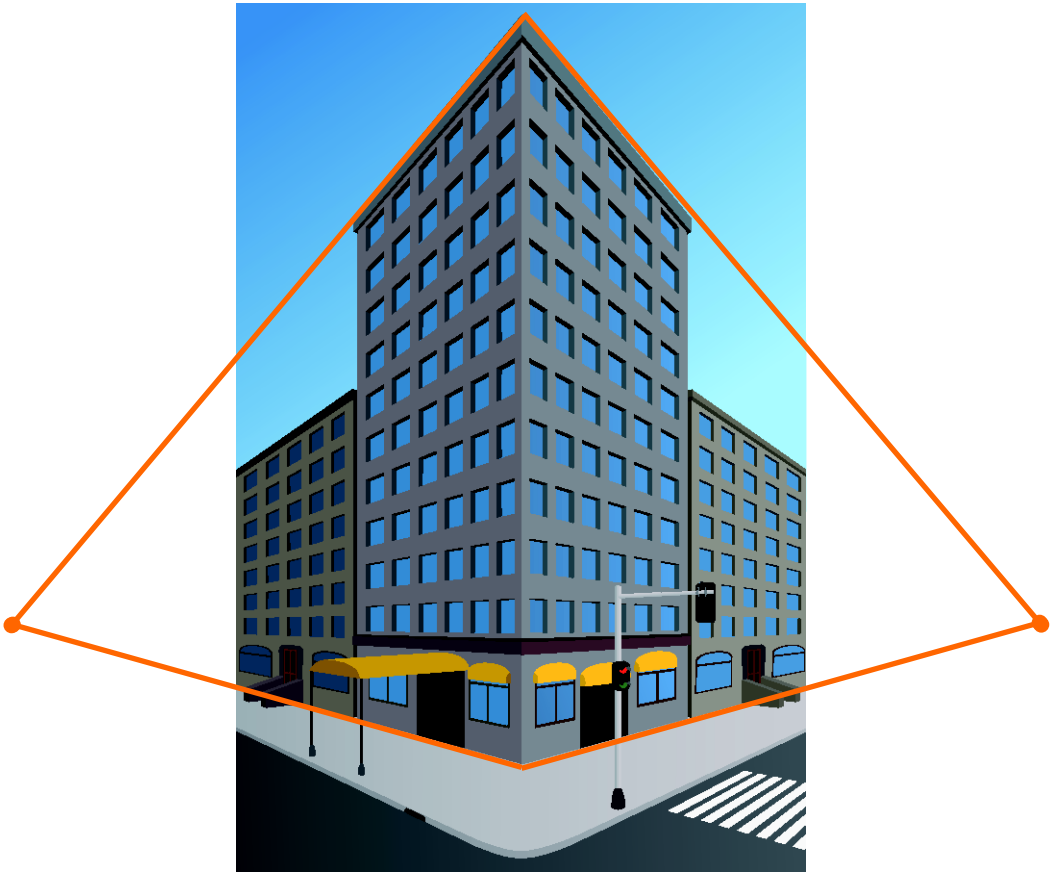
How are centers of dilation similar to vanishing points in perspective drawings?



Finding the center of dilation in math is the same as finding the vanishing point in art.

The center of dilation

How are centers of dilation different to vanishing points in perspective drawings?



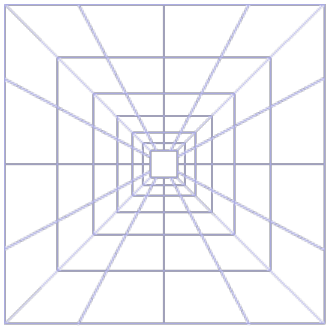
There can be multiple vanishing points in art.

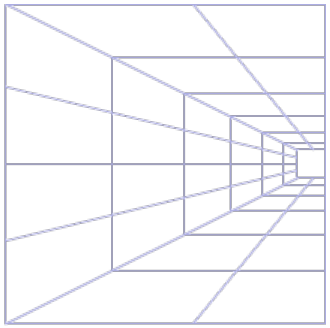
Vanishing point

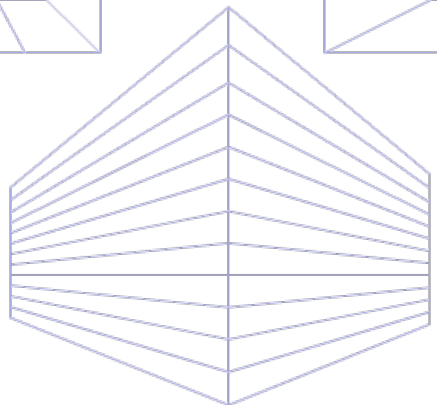
Artists use special grids when doing perspective paintings to help them get the right proportions.

Where is the vanishing point in each grid?

Which type of perspective grid would be best when painting this scene?







Scale plans

An interior designer draws a scale plan of a room with a scale of 14.
The model measures 35 cm by 22.5 cm with a height of 15cm.
Fill in the measurements to the nearest tenth on the dotted lines.

	room	scale plan
floor area	_____ m^2	_____ cm^2
volume	_____ m^3	_____ cm^3

	area	volume
scale factor	_____	_____