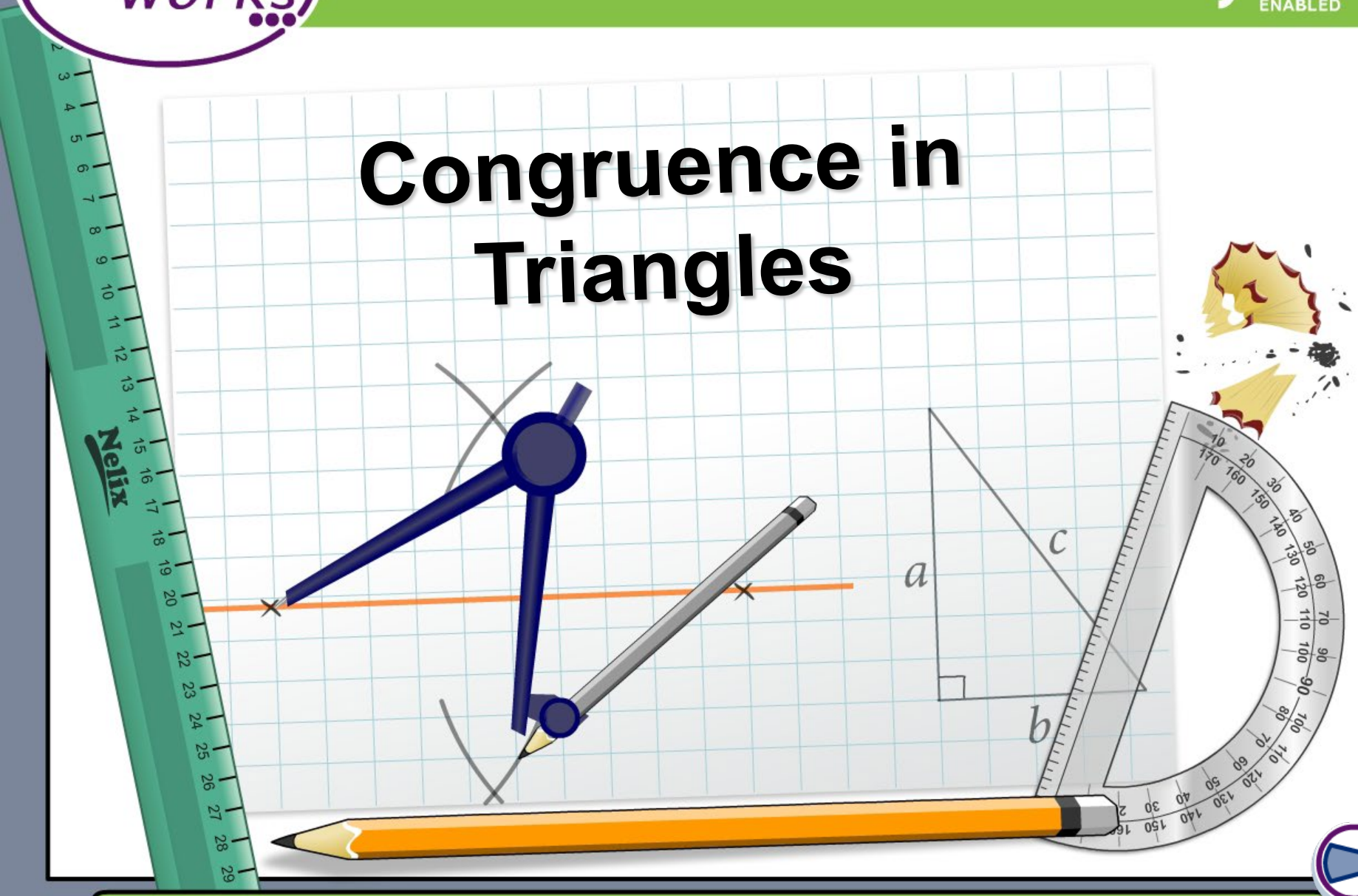
Congruence in Triangles



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Proving triangles congruent

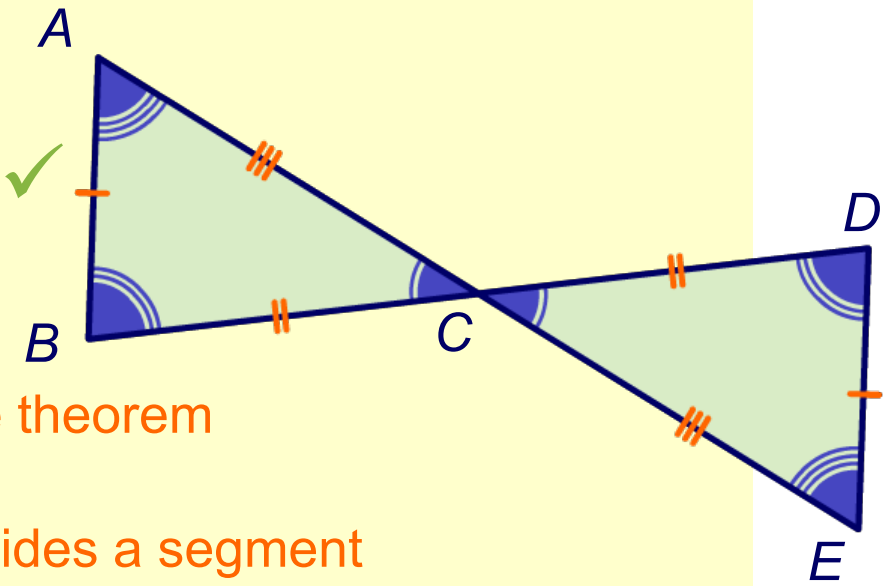
Triangles are congruent when **corresponding angles** and **corresponding sides** are congruent.

A triangle has three sides and three angles, so there are *six* relationships that must be true for two triangles to be congruent.

$\overline{AE}$ and $\overline{BD}$ bisect each other. Prove that $\triangle ABC \cong \triangle DEC$.

given from the figure:

$\angle A \cong \angle E$ ✓ $\angle B \cong \angle D$ ✓ $\overline{AB} \cong \overline{DE}$ ✓



need to show:

$\angle ACB \cong \angle DCE$ ✓ vertical angle theorem

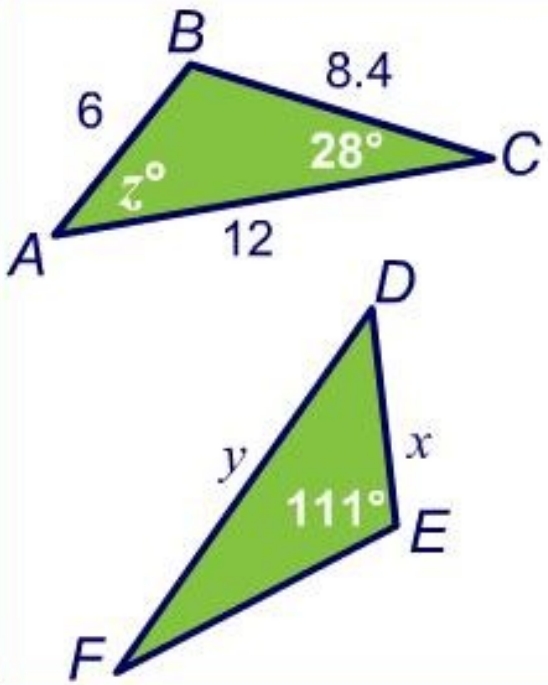
$\overline{BC} \cong \overline{CD}$ ✓

$\overline{AC} \cong \overline{CE}$ ✓ a bisector divides a segment into two congruent segments

Using congruence in triangles

Question: 1/4 $\triangle ABC$ and $\triangle DEF$ are congruent.

How can you determine the missing quantities x, y and z?

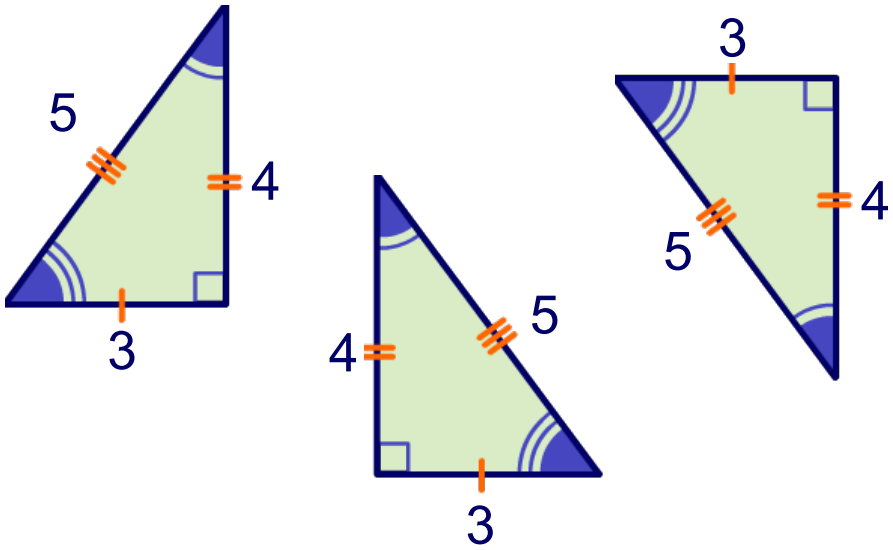


Press the "=" button to show the calculations step-by-step.

Triangle rigidity

Triangle **rigidity** gives a shortcut for proving two triangles congruent. It means that if the side lengths of a triangle are given, the triangle can have only one shape.

How many triangles can you construct with side of lengths 3 cm, 4 cm and 5 cm?

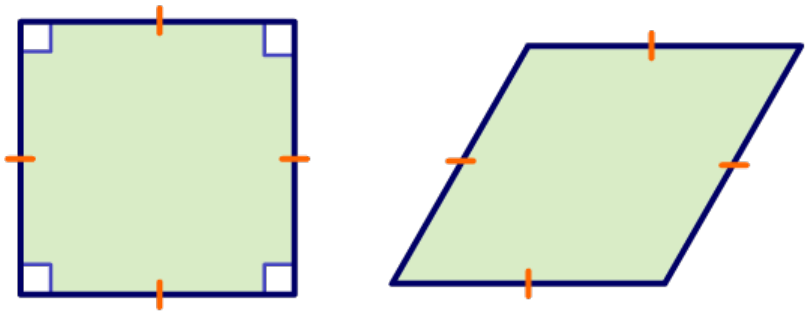


The triangles may appear different, but they are congruent.

If the sides of a triangle are fixed, so are the angles.

Rigidity in other polygons

How many quadrilaterals can you construct with four congruent sides?



Infinitely many different quadrilaterals can have four congruent sides.

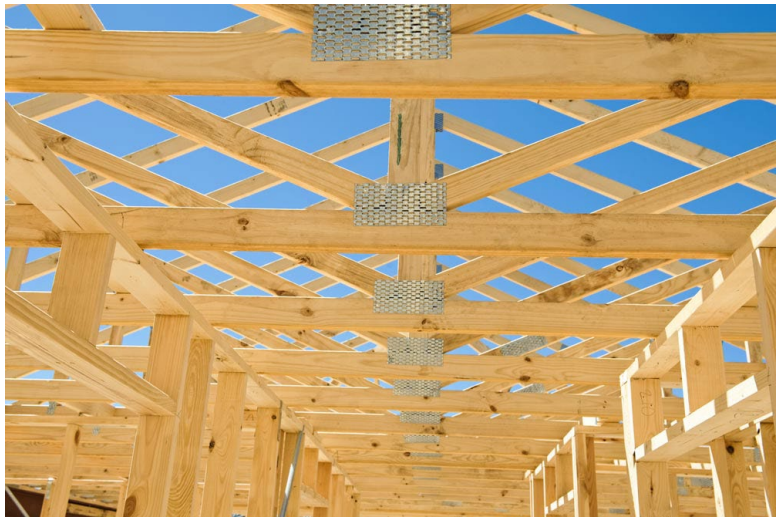
Different quadrilaterals can have congruent sides, but have different angles. Unlike triangles, the sides do not determine the angles.

This is because quadrilaterals are not rigid. In fact, triangles are the only rigid polygon.

Triangle congruence postulates

Because triangles are rigid, if the side lengths are fixed, the triangle can have only one shape.

This means that to prove that two triangles are congruent, you only need to show that corresponding sides are congruent.

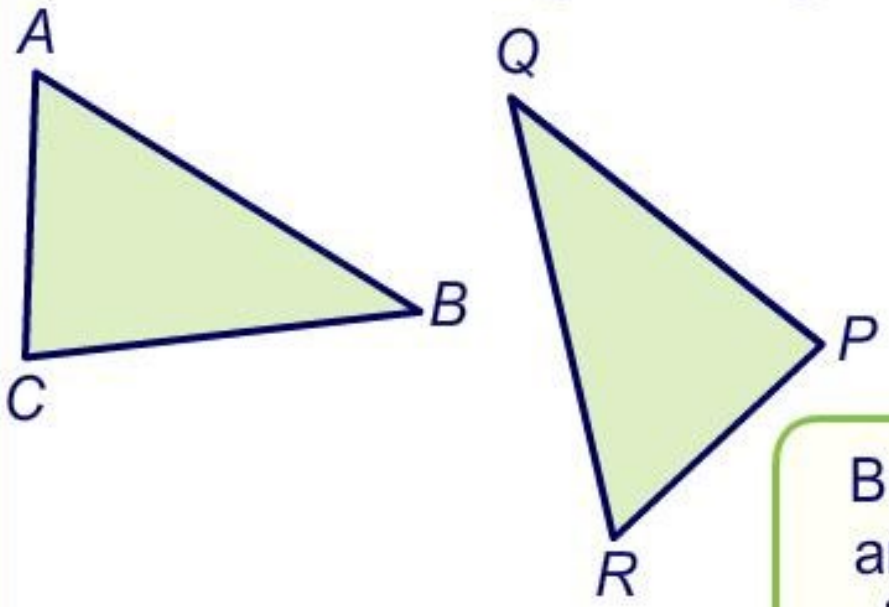


This is the ***side-side-side congruence postulate***.

There are several similar postulates involving angles that can be used to prove triangles congruent, such as:

- ***side-angle-side congruence postulate***
- ***angle-side-angle congruence postulate***
- ***angle-angle-side congruence postulate***.

Triangle congruence postulates



$\triangle ABC \cong \triangle RQP$?

- SSS congruence
- SAS congruence
- ASA congruence
- AAS congruence

Because triangles are rigid, there are several postulates that prove triangle congruence using only three criteria, rather than six side and angle criteria. Press on the postulates above to learn more.

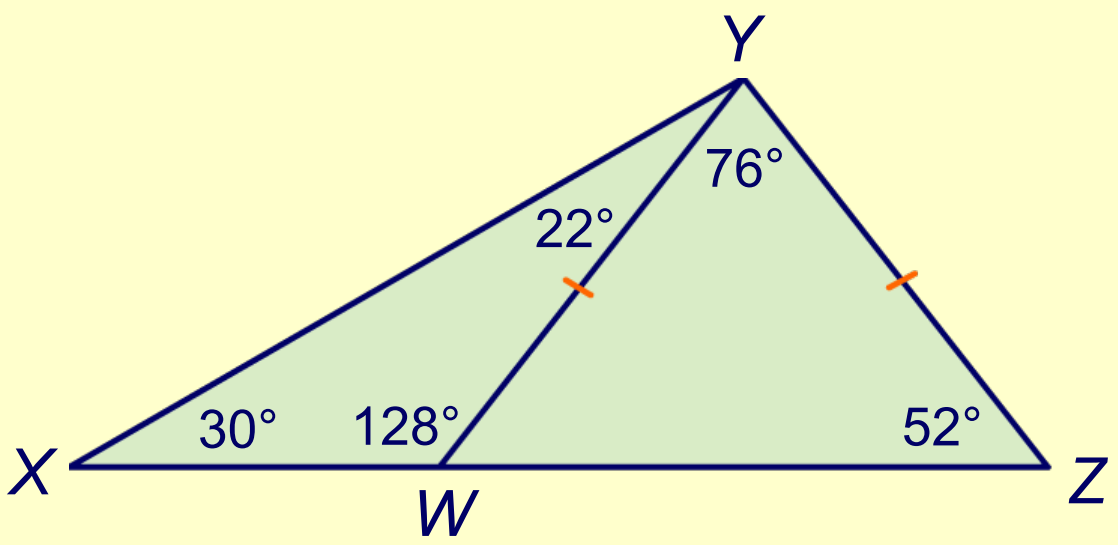
Side-side-angle?

Side-side-angle (SSA) cannot be used to prove triangles congruent. Find an example to illustrate this.

	$\triangle ZYX$		$\triangle WYX$
side:	$\overline{ZY}$	$\cong$	$\overline{WY}$
side:	$\overline{YX}$	$\cong$	$\overline{YX}$
angle:	$\angle X$	$\cong$	$\angle X$

but $\triangle ZYX \ncong \triangle WYX$ ✓

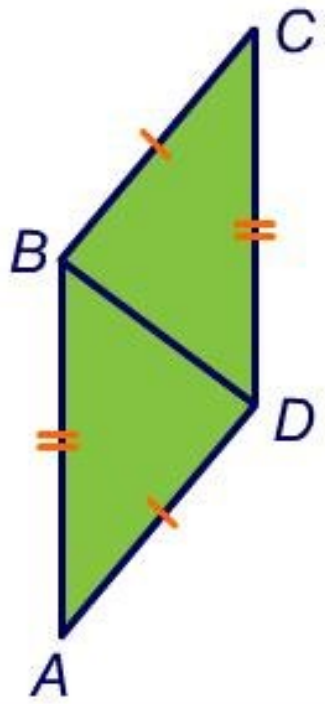
because, $\angle YZW \ncong \angle YWX$.



Even though two sides and a nonincluded angle are congruent, the triangles are not congruent.

Congruent triangles summary quiz

Question: 1/3 Prove that $\angle A$ and $\angle C$ are congruent.



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