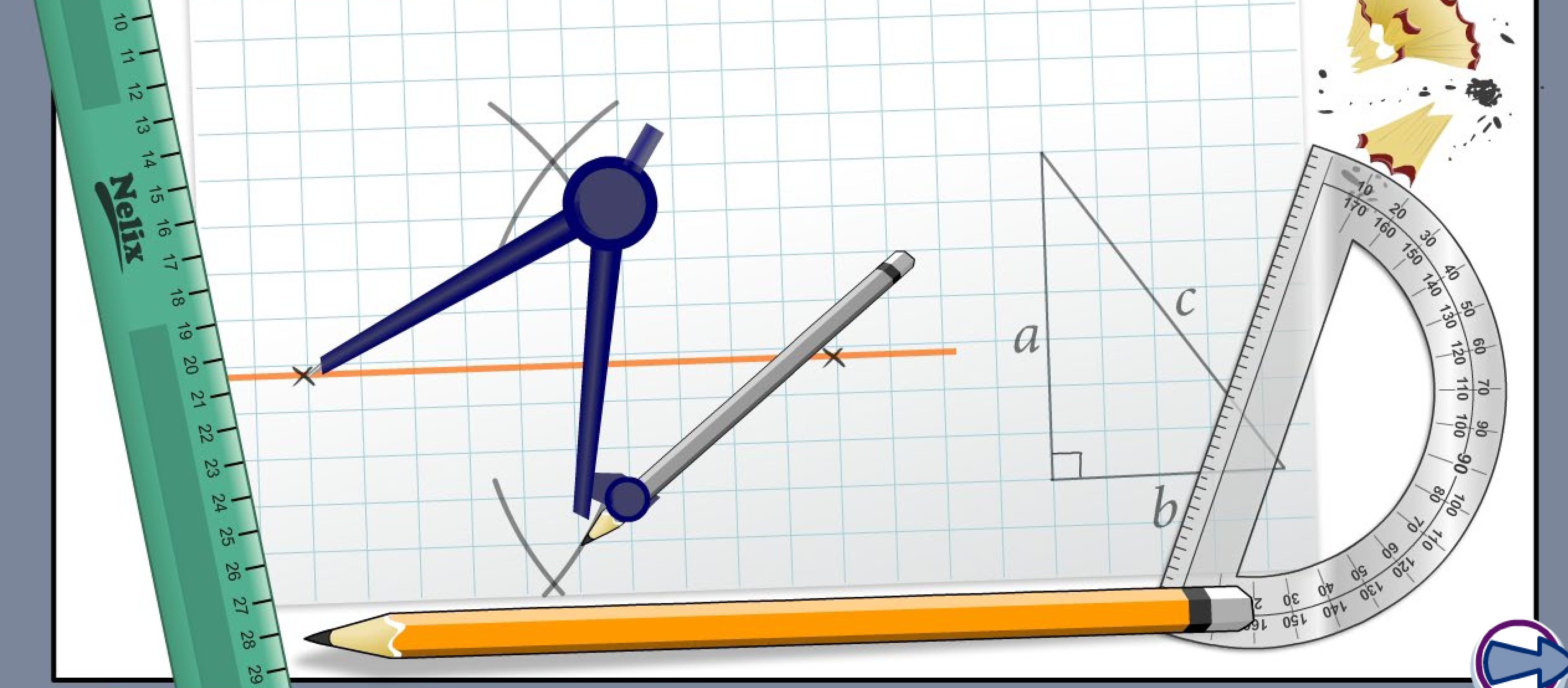
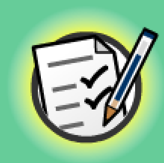
Conditional Probability

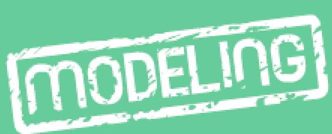


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

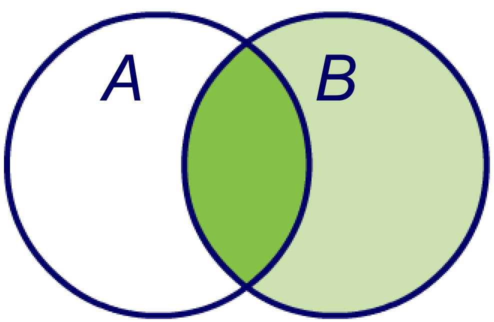
This icon indicates teacher's notes in the Notes field.

Conditional probability

Conditional probability is the probability that one event, A, will occur, given another event, B, occurs.

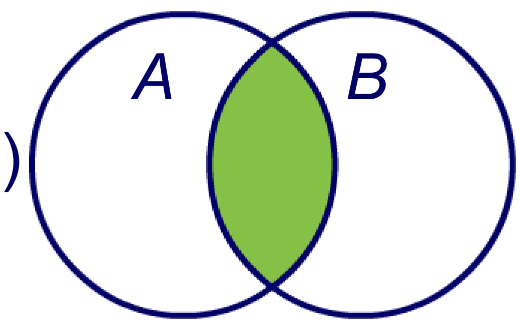
A conditional probability is denoted $P(A \mid B)$.

$$\boldsymbol{P(A \mid B)} = \frac{P(A \cap B)}{P(B)}$$

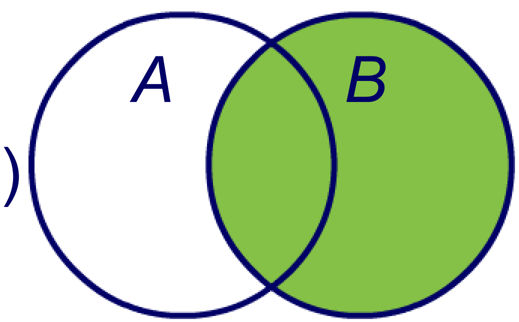


$P(A \mid B) =$

$P(A \cap B)$



$P(B)$



A school district does a survey of teacher age and gender:

age	male	female	total
under 29	9	37	**46**
30-39	45	60	**105**
40-49	32	78	**110**
50+	121	181	**302**
total	**207**	**356**	**563**

Find the probability to the nearest thousandth:

a) P(male) b) P(29 or under) c) P(male and 30 or over)

a) P(male) = 207 ÷ 563 = **0.368**

b) P(under 29) = 46 ÷ 563 = **0.082**

c) P(male and over 30) = (45 + 32 + 121) ÷ 563 = **0.352**

age	male	female	total
under 29	9	37	**46**
30-39	45	60	**105**
40-49	32	78	**110**
50+	121	181	**302**
total	**207**	**356**	**563**

If a teacher is female, find the probability that she is under 29, P(under 29 | female).

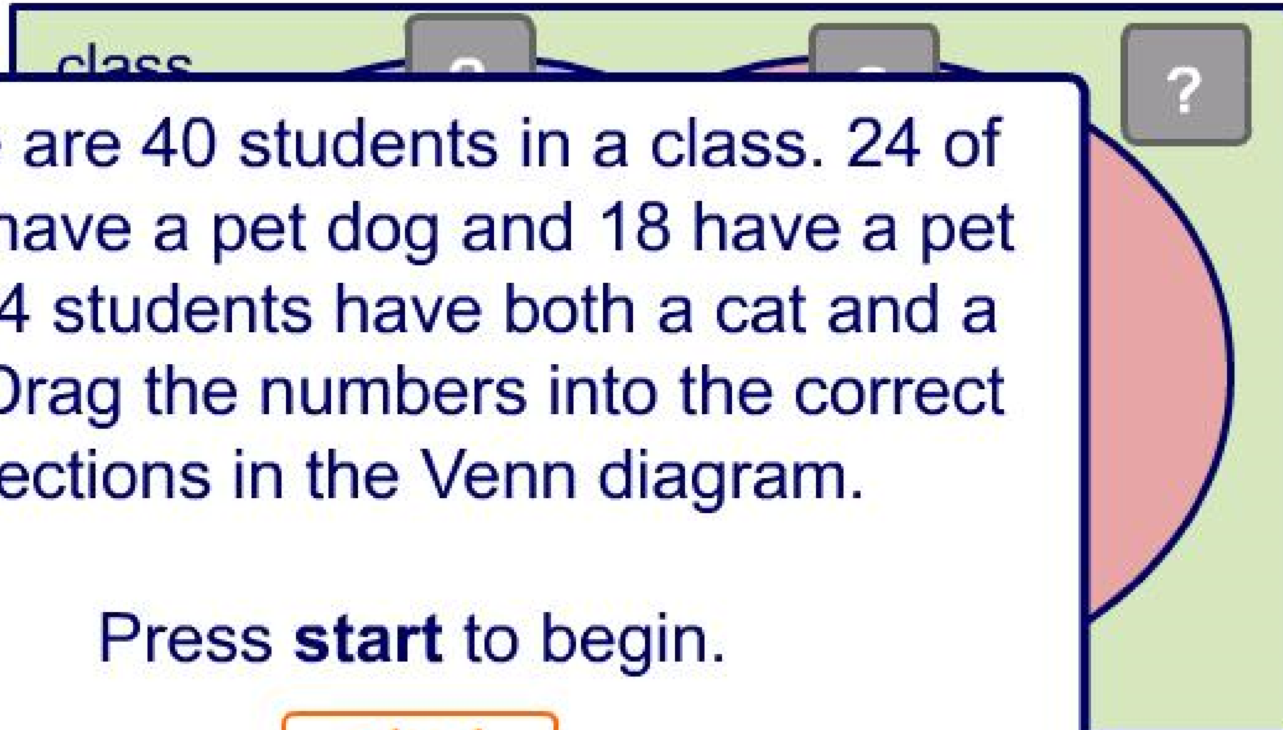
$P(\text{under } 29 \cap \text{female}) = 37 \div 563 = 0.066$

$P(\text{female}) = 356 \div 563 = 0.632$

$P(\text{under } 29 \cap \text{female}) \div P(\text{female}) = 0.066 \div 0.632$ $\mathbf{= 0.104}$

Is it more likely for a teacher to be under 29 if they are male or female?

Venn diagram

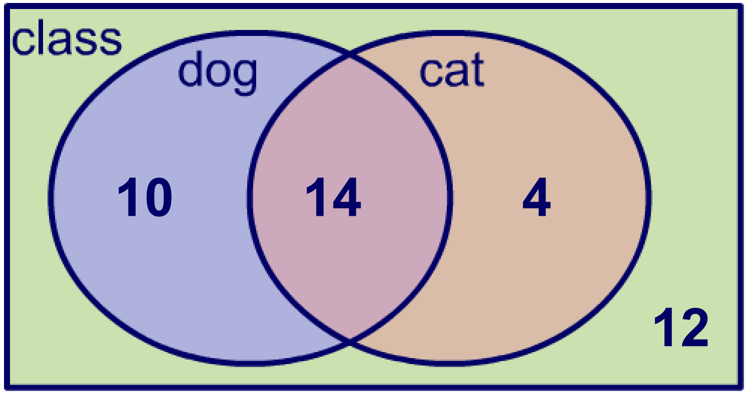


There are 40 students in a class. 24 of them have a pet dog and 18 have a pet cat. 14 students have both a cat and a dog. Drag the numbers into the correct sections in the Venn diagram.

Press **start** to begin.

start

Given that I choose someone in the class who has a cat, what is the probability that they also have a dog?



$$P(\text{dog} \mid \text{cat}) = P(\text{dog} \cap \text{cat}) \div P(\text{cat}) = \frac{(14/40)}{(18/40)} = \frac{0.35}{0.45} = \mathbf{0.778}$$

Given that I select someone without a dog, what is the probability that they have a cat?

$$P(\text{cat} \mid \text{no dog}) = P(\text{dog}' \cap \text{cat}) \div P(\text{dog}') = \frac{(4/40)}{(16/40)} = \frac{0.10}{0.40} = \mathbf{0.25}$$

Use the percentages from the student survey given to fill in the percentages in the Venn diagram. Then use the diagram to answer the following questions.

Press **start** to begin.

start

Dependent events

There are 20 socks in a drawer. 14 of them are blue and the rest are green.

What is the probability of randomly picking a blue sock?

$$P(\text{blue sock}) = \frac{14}{20} = \mathbf{\frac{7}{10}}$$

What is the probability of picking a second blue sock, $P(2^{nd}\text{ blue} \mid 1^{st}\text{ blue})$?

There are now 19 socks left: 13 blue, 6 green. $P(2^{nd}\text{ blue} \mid 1^{st}\text{ blue}) = \mathbf{\frac{13}{19}}$

Or use the formula:

$$P(2^{nd}\text{ blue} \mid 1^{st}\text{ blue}) = \frac{P(2^{nd}\text{ blue} \cap 1^{st}\text{ blue})}{P(1^{st}\text{ blue})} = \frac{(13/19) \times \cancel{(14/20)}}{\cancel{(14/20)}} = \mathbf{\frac{13}{19}}$$

Dependent events

A student has given the following answer to the question "what is the probability of the first two socks drawn being blue?" What is wrong with his argument?

$$P(2\text{ blue}) = P(1^{st}\text{ blue}) \times P(2^{nd}\text{ blue}) = \frac{14}{20} \times \frac{13}{20} = \frac{182}{400} = \frac{91}{200}$$

There are not twenty socks left to choose from on the second draw. After the first draw there are only 19 socks left in the drawer.

What is the correct answer?

$$P(2\text{ blue}) = P(1^{st}\text{ blue}) \times P(2^{nd}\text{ blue} \mid 1^{st}\text{ blue})$$

$$= \frac{14}{20} \times \frac{13}{19} = \frac{182}{380} = \mathbf{\frac{91}{190}}$$

What color socks will Aaron wear today?
Will it be a pair?
He reaches into a drawer of blue and red socks and chooses two at random. Vary the contents of the drawer and investigate the probabilities in the tree diagram.
Press **start** to begin.

start

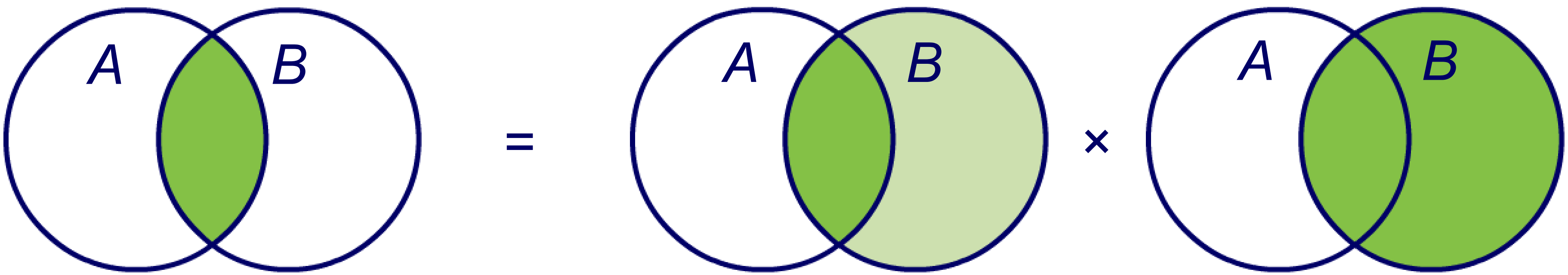
Total number of socks = 8

General multiplication law

When events depend on each other, to find the probability of combined events are not just multiplied together.

The ***general multiplication law*** gives the relationship between two dependent events, A and B.

$$P(A \cap B) = P(A) \times P(B \mid A) = P(B) \times P(A \mid B)$$



$$P(A \cap B) = P(A \mid B) \times P(B)$$

If A and B are independent events, the general multiplication law reduces to simple multiplication: $P(A \cap B) = P(A) \times P(B)$.

When cards are drawn and not replaced before another draw, the events are **dependent**. Suppose you pick two cards at random from a full deck without replacement.

What is the probability of drawing exactly one king?

Press **start** to begin.

start

Dependent and independent events

Question 1/4

A woman has three nickels and two dimes in her pocket. She draws one coin and then another one without replacing the first. Are these two events dependent or independent?

dependent

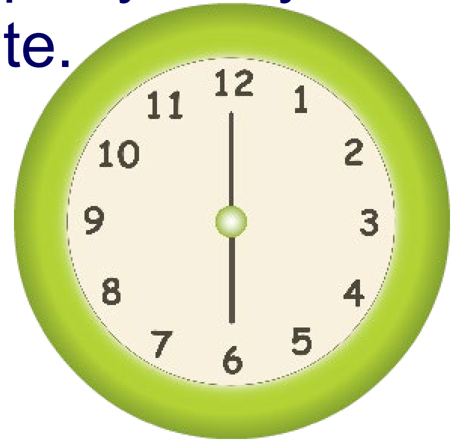
independent

Tree diagrams for dependent events

Marie either drives to work or takes the bus. The probability of her driving depends when she wakes up.

- If she wakes up early, the probability she drives is 0.2
- If she wakes up on time, the probability she drives is 0.4.
- If she wakes up late, the probability she drives is 0.8.

Marie wakes up on time 60% of the time. It is equally likely that she wakes up early or that she wakes up late.



- P(on time) = 0.6
- P(early) = (1 – 0.6) ÷ 2 = 0.2
- P(late) = P(early) = 0.2

What is the probability that she takes the bus to work?

Tree diagrams for dependent events

Press the clocks, bus or car to show the probability of the event. Press the green boxes to show the probability of the combined events

What is the probability that Marie will take the bus to work?

Press **start** to begin.

start