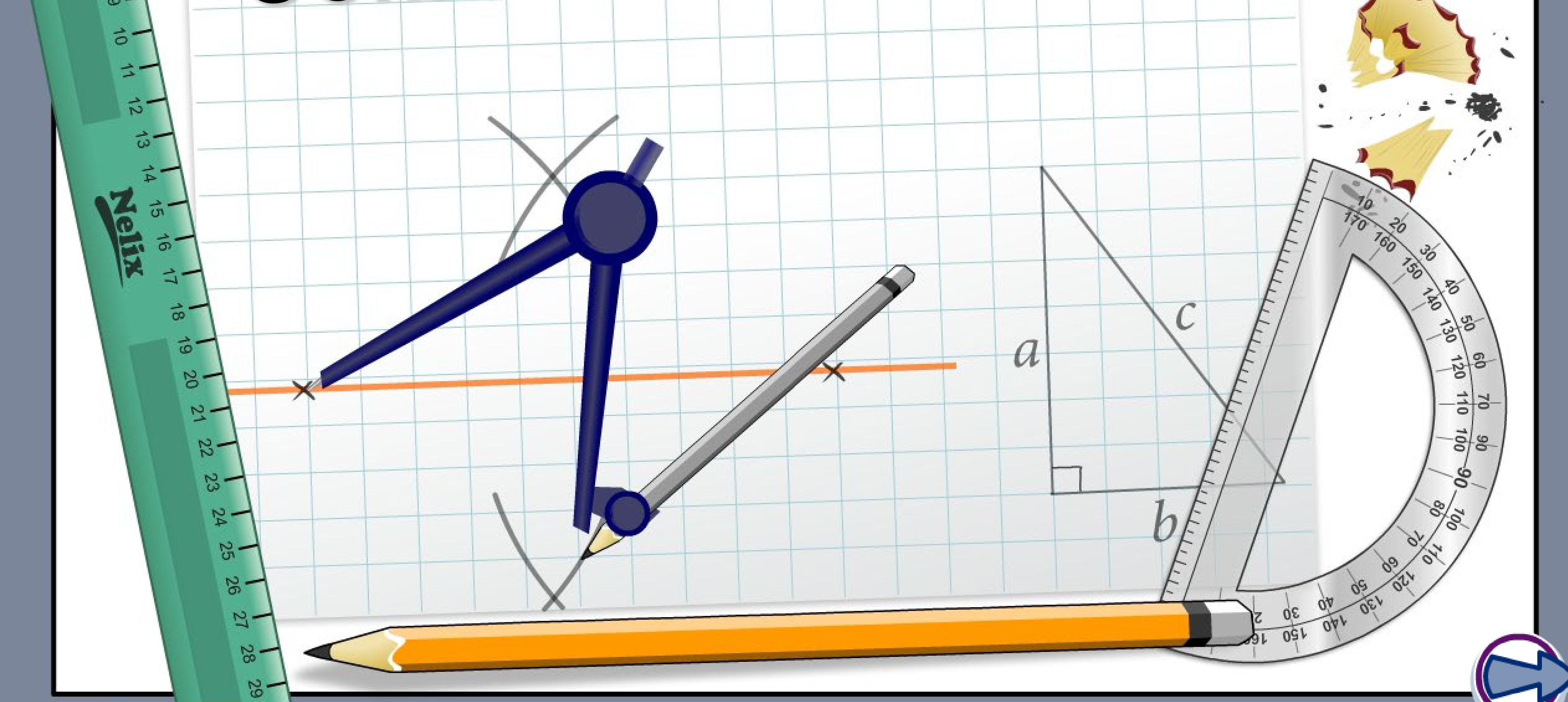
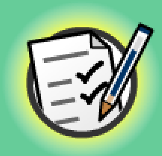
Combined Probability



Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Auto move on

Combinations

Complete the table to find the probability of each horse winning.

The # column refers to how many successful combinations there are for each horse.

horse	combinations	#	probability
1	no combinations	0	0
2	1,1	1	$\frac{1}{36}$
3	1,2 2,1	2	$\frac{2}{36} = \frac{1}{18}$
4	1,3 3,1 2,2	3	$\frac{3}{36} = \frac{1}{12}$
5	1,4 4,1 2,3 3,2	4	$\frac{4}{36} = \frac{1}{9}$
6	1,5 5,1 2,4 4,2 3,3	5	$\frac{5}{36}$
7	1,6 6,1 2,5 5,2 3,4 4,3	6	$\frac{6}{36} = \frac{1}{6}$
8	2,6 6,2 3,5 5,3 4,4	5	$\frac{5}{36}$
9	3,6 6,3 4,5 5,4	4	$\frac{4}{36} = \frac{1}{9}$
10	4,6 6,4 5,5	3	$\frac{3}{36} = \frac{1}{12}$
11	5,6 6,5	2	$\frac{2}{36} = \frac{1}{18}$
12	6,6	1	$\frac{1}{36}$
total	n/a	36	$\frac{36}{36} = 1$

Sample space diagrams

Another way of displaying all the outcomes from throwing two dice is a **sample space diagram**. The grid squares show the sum of the dice.

second die

first die

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Fill in the rest of the cells in the table and color in all the twos one color, the threes another color, and so on.

What patterns do you notice in the table?

Horse race questions

Use either the table of combinations or the sample space diagram to answer these questions.

Which horse has the best chance of winning?

Horse seven has the best chance of winning. There are more outcomes that sum to seven than any other outcome so this horse has the highest probability.

Are there any horses that cannot win?

Horse 1 can't win because no two numbers on the dice will sum to 1.

How would the game change if you used ten-sided dice?

The denominator of the probability will be higher because there are more possible outcomes. Horses with higher numbers, such as 8, 9, and 10, would have more possible outcomes and a higher probability.

Use the table find:

a) *P*(total is a square number)
b) *P*(an even total)
c) *P*(a total less than 7)
d) *P*(number on first die < number on second die)

second die

first die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a) $P(\text{square number}) = \mathbf{\frac{7}{36}}$

b) $P(\text{even total}) = \frac{18}{36} = \mathbf{\frac{1}{2}}$

c) $P(\text{total} < 7) = \frac{15}{36} = \mathbf{\frac{5}{12}}$

d) $P(\text{first die} < \text{second die}) = \frac{15}{36} = \mathbf{\frac{5}{12}}$

Calculating the number of outcomes

die 1 6

die 2 6

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Make a sample space diagram for the outcomes of the first roll of this game, which uses two four-sided dice.

Starting at "GO", what is the probability of:

a) landing on Circle Station?

b) rolling a double?

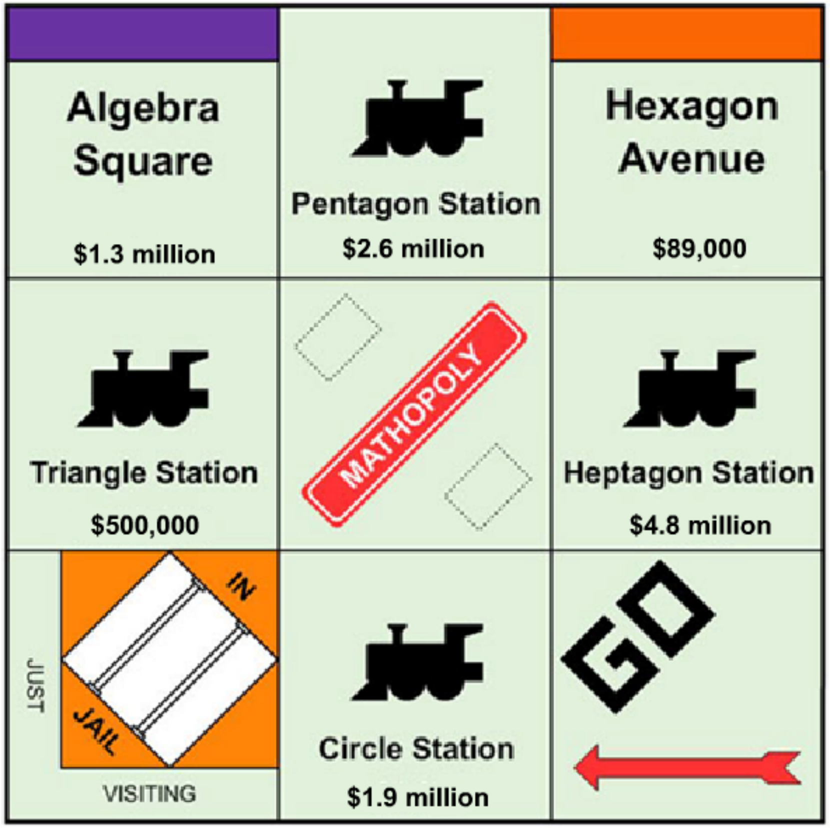
c) visiting jail?

a) P(circle station) = P(total on dice = 1)

$$= \frac{0}{16} = \mathbf{0}$$

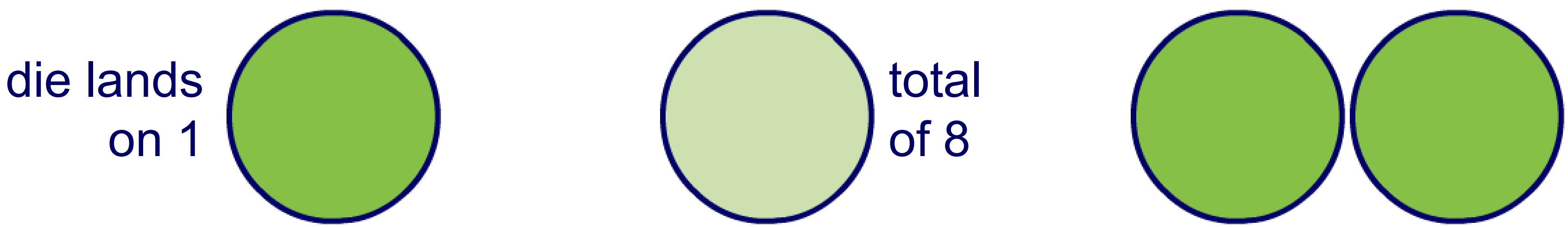
b) P(doubles) $= \frac{4}{16} = \mathbf{\frac{1}{4}}$

c) P(visiting jail) $= \mathbf{\frac{1}{16}}$



Mutually exclusive events

When throwing two six-sided dice, if one die lands on 1, it is impossible to get a total score of 8. Rolling 1 and getting a total score of 8 are **mutually exclusive**.



$P(1) + P(\text{total of } 8) = P(1 \cup \text{total of } 8)$

When events are mutually exclusive, the probability of one or the other occurring can be found by adding their probabilities.

Coins and dice

A coin and a ten-sided die are thrown and the outcomes are recorded in the two-way table below.

ten-sided die

coin	1	2	3	4	5	6	7	8	9	10
H	1,H	2,H	3,H	4,H	5,H	6,H	7,H	8,H	9,H	10,H
T	1,T	2,T	3,T	4,T	5,T	6,T	7,T	8,T	9,T	10,T

How many outcomes are there? What is the probability:

a) P(heads ∩ even)

b) P(tails ∩ square number)

a) $P(\text{heads} \cap \text{even}) = \frac{5}{20} = \frac{1}{4}$

b) $P(\text{tails} \cap \text{square}) = \frac{3}{20}$

If events are mutually exclusive, the probability of one event or the other occurring can be found by adding their probabilities:

$$P(A \cup B) = P(A) + P(B)$$

What is the probability of rolling a sum of 3 or 4?

$$P(3 \cup 4) = P(3) + P(4) = \frac{2}{36} + \frac{3}{36}$$

$$= \mathbf{\frac{5}{36}}$$

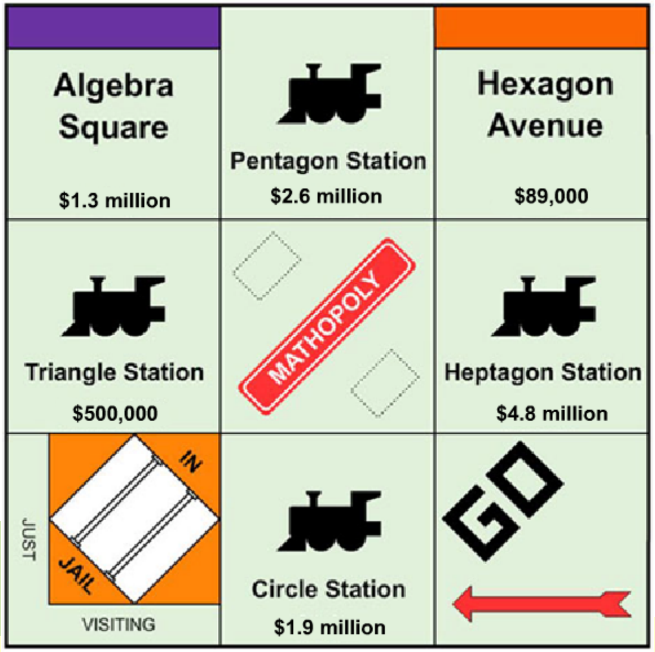
What is the probability of rolling a sum of 5 or 6?

$$P(5 \text{ or } 6) = P(5) + P(6) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36} = \mathbf{\frac{1}{4}}$$

second die

first die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Lamorna wants to know the probability of landing on Triangle Station or Pentagon Station using two four-sided dice. She cannot possibly land on both stations at the same time, so they are mutually exclusive events.



The probability of landing on either station is the same as the probability of landing on Triangle Station plus the probability of landing on Pentagon Station.

Find P(triangle ∪ pentagon).

$$P(\text{triangle} \cup \text{pentagon}) = P(\text{triangle}) + P(\text{pentagon})$$

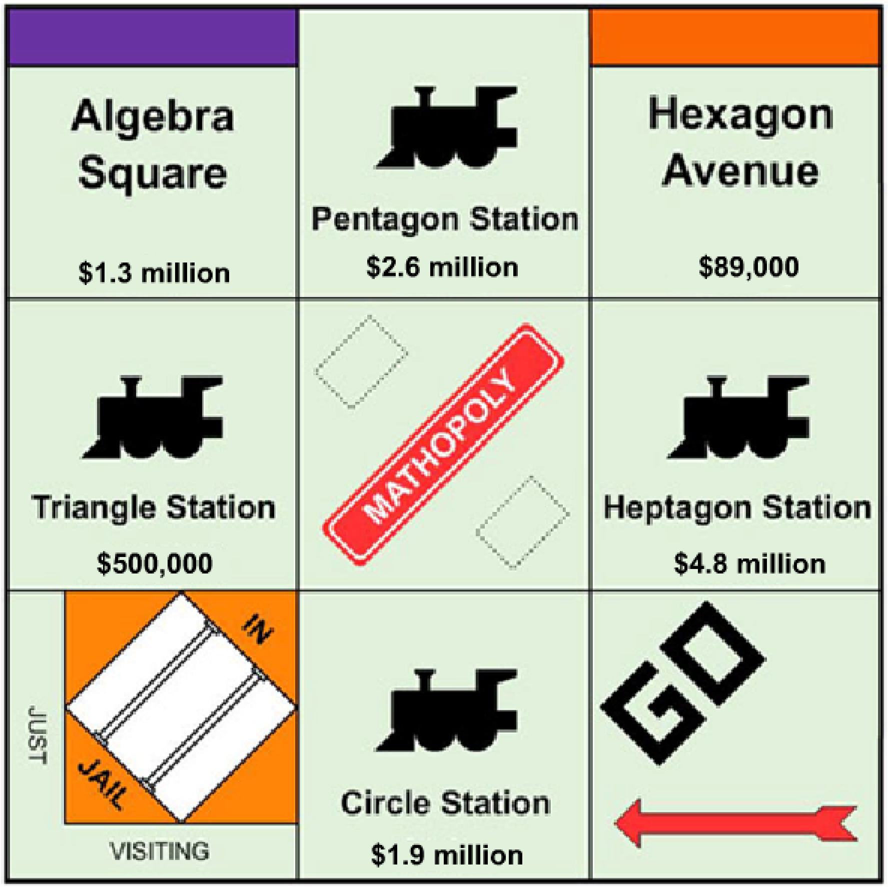
$$= \frac{2}{16} + \frac{4}{16} = \frac{6}{16} = \mathbf{\frac{3}{8}}$$

Use two four-sided dice to find the probability of these events in Mathopoly on the first roll.

Starting at "GO" what is the probability of each event:

a) landing on a space over $1m or visiting jail?

b) landing on a space under $1m or landing on GO?



a) P(space over \$1m ∪ jail) =

$$\frac{3}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16} = \frac{10}{16} = \mathbf{\frac{5}{8}}$$

b) P(space under \$1m ∪ GO) =

$$\frac{2}{16} + \frac{3}{16} + \frac{1}{16} = \frac{6}{16} = \mathbf{\frac{3}{8}}$$

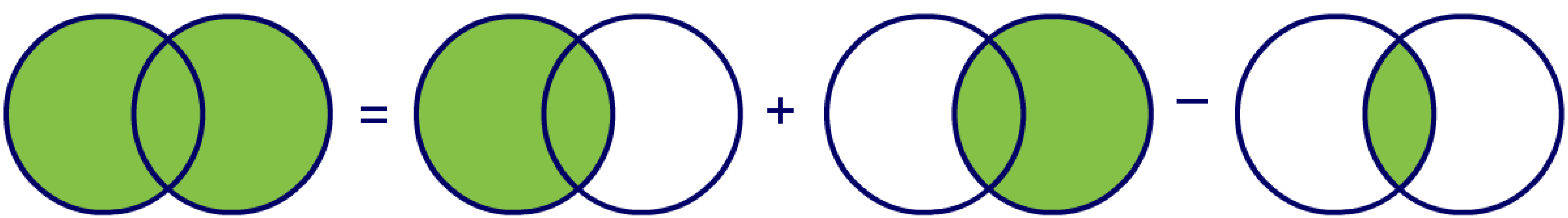
General addition law

When events are not mutually exclusive, to find the combined probability, the intersection must be subtracted to avoid counting the overlapping outcomes twice.

The ***general addition law*** gives the relationship between two non-mutually exclusive events, A and B.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cup B)$ = $P(A)$ + $P(B)$ − $P(A \cap B)$



Mutually exclusive or not?

You are playing a game with two fair six-sided dice. Decide whether each set of events is mutually exclusive or not. If events are not mutually exclusive, give examples where both occur.

Press **play** to start.

start

Shakil's experiment

Shakil is trying to work out the probability of getting an even number or a prime number from the sum of two dice.

Here is Shakil's answer, explain why it is wrong.

$P(\text{even}) + P(\text{prime}) = \frac{18}{36} + \frac{15}{36} = \mathbf{\frac{33}{36}}$

The number 2 is even *and* prime. It has been counted twice.

What is the correct answer?

$P(\text{even} \cup \text{prime}) =$

$P(\text{even}) + P(\text{prime}) - P(\text{even} \cap \text{prime})$

$= \frac{18}{36} + \frac{15}{36} - \frac{1}{36} = \frac{32}{36} = \mathbf{\frac{8}{9}}$

second die; first die

first die \ second die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Exclusive or not exclusive events

Are P(total that is a multiple of 3) and P(total that is an even number) exclusive or not exclusive? Find P(multiple of 3 ∪ even number).

second die

first die

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

These are not exclusive events.

P(multiple of 3 ∪ even number) =

$$\frac{12}{36} + \frac{18}{36} - \frac{6}{36} = \frac{24}{36} = \mathbf{\frac{2}{3}}$$

Find P(total that is odd ∪ one die showing a 4).

P(odd number ∪ 4 on one die) =

$$\frac{18}{36} + \frac{11}{36} - \frac{6}{36} = \mathbf{\frac{23}{36}}$$

Complete the formula

A and *B* are mutually exclusive events. Complete the formula.

? = $P(A)$ + ?

A and *B* are not mutually exclusive. Complete the formula.

$P(A \cup B)$ = ? + $P(B)$ – ?

$P(A \cup B)$ | $P(A)$ | $P(A \cap B)$ | $P(B)$