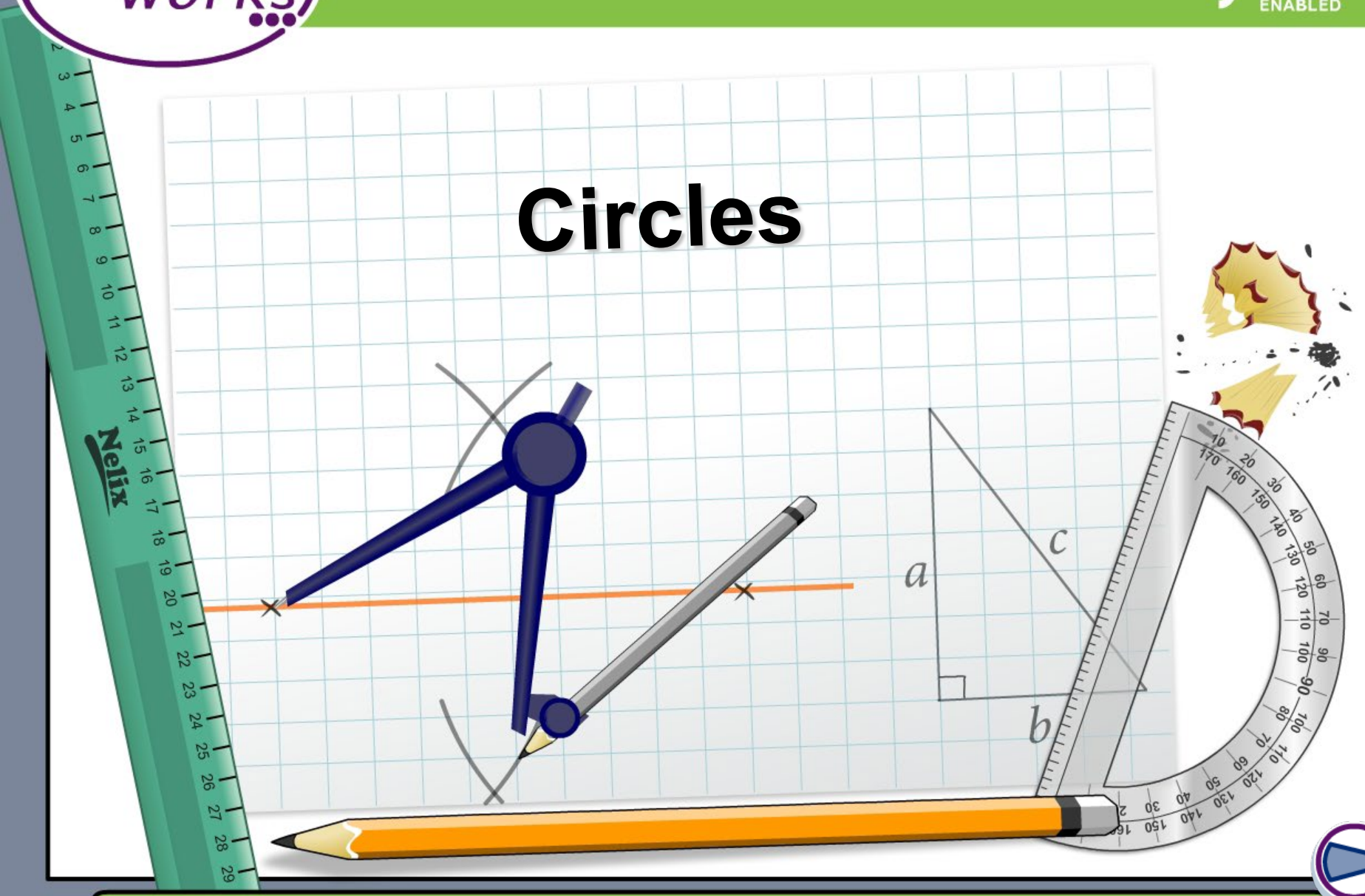
Circles



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

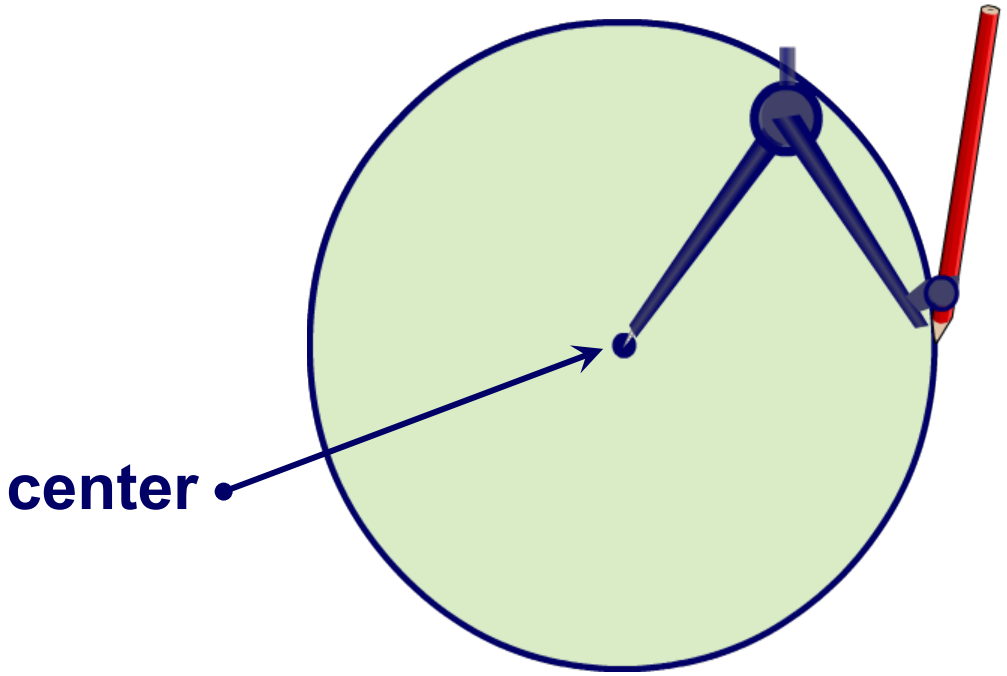
8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Definition of a circle

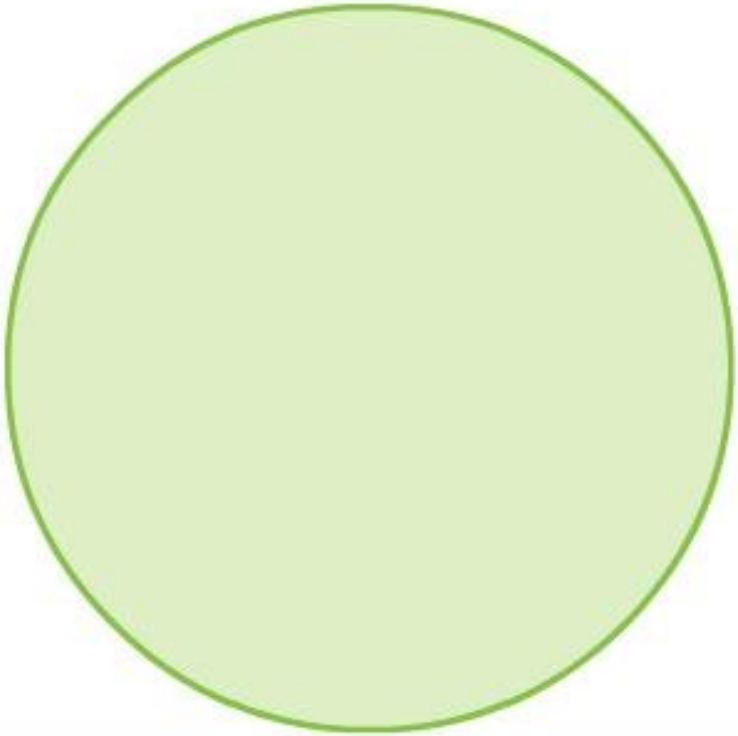
A circle is a set of all points in a plane that are equidistant from a given point. This given point is the center of the circle.



This is why a compass is used to draw a circle – the pencil is always held at an equal distance from the center of the circle.

Parts of circle

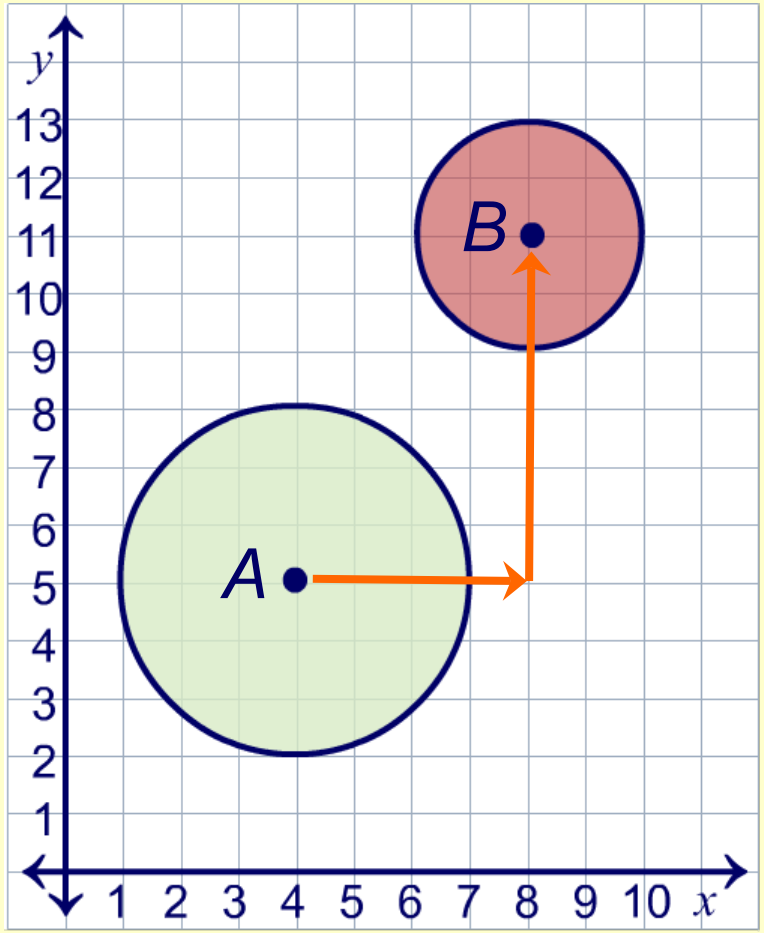
Circles



- center and circumference
- interior and exterior
- radius and diameter
- chords and secants
- tangents

Press the yellow buttons to reveal an example and more information about each of the terms.

Are circles A and B similar? Prove your answer.



Circle A can be mapped to circle B by:

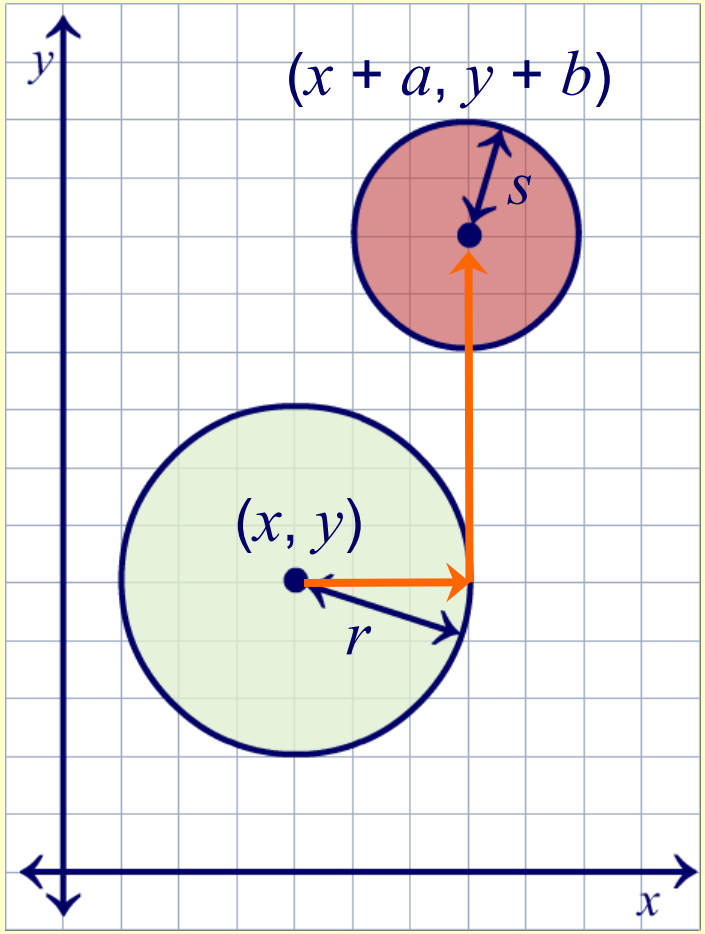
- a translation $(x, y) \rightarrow (x + 4, y + 6)$
- and a dilation with center (8, 11) and scale factor ⅔.

To prove that two shapes are similar, it must be shown that they are related by similarity transformations.

So circles A and B are similar.

Circles and similarity proof

Are all circles similar? Prove your answer.



Any circle with center (x, y) and radius r can be mapped to another circle with center $(x + a, y + b)$ and radius s by:

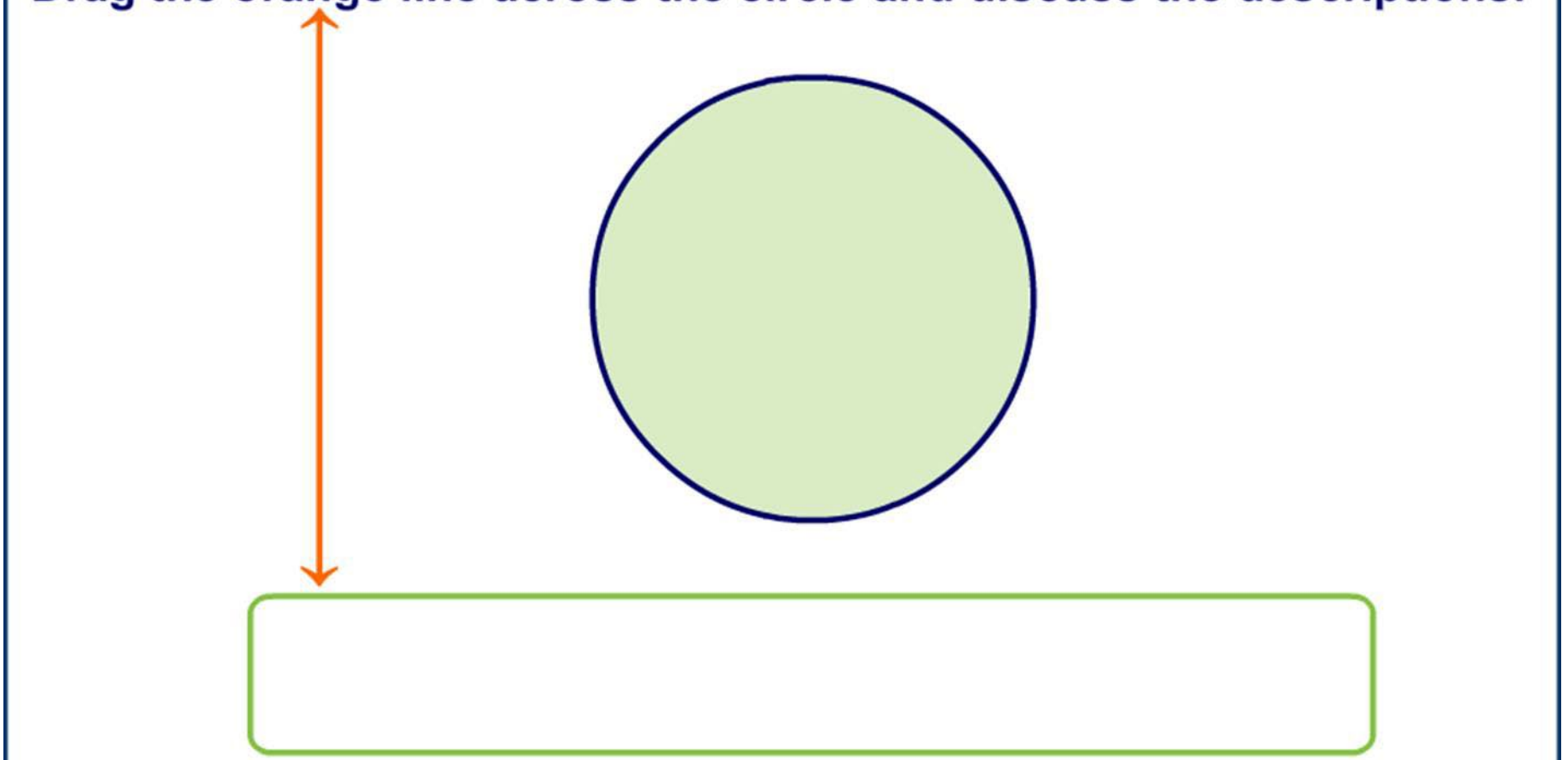
- a translation $(x, y) \rightarrow (x + a, y + b)$
- and a dilation with center $(x + a, y + b)$ and scale factor $\frac{s}{r}$.

The new circle is a dilation and a translation of the original circle. Dilations and translations are both similarity transformations.

So any circle is similar to any other circle.

Lines in a circle

Drag the orange line across the circle and discuss the descriptions.

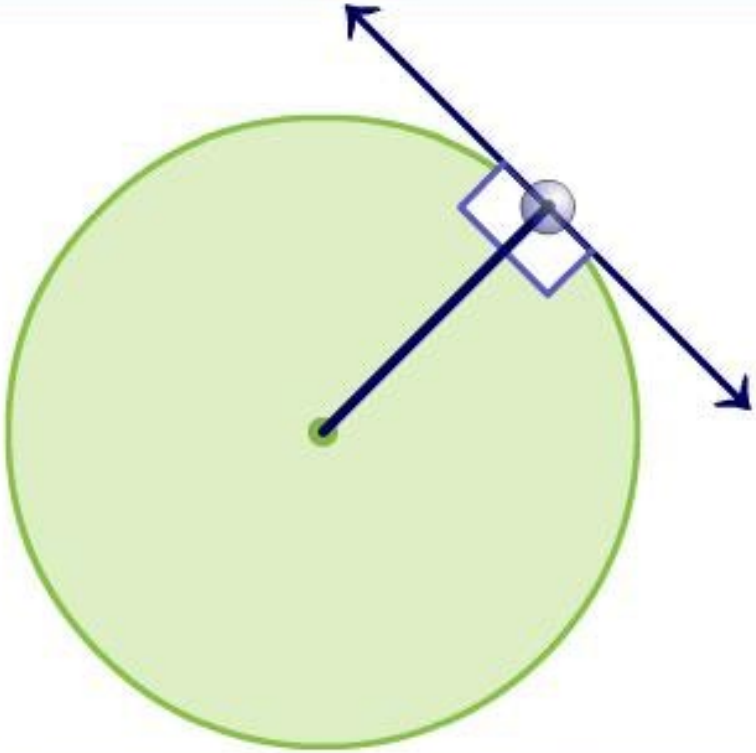


Pairs of circles

congruent circles | concentric circles | tangent circles

Press the terms above to find out more about pairs of circles.

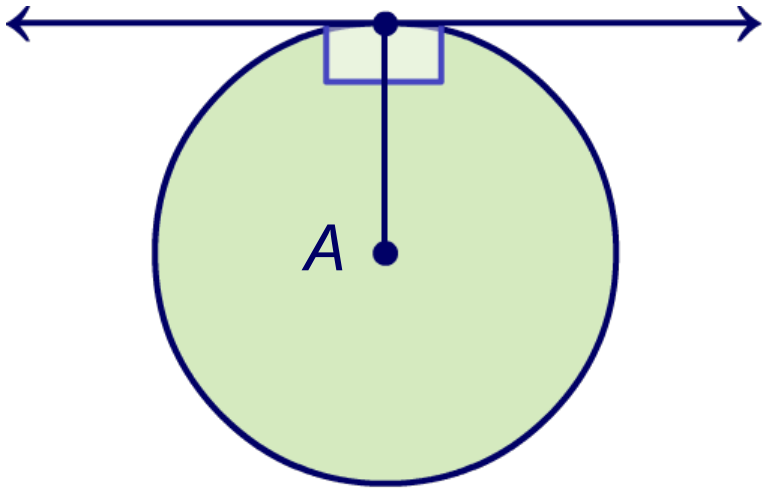
Exploring radius and tangent



The angle between a tangent and the radius at the point of tangency is always 90°.

Radius and tangent

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.



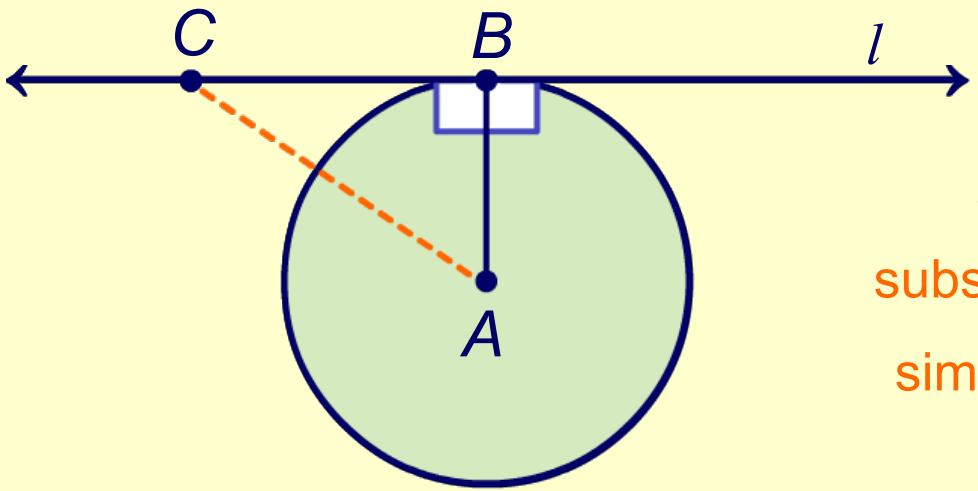
If a line is perpendicular to a radius of a circle at a point on a circle, then the line is tangent to the circle.

The first theorem is the converse of the second theorem.

Radius and tangent proof

The line, l, lies on the circle A and is perpendicular to the radius at that point. Is l also a tangent to the circle?

Choose any point on l that is not B. Call this point C.
We are told that $\overline{CB}$ is perpendicular to $\overline{AB}$. This means that $\overline{CB}$ and $\overline{AB}$ form a right angle in $\triangle ABC$.



apply the Pythagorean theorem:

$$AB^2 + BC^2 = AC^2 \quad (1)$$

$BC > 0$:

$$AB^2 + BC^2 > AB^2 \quad (2)$$

substituting:

$$AC^2 > AB^2$$

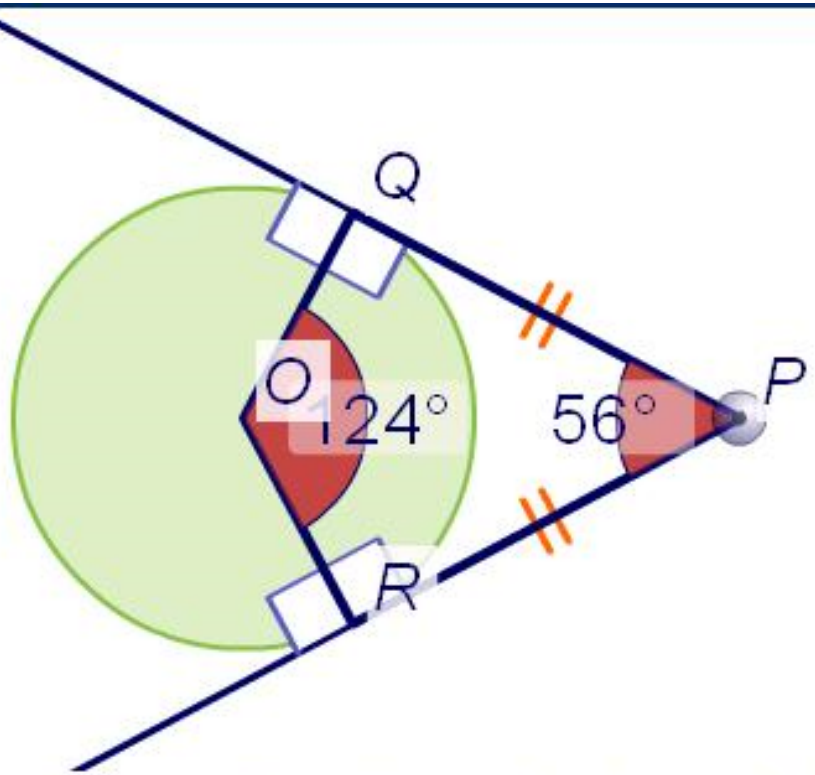
simplifying:

$$AC > AB$$

Therefore $AC \neq AB$

C cannot lie on the circle. The line l intersects with the circle at one point only, meaning that **l is a tangent to the circle**.

Exploring intersecting tangents



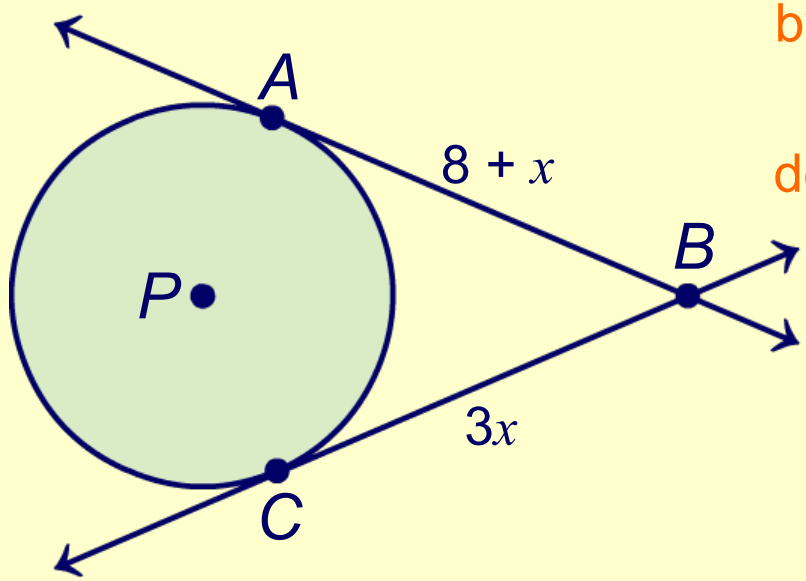
Here are two intersecting tangents to a circle.

What do you notice about the distances from their points of intersection to each of their points of tangency?

Intersecting tangents

If two segments are tangent to a circle from the same external point, then the segments are congruent.

The figure below shows two tangents to a circle, P. Use this theorem to find the value of x.



by the theorem: $\overline{AB}$ is congruent to $\overline{BC}$.

definition of congruence: $AB = BC$

substituting values: $8 + x = 3x$

simplifying: $8 = 2x$

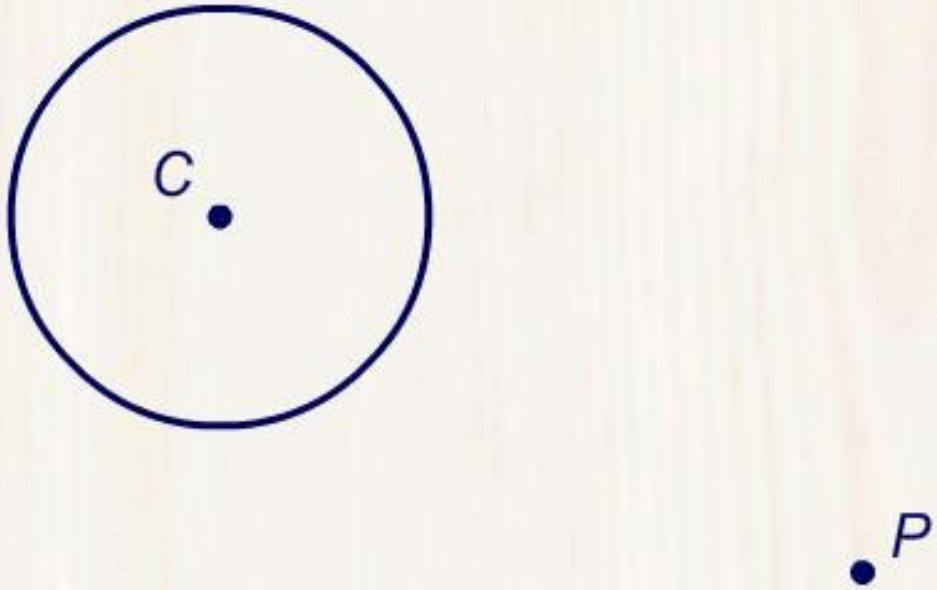
$x = 4$

Constructing a tangent line

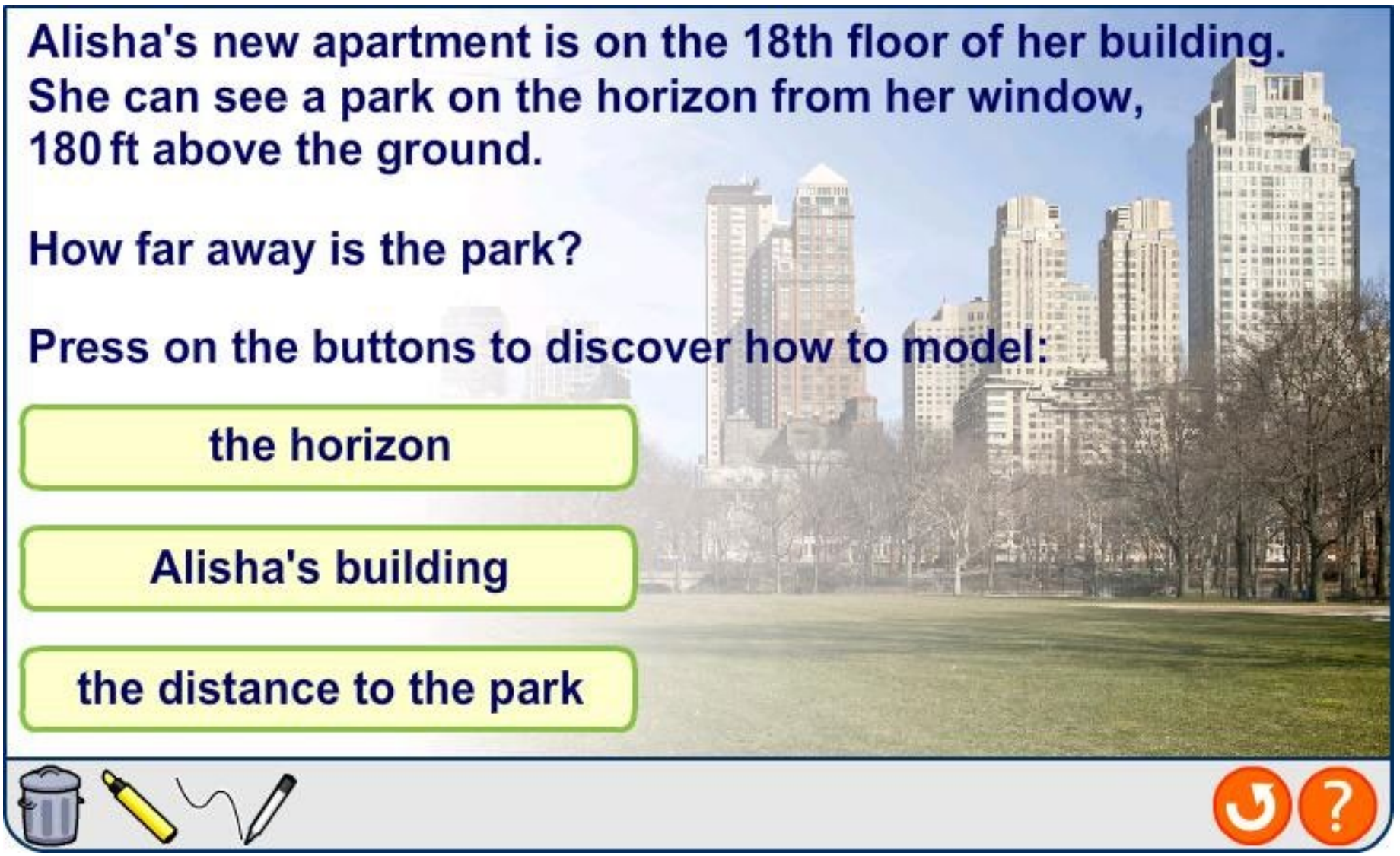
Constructing a tangent from an exterior point

How can you draw a tangent to a circle through a given exterior point using only a straightedge and compass?

Press "**play**" to learn more.



Modeling with circles



Alisha's new apartment is on the 18th floor of her building. She can see a park on the horizon from her window, 180 ft above the ground.

How far away is the park?

Press on the buttons to discover how to model:

- the horizon
- Alisha's building
- the distance to the park