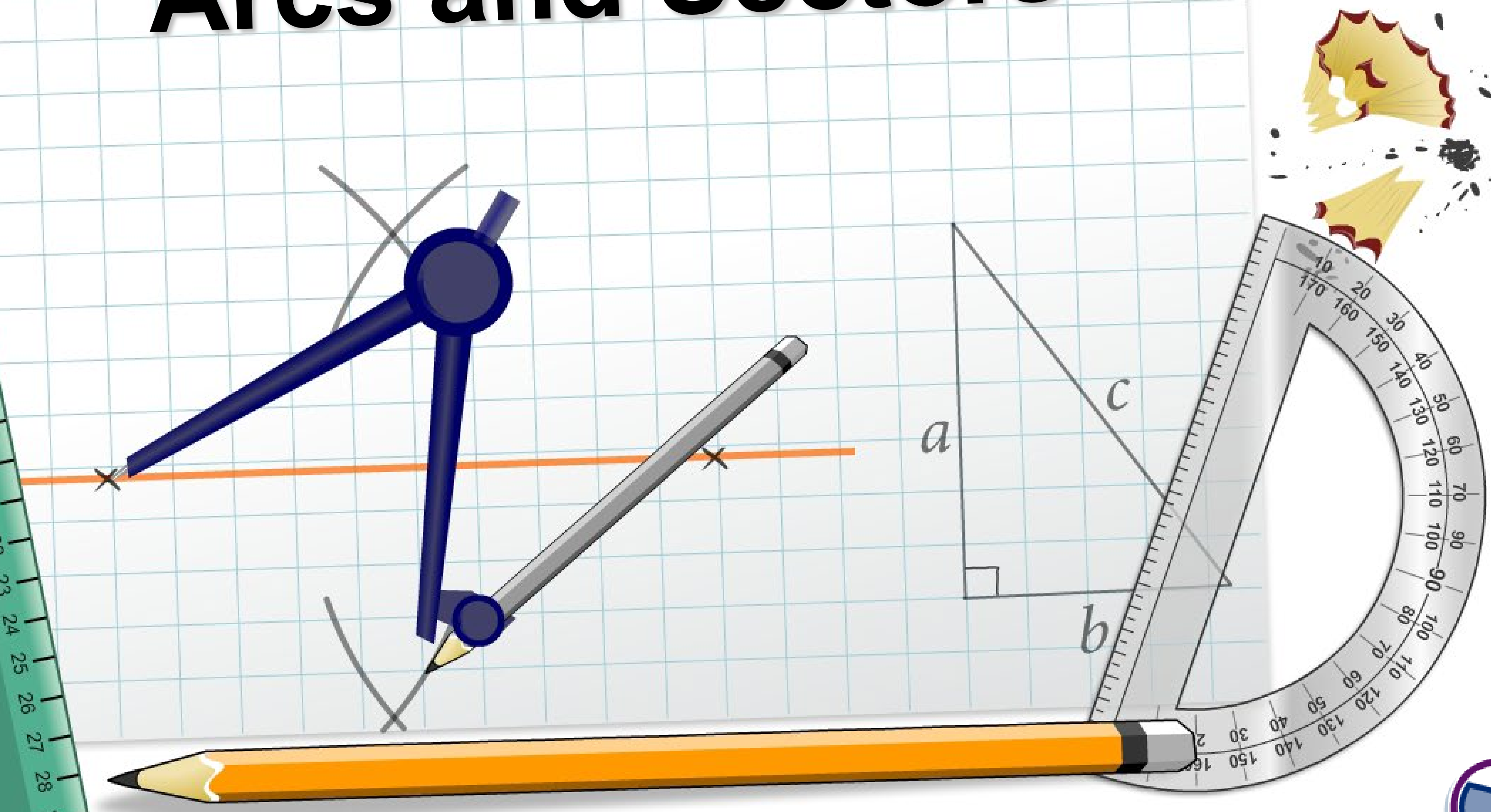
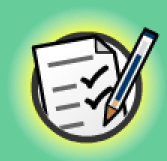
Arcs and Sectors

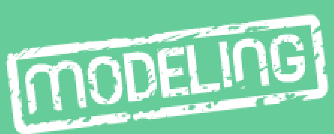


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

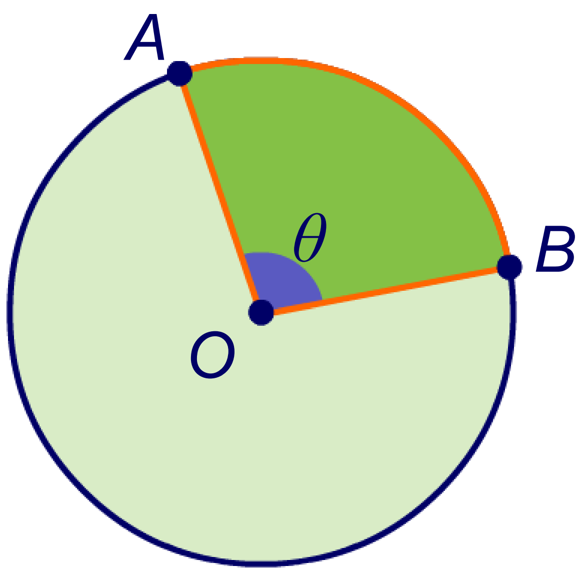
This icon indicates teacher's notes in the Notes field.

Arcs and sectors

An **arc** is all of the points on a circle between two endpoints.

$\overset{\frown}{AB}$ is the arc between the endpoints A and B.

It is **intercepted** by the angle at the center O between the radius to B and the radius to A.



A **central angle** of a circle is an angle whose vertex is the center of a circle. $\angle AOB$ is a central angle. The angle is represented by the variable θ.

The region contained between the arc and the two radii is called a **sector**. It is **sector AOB**.

Arc length and arc measure

The size of an arc can be evaluated in two different ways.

Press the buttons to find out more and see how to evaluate the size of the arc of paving stones around this pond.

arc length

arc measure

Types of arc

minor arc

arc in the interior of a central angle

draw

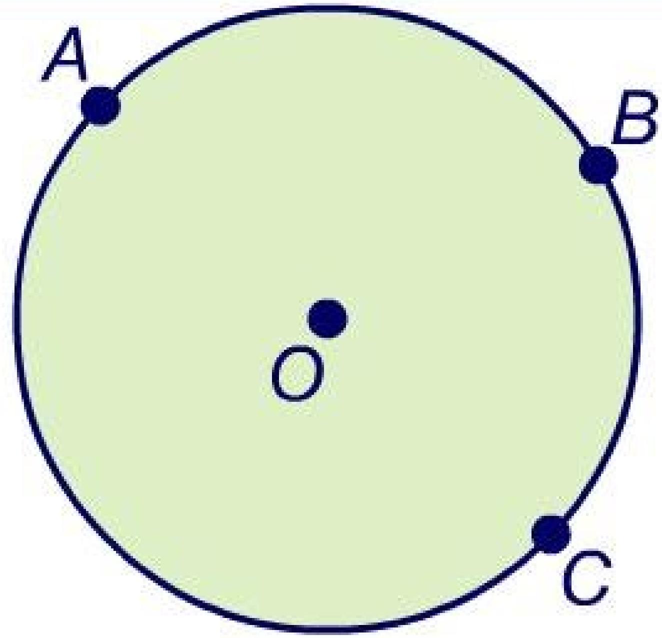
measure

major arc

arc in the exterior of a central angle

draw

measure



semicircle

arc with endpoints on a diameter

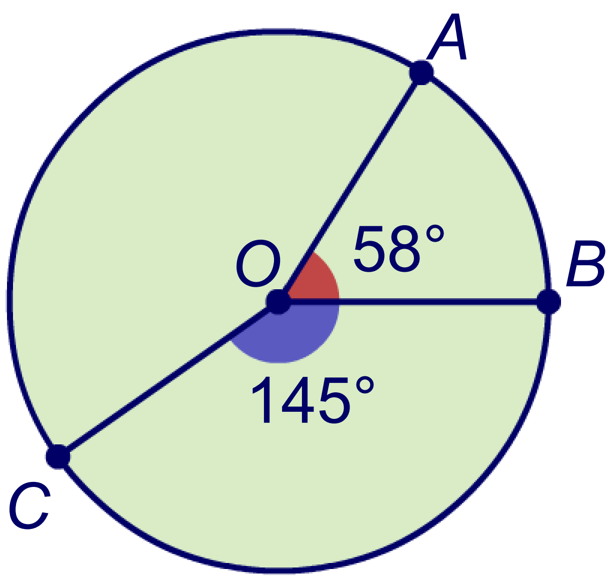
measure

draw

The arc addition postulate

The arc addition postulate: the measure of an arc formed by adjacent arcs is the sum of the measures of the two arcs.

$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$



A ride at a fun fair has seats facing each other around the circumference of a wheel. The arc between Aliyah and Bea measures 58° and the arc between Bea and Chelsea is 145°.

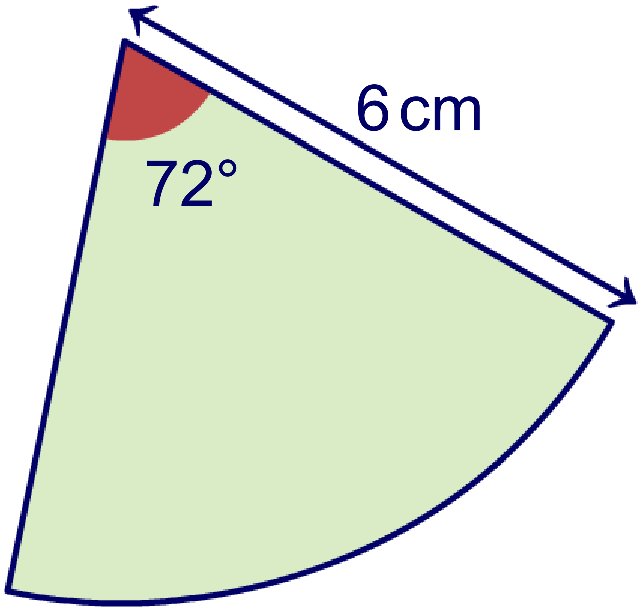
What is the measure of the major arc between Aliyah and Chelsea?

58° + 145° = **203°**

Finding the area of a sector

The area of a sector is a fraction of the area of a full circle. We can find this fraction by dividing the arc measure by 360°.

What is the area of this sector?



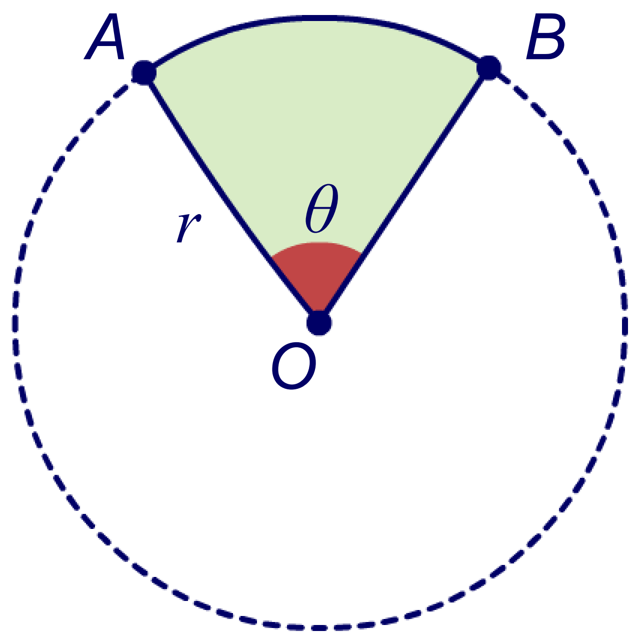
$$\text{Area of the sector} = \frac{72°}{360°} \times \pi \times 6^2$$

$$= \frac{1}{5} \times \pi \times 6^2$$

$$= \mathbf{22.62\ cm^2}$$

(to nearest hundredth)

This method can be used to find the area of any sector.

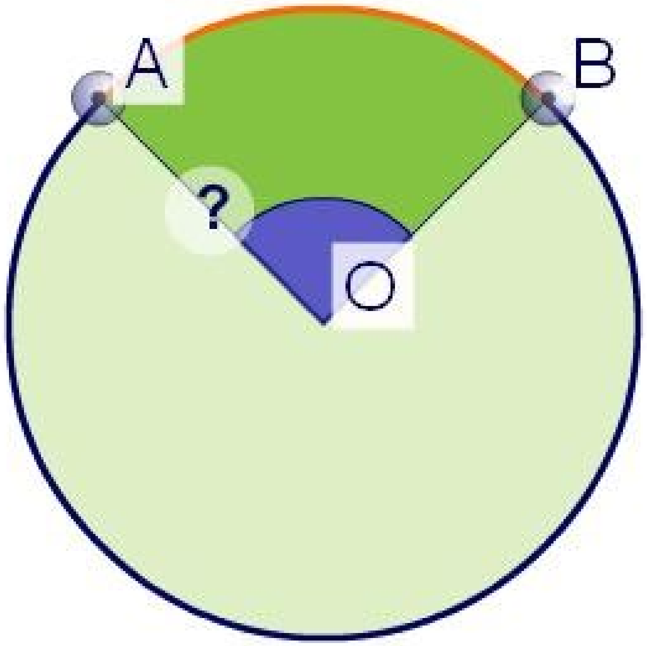


For any circle with radius r and angle at the center θ,

Area of sector $AOB = \frac{\theta}{360} \times \pi r^2$ ← This is the area of the circle.

Area of sector $AOB = \frac{\pi r^2 \theta}{360}$

Area of a sector



area of sector $AOB = \frac{\pi r^2 \theta}{360}$

= ?

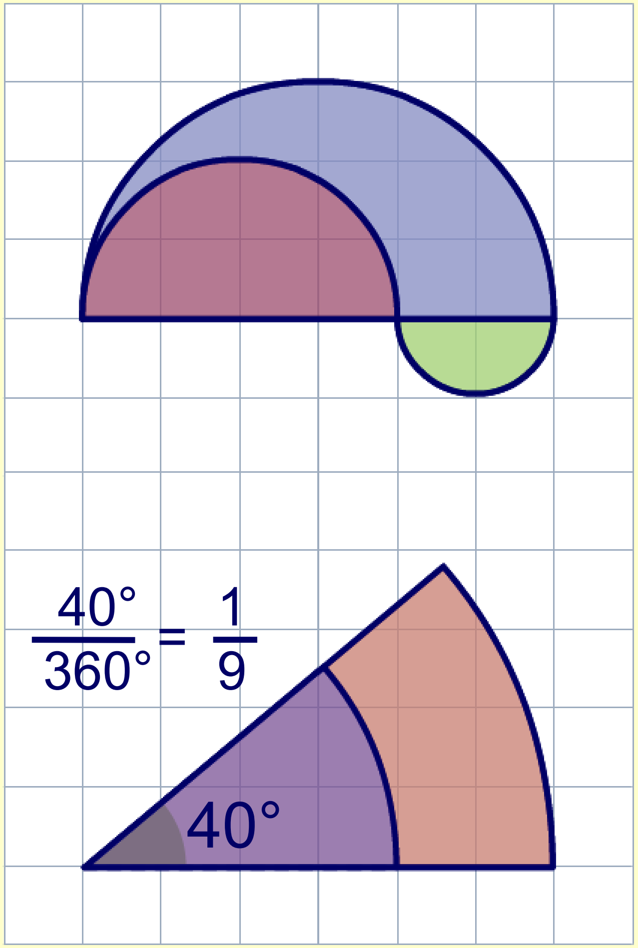
= ?

radius

?

The area of shapes made from sectors

Find the area of these shapes on a cm square grid.



$$\text{area} = \frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times \pi \times 1^2 - \frac{1}{2} \times \pi \times 2^2$$

$$= 3\pi \text{ cm}^2$$

$$= \mathbf{9.42\ cm^2}$$

(to nearest hundredth)

$$\text{area} = \frac{1}{9} \times \pi \times 6^2 - \frac{1}{9} \times \pi \times 4^2$$

$$= \frac{20}{9} \times \pi \text{ cm}^2$$

$$= \mathbf{6.98\ cm^2}$$

(to nearest hundredth)

The Pizza Shop wants to make a new pizza called the "Eight Taste Pizza" where there is one slice of each topping.

There are 3 sizes of pizza with different diameters: small (6 inch), medium (9 inch) and large (12 inch).

Calculate the area of each slice so the amount of topping required can be determined.

Each slice is $1/8^{th}$ of the whole pizza, so we do not need to find the angle.

Area of pizza sector $= \frac{1}{8} \times \pi r^2$

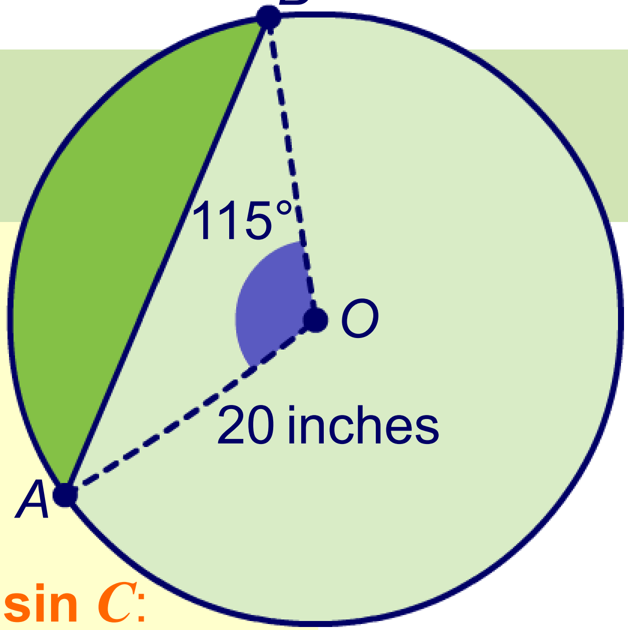
Area of small slice $= \frac{1}{8} \times \pi \times 3^2$

$= 3.53 \text{ in}^2$

$= \mathbf{4\ in^2}$ (to nearest square inch)

Now find the medium and large.

A segment is a region of a circle contained between an arc and the chord between its endpoints.



How can you find the area of the marked segment?

find the area of the sector using the radius and central angle:

$$\frac{\theta}{360} \times \pi r^2 = \frac{115}{360} \times \pi \times 20^2 = 401.4 \text{ in}^2$$

find the area of the triangle OAB using **area = ab sin C**:

$$ab \sin C = r^2 \sin \theta = 20^2 \times \sin 115° = 362.5 \text{ in}^2$$

subtract the area of the triangle from the area of the sector:

$$401.4 - 362.5 = \mathbf{38.9 \text{ in}^2}$$

In congruent circles, **congruent arcs** are arcs that have the same measure.

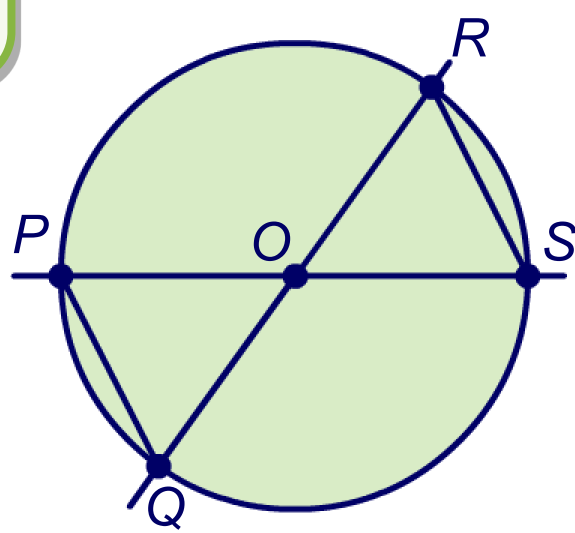
This means that the central angles that intercept the arcs are also congruent. If $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$, then $\angle POQ \cong \angle ROS$.

Congruent arcs have congruent chords.

$\overset{\frown}{PQ} \cong \overset{\frown}{RS}$

implies

$\overline{PQ} \cong \overline{RS}$



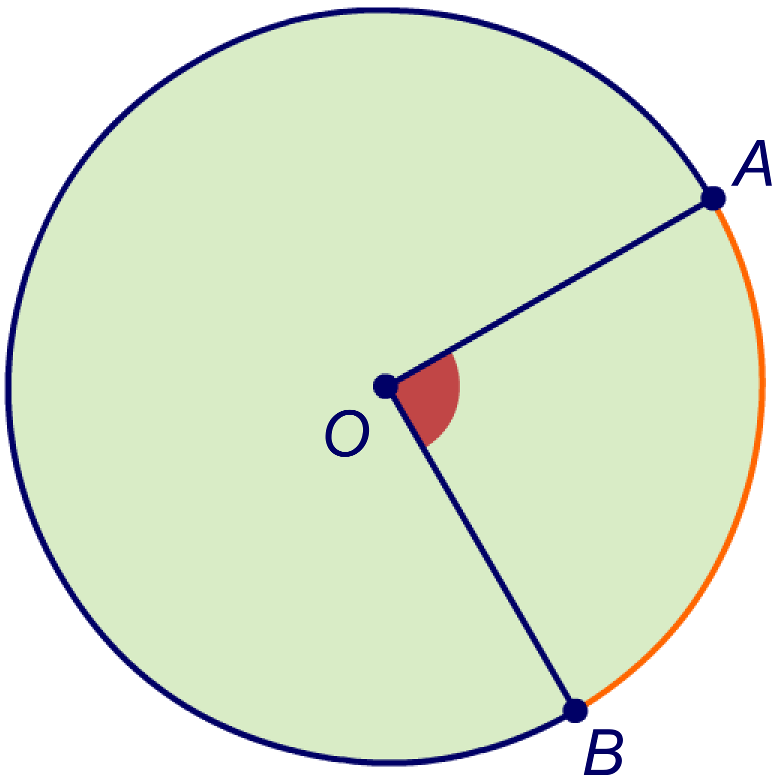
Congruent chords intercept congruent arcs.

$\overline{PQ} \cong \overline{RS}$

implies

$\overset{\frown}{PQ} \cong \overset{\frown}{RS}$

How do you find the length of an arc?

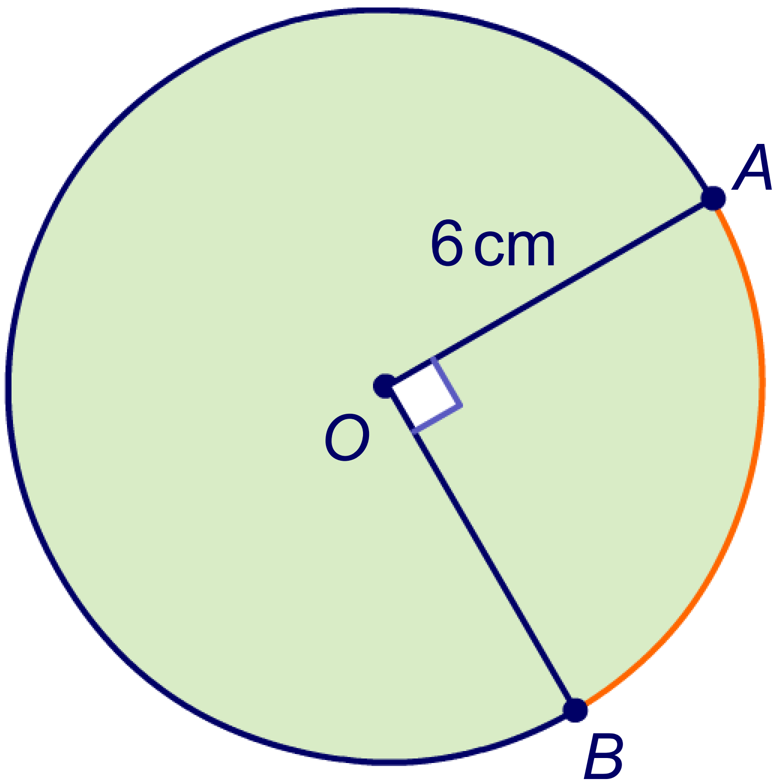


The length of an arc can be measured directly using a string or flexible ruler.

The length of an arc is a fraction of the length of the circumference, so it can also be calculated by finding the circumference and then finding the fraction using the central angle.

The central angle is $\theta/360$ of the circle, so the arc is $\theta/360$ of the circumference.

What is the length of arc AB?



The central angle is 90 degrees.
So the sector is:

$$\frac{90°}{360°} = \frac{1}{4} \text{ of the circle.}$$

The arc length, L, is ¼ of the circumference of the circle, which is $C = 2\pi r$:

$$L = \frac{1}{4} \times 2\pi r$$

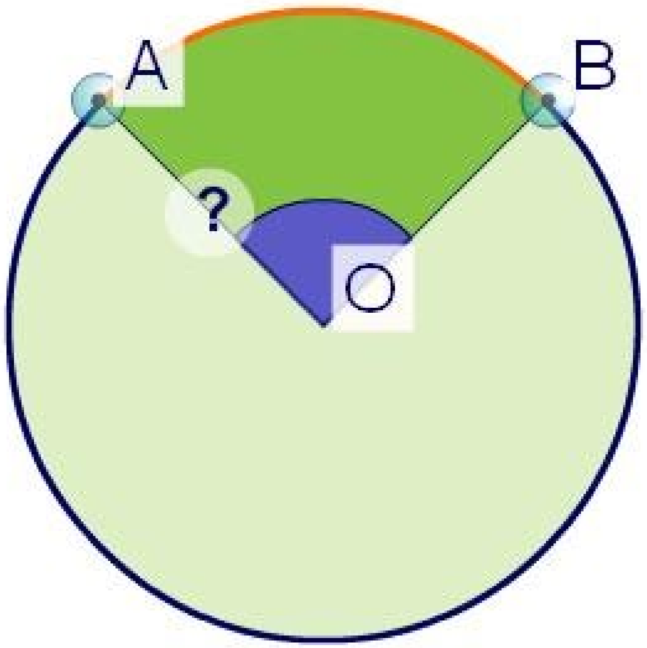
$$= \frac{1}{4} \times 2\pi \times 6$$

$$L = 9.42 \text{ cm}$$

(to nearest hundredth)

Finding the length of an arc

Arc length



L of $\overset{\frown}{AB}$ = $\dfrac{2\pi r\theta}{360} = \dfrac{\pi r\theta}{180}$

L of $\overset{\frown}{AB}$ = ?

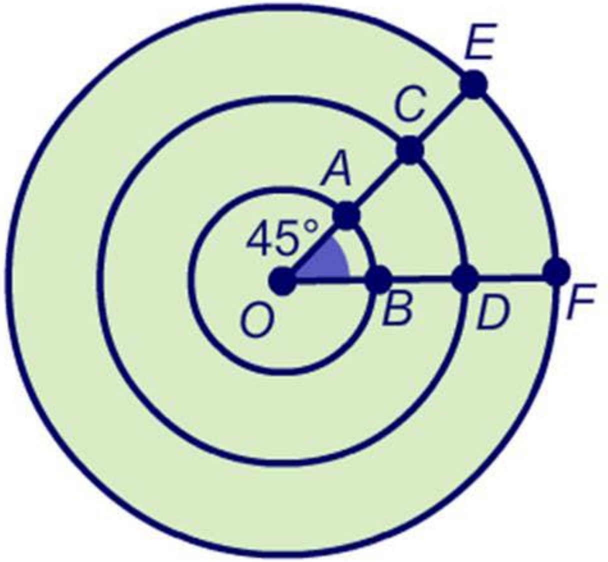
= ?

radius

?

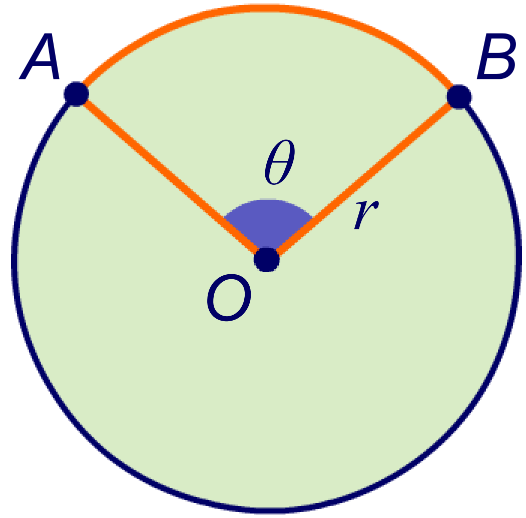
Arc length and radius

These concentric circles have different radii. A central angle of 45° intercepts an arc on each circle.



For any circle with radius r and arc measure θ, the arc length, L, is given by:

$$L = \frac{\theta}{360} \times 2\pi r = \frac{\pi r \theta}{180}$$



This formula gives that the constant of proportionality between arc length and radius for a fixed central angle.

The constant of proportionality is $2\pi\theta/360$, where θ is in degrees. This number is the **radian measure** of the angle.

To convert degrees to radians, divide by 180° and multiply by π.

Radian measurements

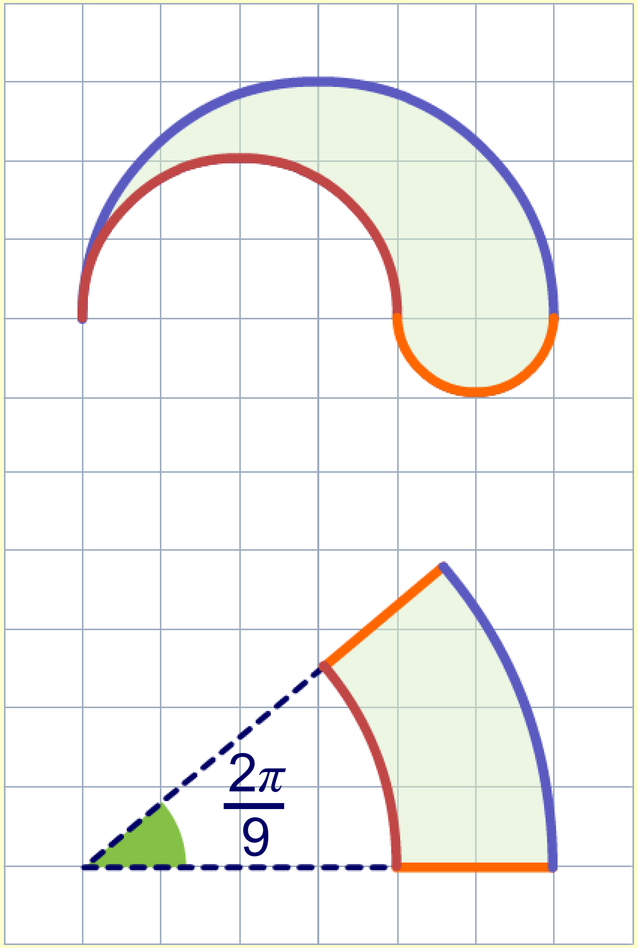
Press to link the angle measures in degrees and radians.

Radians are another method to measure an angle. Just as there are 360 degrees in a circle, there are 2π radians in a circle.

To convert degrees to radians, divide by 180° and multiply by π.
To convert radians to degrees, divide by π and multiply by 180°.

0°	360°	30°	24°	90°	180°	640°	216°
$\frac{32\pi}{9}$	π	2π	$\frac{6\pi}{45}$	$\frac{12\pi}{10}$	0	$\frac{\pi}{2}$	$\frac{\pi}{6}$

Find the perimeter of these shapes on a cm square grid.



The perimeter of this shape is made from three semicircles.

$$\text{perimeter} = \frac{1}{2} \times \pi \times 6 + \frac{1}{2} \times \pi \times 4 + \frac{1}{2} \times \pi \times 2$$

$$= 6\pi \text{ cm}$$

$$= \mathbf{18.85\ cm}$$

(to nearest hundredth)

$$\text{perimeter} = \frac{1}{9} \times \pi \times 6^2 - \frac{1}{9} \times \pi \times 4^2 + 2 + 2$$

$$= 2\pi + 4 \text{ cm}$$

$$= \mathbf{12.28\ cm}$$

(to nearest hundredth)

Patio problem

Keisha is cleaning her patio. The patio is rectangular, extending all the way along the side of the house, which is 25 feet, and then 15 feet in the other direction. She has a hosepipe 10 foot long attached to a tap in center of the exterior wall of the house. The power is sufficient to spray the water 2.5 feet from the nozzle.

How much of the patio can Keisha clean with the hose, and how much will she have to clean by hand?

W

*Press the **W** button to see the solution.*