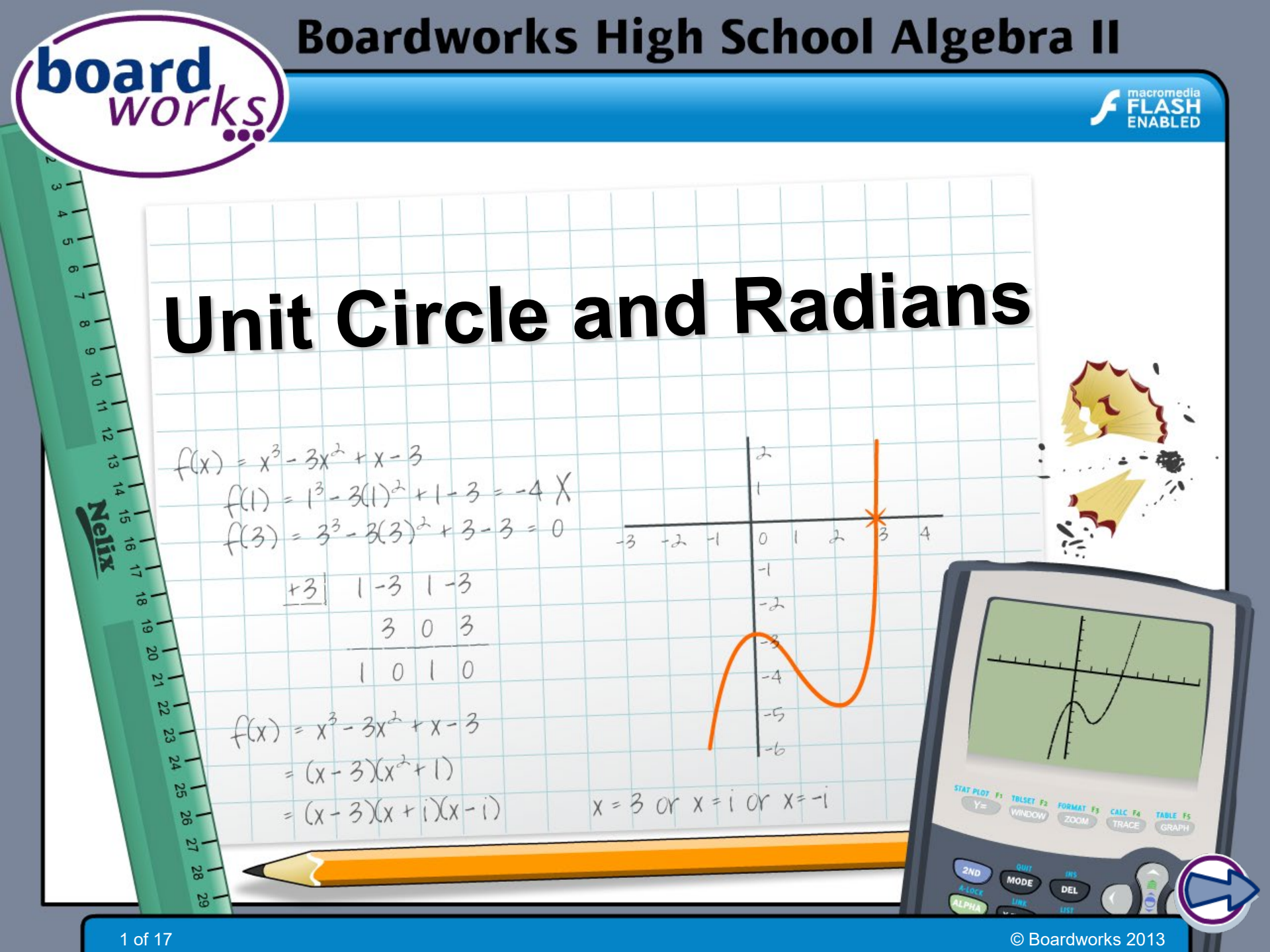
Unit Circle and Radians



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

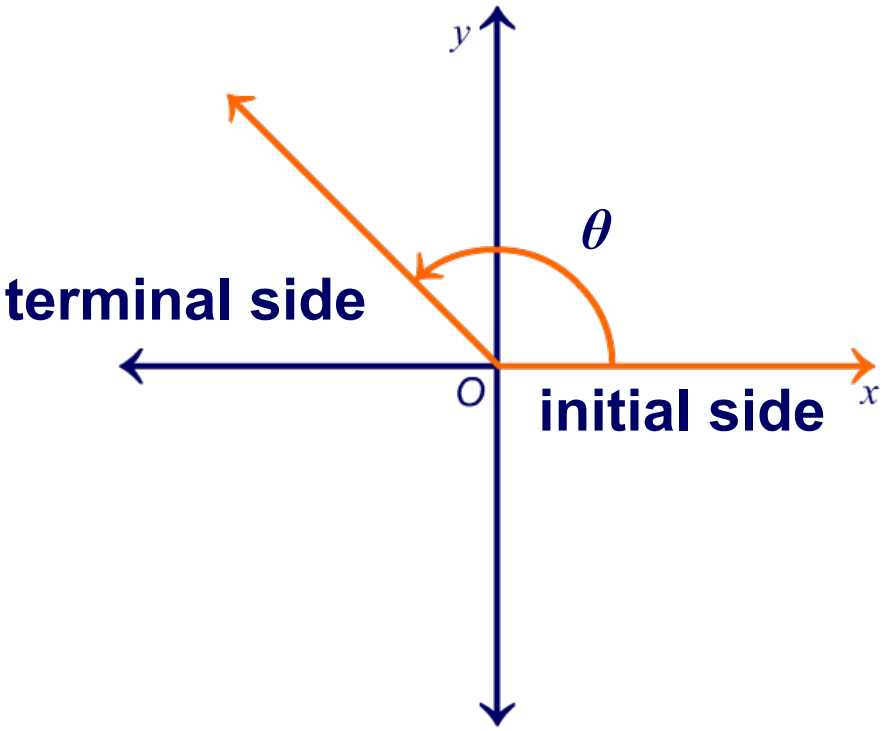
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Angles in standard position

An angle on the coordinate plane is in **standard position** if:

- the origin is the vertex of the angle
- and the positive x-axis is one ray of the angle.



The ray on the x-axis is the **initial side**.

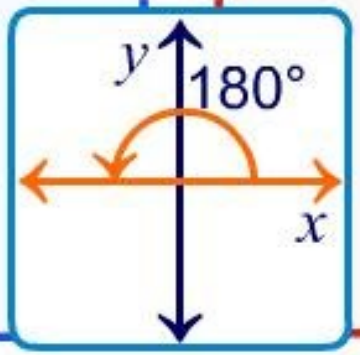
The other ray is the **terminal side**.

Angles are measured as counterclockwise rotations. A negative angle indicates a clockwise rotation.

Standard position practice

Are the angles in standard position?

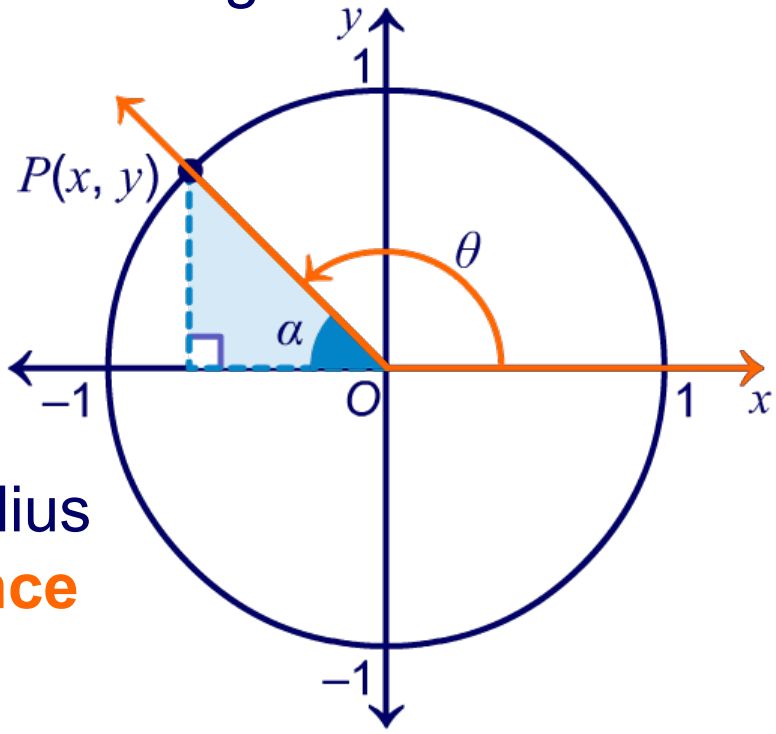
angle in standard position	not in standard position



Unit circle and right triangles

The **unit circle** on the coordinate plane is the circle formed by all the points at 1 unit distance from the origin.

For any point $P(x, y)$ on the unit circle, there is a right triangle with the radius as its hypotenuse and a section of the x-axis as one leg.



The acute angle α between the radius and the x-axis is called the **reference angle** or **associated angle** for the angle θ in standard position.

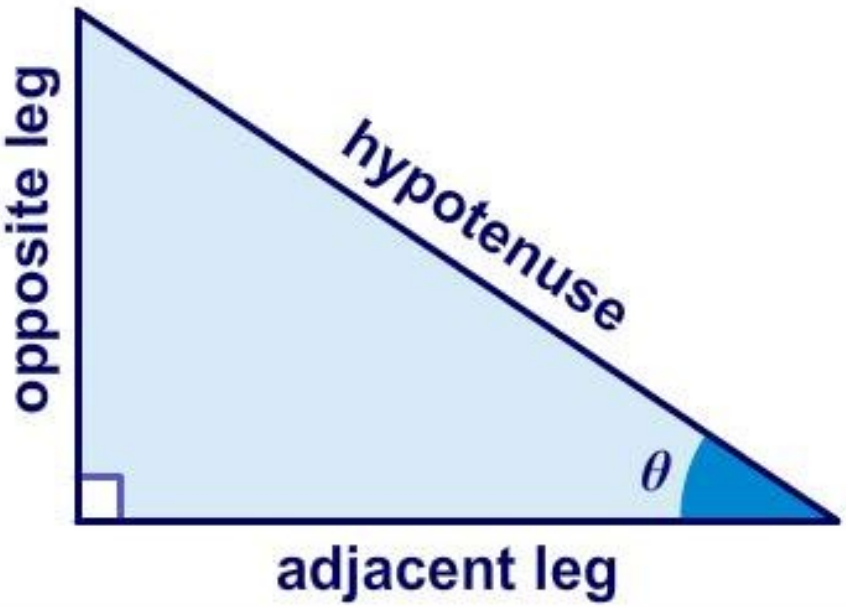
When is α identical with θ?

When α is in standard position (when P is in the first quadrant.)

The three trigonometric ratios

The three trigonometric ratios, **sine**, **cosine** and **tangent**, are defined using the ratios of the side lengths in a right triangle.

Which side lengths form each ratio?



$\sin\theta$ = ?

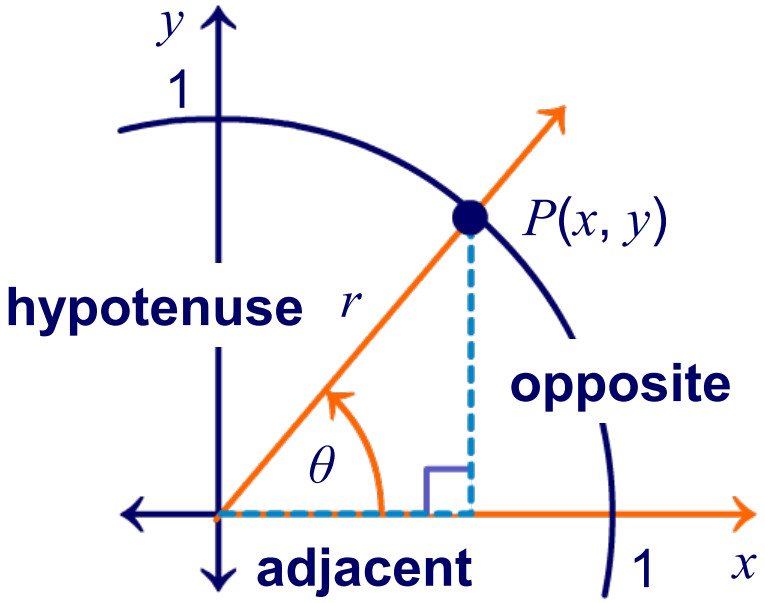
$\cos\theta$ = ?

$\tan\theta$ = ?

Trigonometry on the unit circle

Consider the right triangle formed along the x-axis with a vertex at the origin and a vertex at a point $P(x, y)$ on the unit circle in the first quadrant.

What are the trigonometric ratios for the angle at the origin, θ, in terms of x and y?



adjacent leg = x

opposite leg = y

hypotenuse = radius, $r = 1$

$\cos\theta = x/r = x$

$\sin\theta = y/r = y$

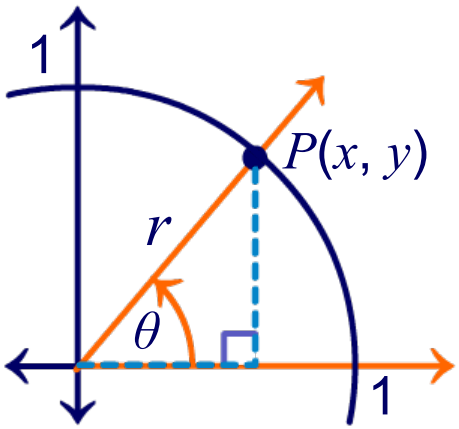
$\tan\theta = y/x$

Trigonometric functions

For any angle θ, the trigonometric functions are defined:

$$\sin\theta = y/r \qquad \cos\theta = x/r \qquad \tan\theta = y/x$$

where x and y are the coordinates of any point $P(x, y)$ on the terminal ray of the angle θ when it is in standard position, and r is the distance from the origin to P.



In particular, if P is on the unit circle:

$$\sin\theta = y$$

$$\cos\theta = x$$

$$\tan\theta = y/x$$

Quadrants

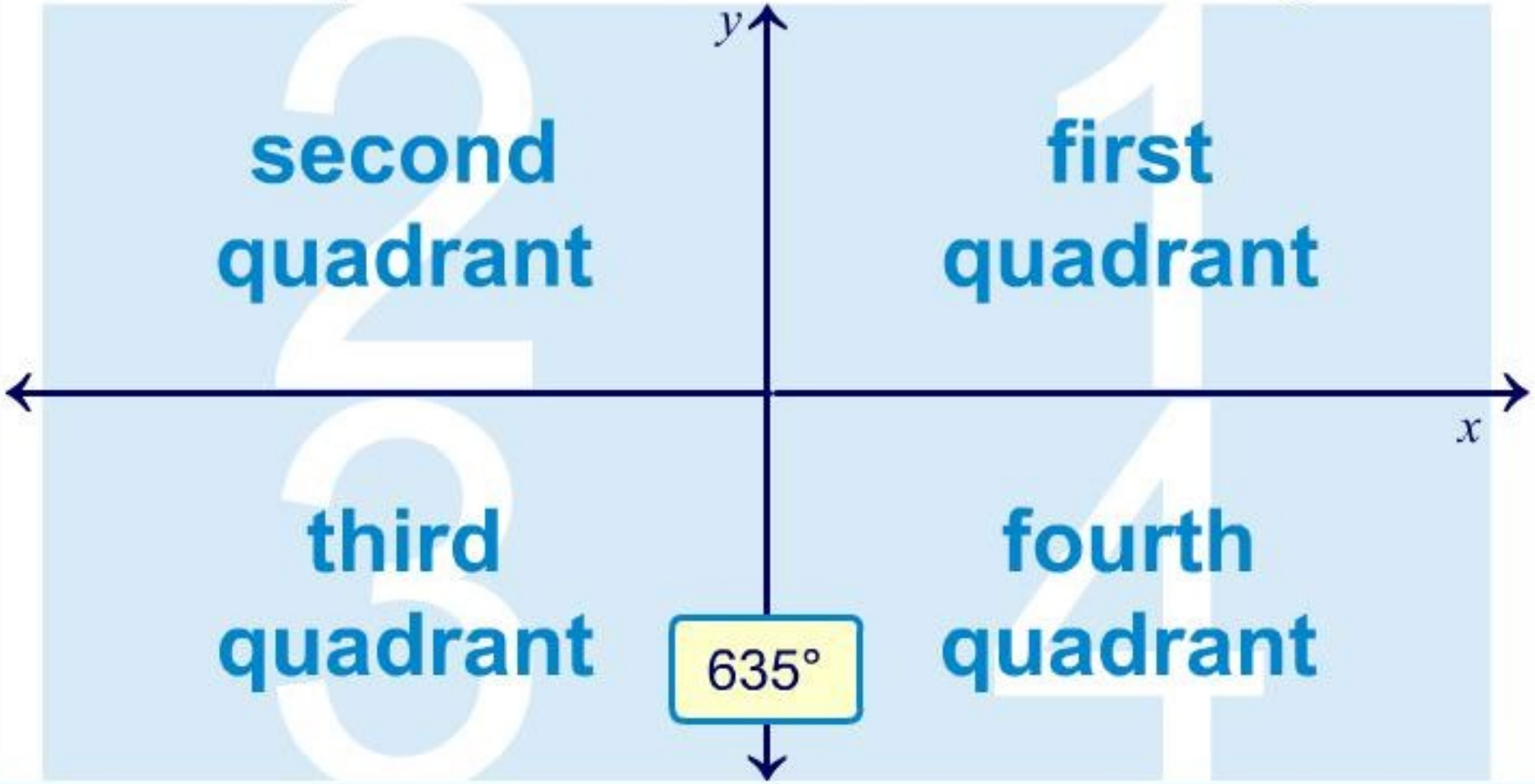
The trigonometric ratios for an angle θ in standard position are related to the ratios for the associated acute angle α, called the **reference angle**.

Press **start** to see how.

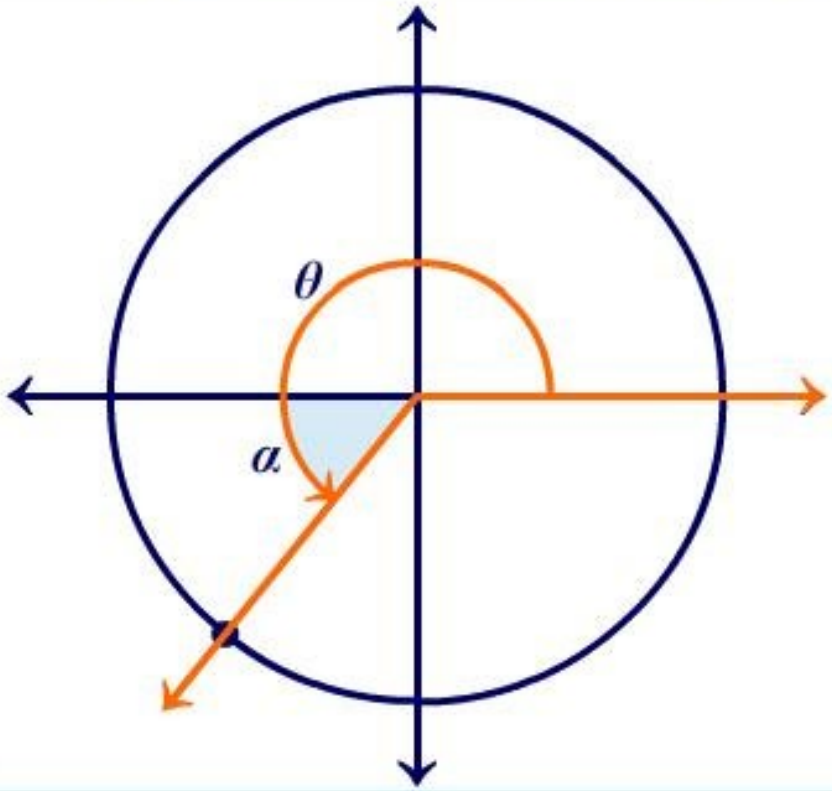
start

Quadrant practice

Which quadrant is P contained in for each angle?



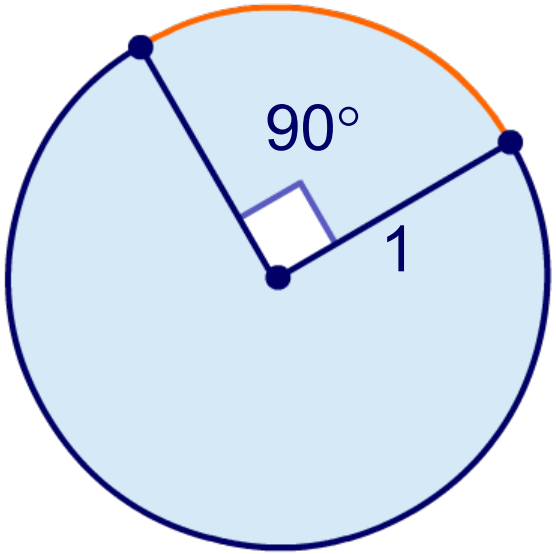
Match the trig functions of θ with the equivalent trig functions with the reference angle α.



θ	α
sin280°	–tan30°
cos690°	–tan50°
tan150°	cos40°
tan–50°	cos30°
sin165°	–sin80°
cos–320°	sin15°

Arc length and angle

What is the length of an arc on the unit circle if it is intercepted by a 90° angle?



total circumference = $2\pi r = 2\pi$

arc length = $\frac{90°}{360°}\ 2\pi = \boldsymbol{\frac{\pi}{2}}$

The length of an arc on the unit circle intercepted by an angle is called the **radian** measure of that angle.

1 radian is the angle intercepted by an arc the same length as the radius. 2π radians is a full turn.

$$\text{angle in radians} = \pi\ \frac{\text{angle in degrees}}{180°}$$

Radian measurements

Match the angle measures in degrees and radians

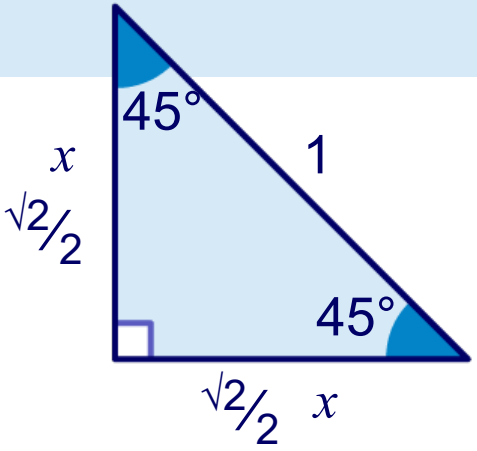
0°	572°	30°	24°	90°	180°	640°	315°
$\frac{2\pi}{15}$	5.50 (2 dp)	0	π	$\frac{32\pi}{9}$	$\frac{\pi}{2}$	$\frac{\pi}{6}$	9.98 (2 dp)

Special triangles

What are the leg lengths in a 45°-45° triangle with hypotenuse of length 1?

The angles are congruent so the legs are equal in length.

$$1^2 = x^2 + x^2$$
$$1^2 = 2x^2$$
$$\tfrac{1}{2} = x^2$$
$$\tfrac{1}{\sqrt{2}} = x$$
$$\tfrac{\sqrt{2}}{2} = x$$

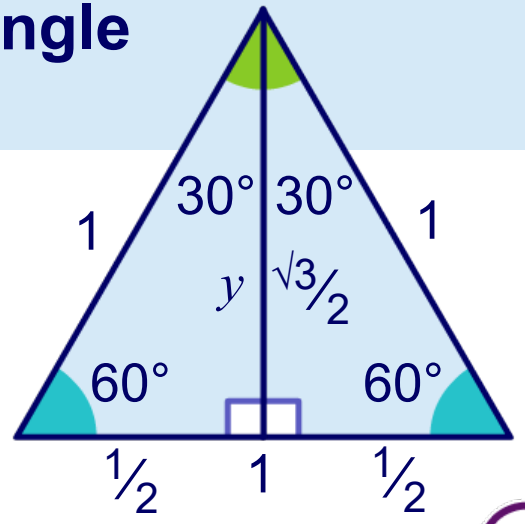


What are the leg lengths in a 30°-60° triangle with hypotenuse of length 1?

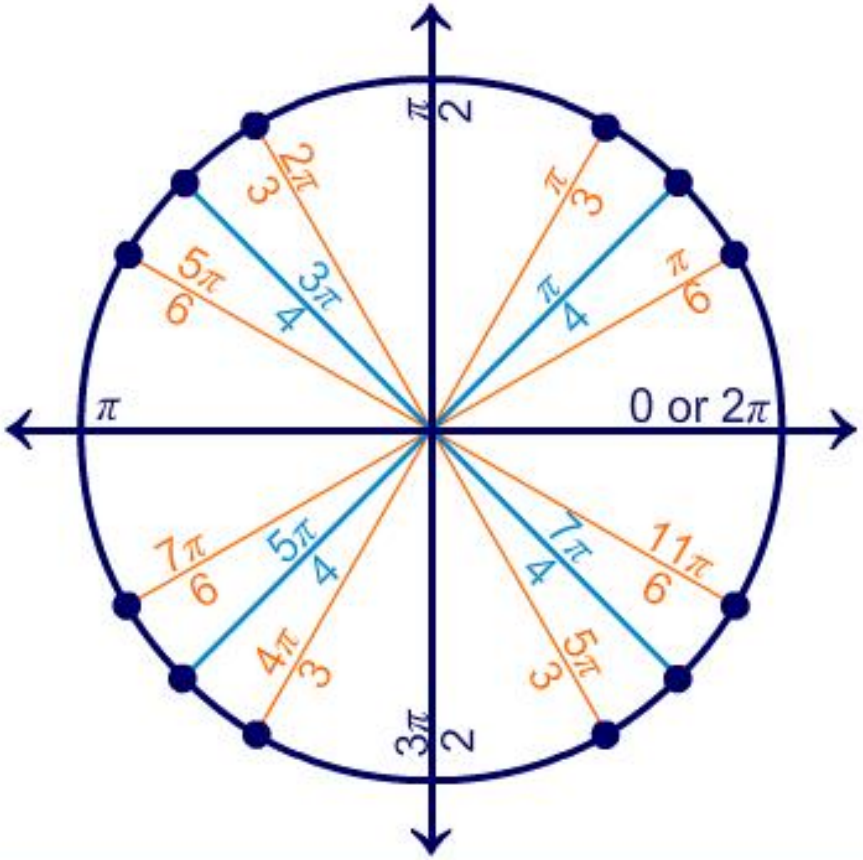
Draw as half of an equilateral triangle.

Then the side adjacent to 60° is $\tfrac{1}{2}$.

$$1^2 = (\tfrac{1}{2})^2 + y^2$$
$$1 - (\tfrac{1}{4}) = y^2$$
$$\tfrac{3}{4} = y^2$$
$$\sqrt{(\tfrac{3}{4})} = y^2$$
$$\tfrac{\sqrt{3}}{2} = y$$



Special points in the unit circle



Press the points to reveal their coordinates and the associated right triangle.

Sin, cos and tan of 30°, 45° and 60°

The exact values of the sine, cosine and tangent of angles between 0° and 90° can be summarized as follows:

	0° 0	30° $\frac{\pi}{6}$	45° $\frac{\pi}{4}$	60° $\frac{\pi}{3}$	90° $\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Exact trigonometric ratios

Write the following ratios exactly:

1) $\cos\frac{5\pi}{3}$	=	$\frac{1}{2}$	2) $\tan\frac{7\pi}{4}$	=	-1
3) $\tan\frac{5\pi}{4}$	=	1	4) $\sin-\frac{11\pi}{6}$	=	$\frac{1}{2}$
5) $\cos-\frac{\pi}{6}$	=	$\frac{\sqrt{3}}{2}$	6) $\tan-\frac{3\pi}{4}$	=	1
7) $\sin\frac{7\pi}{6}$	=	$-\frac{1}{2}$	8) $\cos\frac{7\pi}{4}$	=	$\frac{\sqrt{2}}{2}$