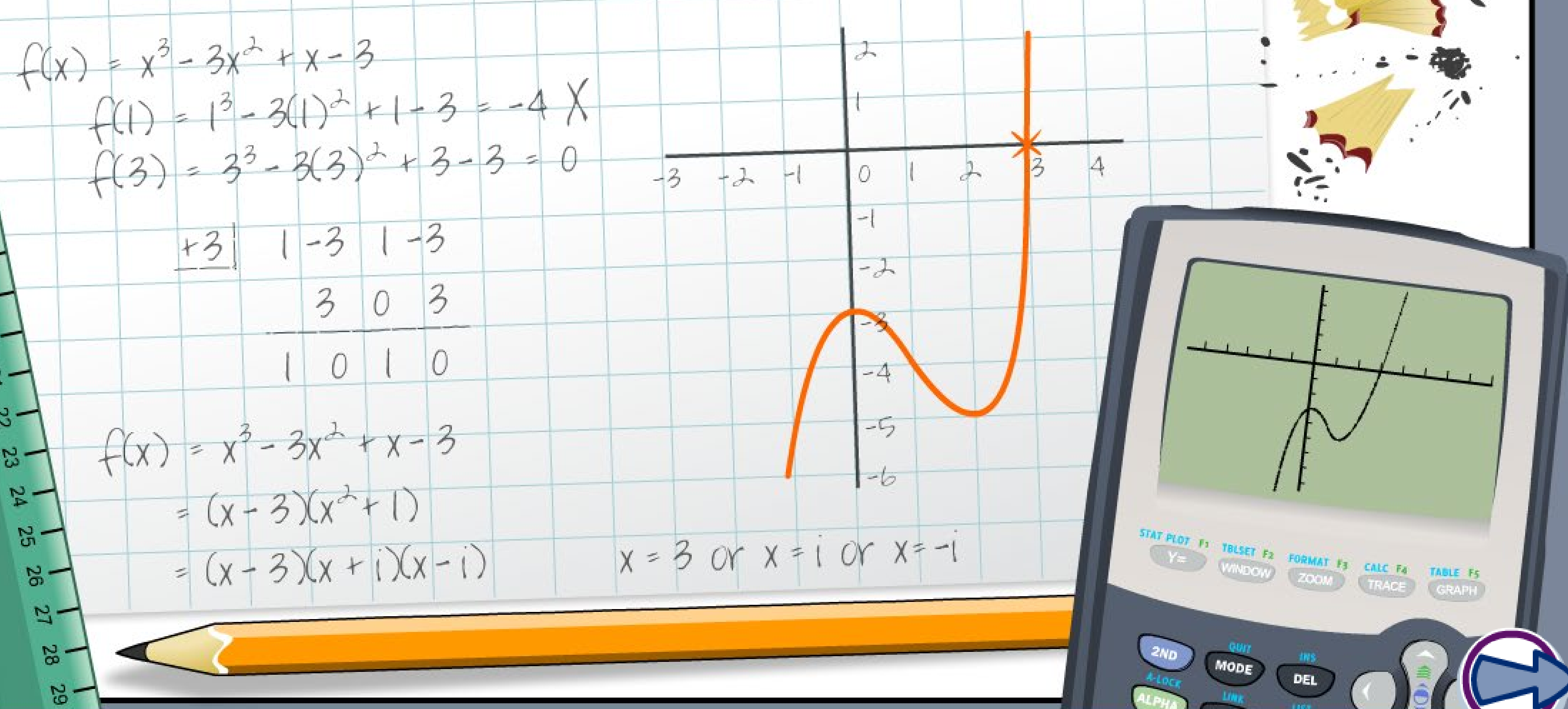
Trigonometric Identities



Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

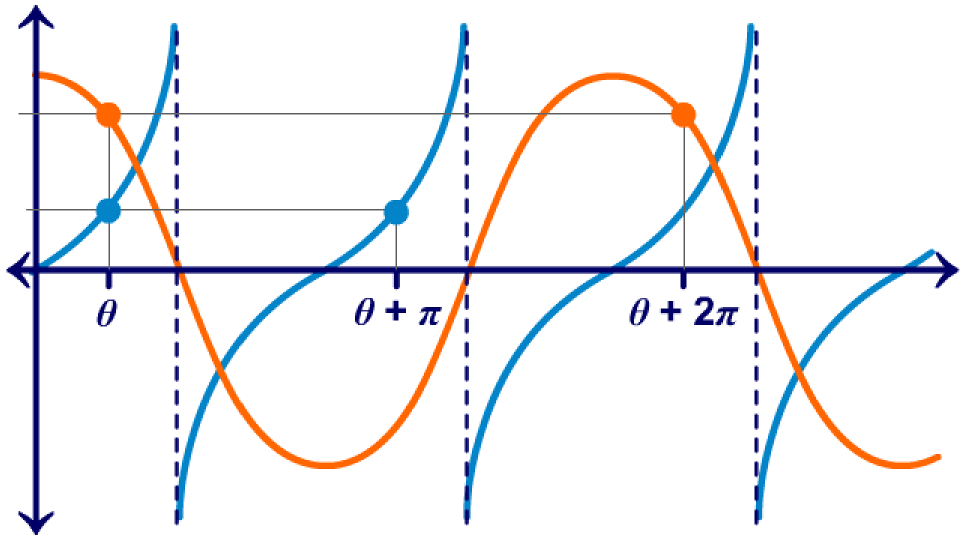
Identities

A **trigonometric identity** is an identity between expressions containing trigonometric functions.

It holds true for every value of the variables.

For example, because sine is periodic with a period of 2π, $\sin\theta = \sin(\theta + 2\pi)$ is true for every value of θ.

What similar identities are there for cosine and tangent?



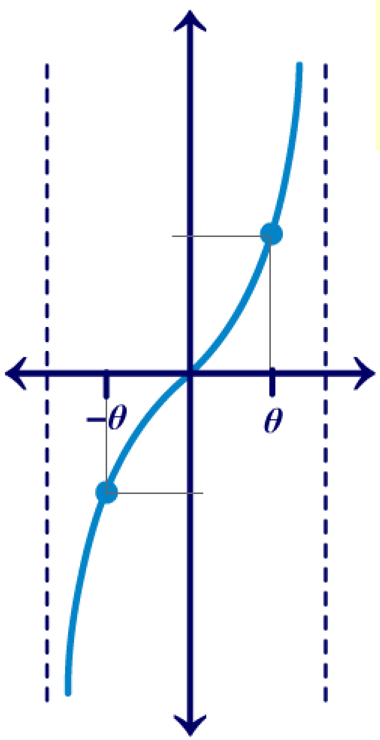
$\cos\theta = \cos(\theta + 2\pi)$

$\tan\theta = \tan(\theta + \pi)$

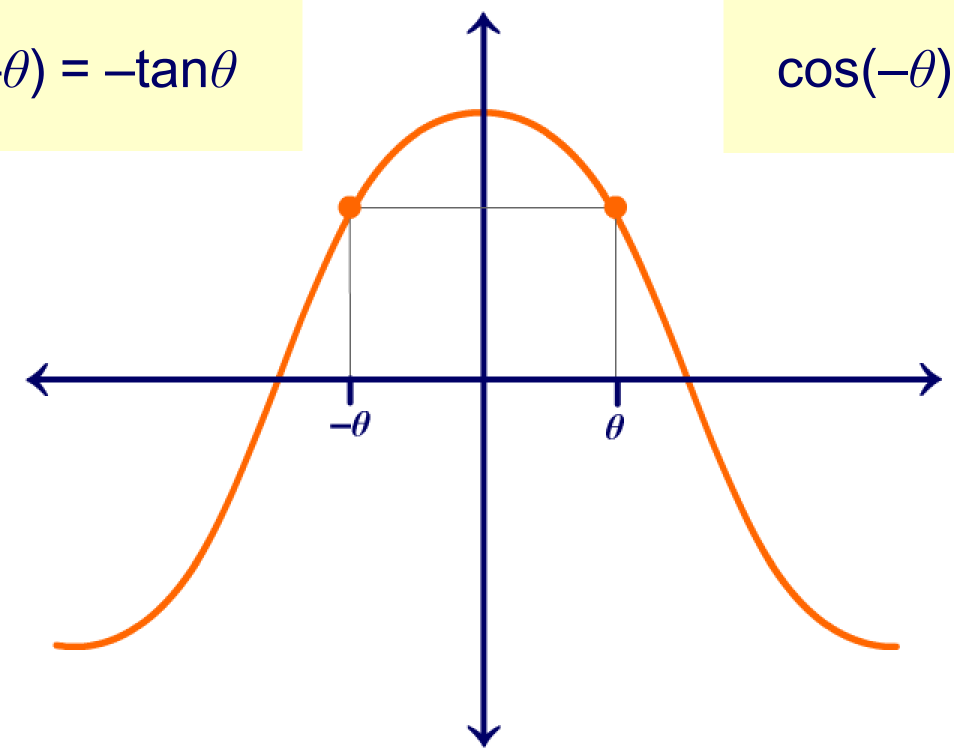
Even and odd functions

Sine is an **odd function**, meaning it has rotational symmetry of order 2 about the origin. This gives the identity $\sin(-\theta) = -\sin\theta$.

What similar identities are there for cosine and tangent?

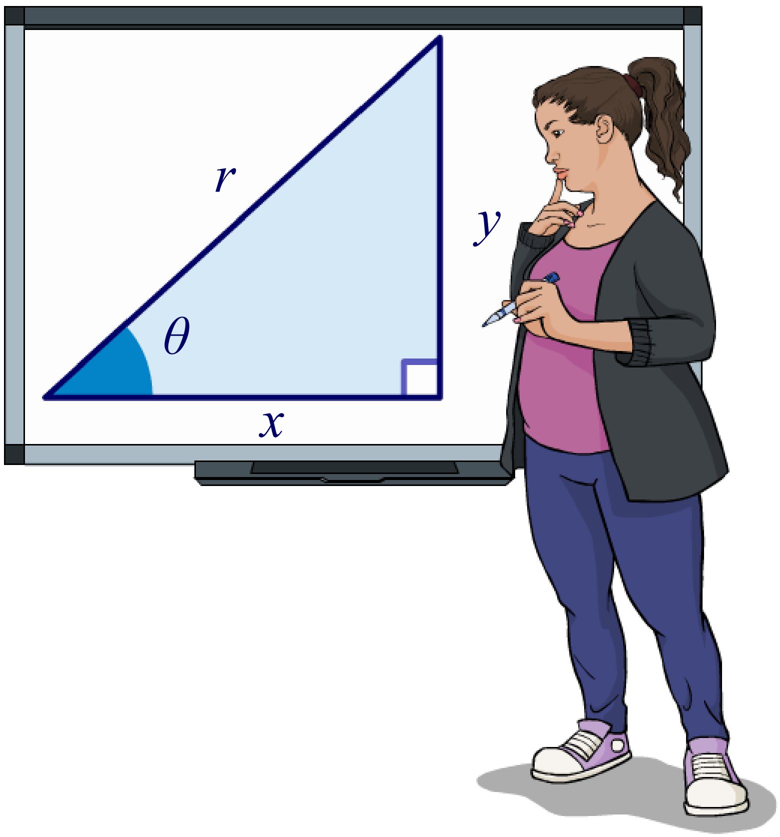


$\tan(-\theta) = -\tan\theta$



$\cos(-\theta) = \cos\theta$

Show that tanθ = sinθ / cosθ is a trigonometric identity.



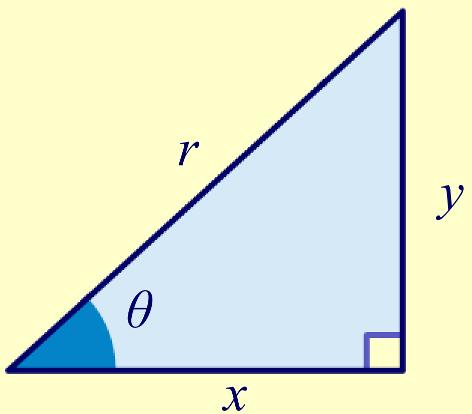
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r}$$

$$= \frac{y}{x}$$

$$= \tan\theta$$ ✓

Show that $\sin^2\theta + \cos^2\theta = 1$ is a trigonometric identity.



$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\sin^2\theta + \cos^2\theta = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{x^2 + y^2}{r^2}$$

Pythagorean theorem:

$$x^2 + y^2 = r^2$$

$$\sin^2\theta + \cos^2\theta = \frac{r^2}{r^2}$$

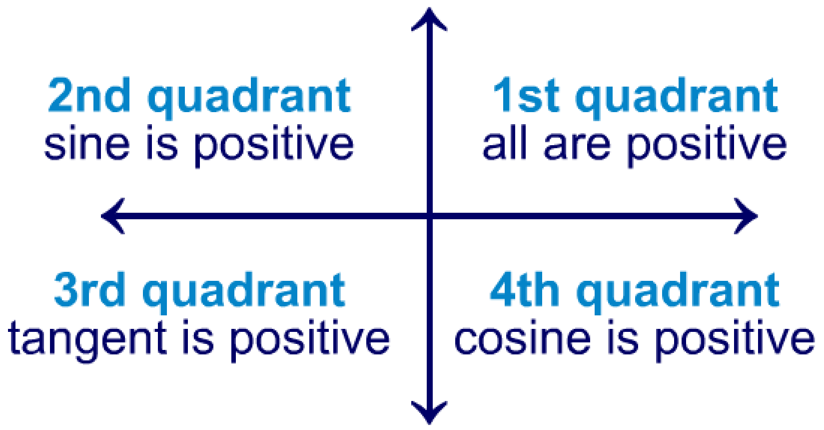
$$= 1$$ ✓

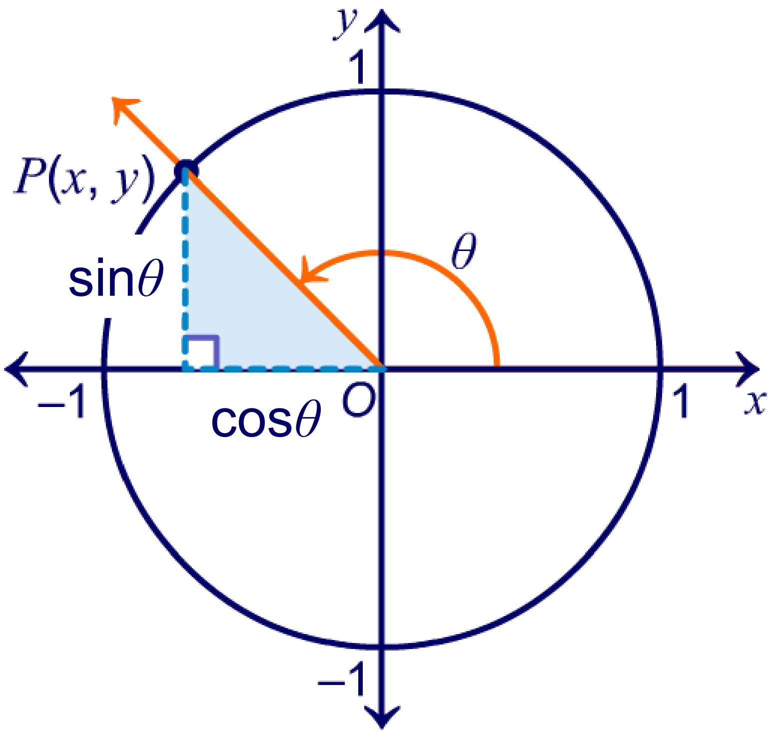
To find the value of one trigonometric function from the value of another, use the identities:

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

It is also important to get the sign right. Use the chart:





Find $\cos\theta$ if $\sin\theta = 0.6$ is in the second quadrant.

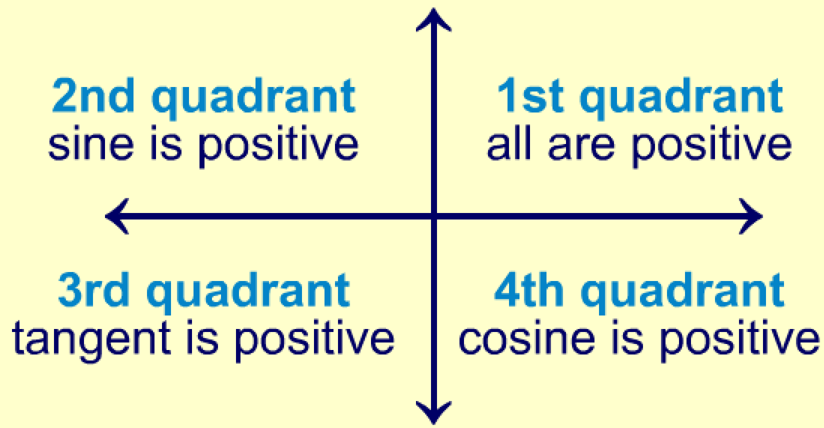
identity: $\cos^2\theta + \sin^2\theta = 1$

substitute $\sin\theta = 0.6$: $\cos^2\theta + (0.6)^2 = 1$

solve for $\cos\theta$: $\cos^2\theta + 0.36 = 1$

$\cos^2\theta = 0.64$

$\cos\theta = \pm 0.8$



cosine is negative in the second quadrant: $\mathbf{\cos\theta = -0.8}$

Using trigonometric identities (2)

Find $\sin\theta$ if $\tan\theta = 0.4$ is in the fourth quadrant.

identity: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

substitute $\tan\theta = 0.4$: $0.4 = \dfrac{\sin\theta}{\cos\theta}$

square: $0.16 = \dfrac{\sin^2\theta}{\cos^2\theta}$

substitute $\cos^2\theta = 1 - \sin^2\theta$: $0.16 = \dfrac{\sin^2\theta}{1 - \sin^2\theta}$

solve for $\sin\theta$:

$$\sin^2\theta = 0.16(1 - \sin^2\theta)$$

$$\sin^2\theta = 0.16 - 0.16\sin^2\theta$$

$$1.16\sin^2\theta = 0.16$$

$$\sin^2\theta = 0.138$$

sine is negative in the fourth quadrant: $\mathbf{\sin\theta = -0.37}$

Practice

1. If $\tan\theta$ is –12 in the 4th quadrant, find $\sin\theta$. ?
2. If $\cos\theta$ is –0.39 in the 3rd quadrant, find $\tan\theta$. ?
3. If $\sin\theta$ is 0.18 in the 1st quadrant, find $\cos\theta$. ?
4. If $\tan\theta$ is –0.5 in the 2nd quadrant, find $\cos\theta$. ?
5. If $\cos\theta$ is –0.6 in the 2nd quadrant, find $\sin\theta$. ?
6. If $\sin\theta$ is –0.25 in the 3rd quadrant, find $\tan\theta$. ?

Give answers to the nearest hundredth.
Press the gray boxes to reveal the answers.