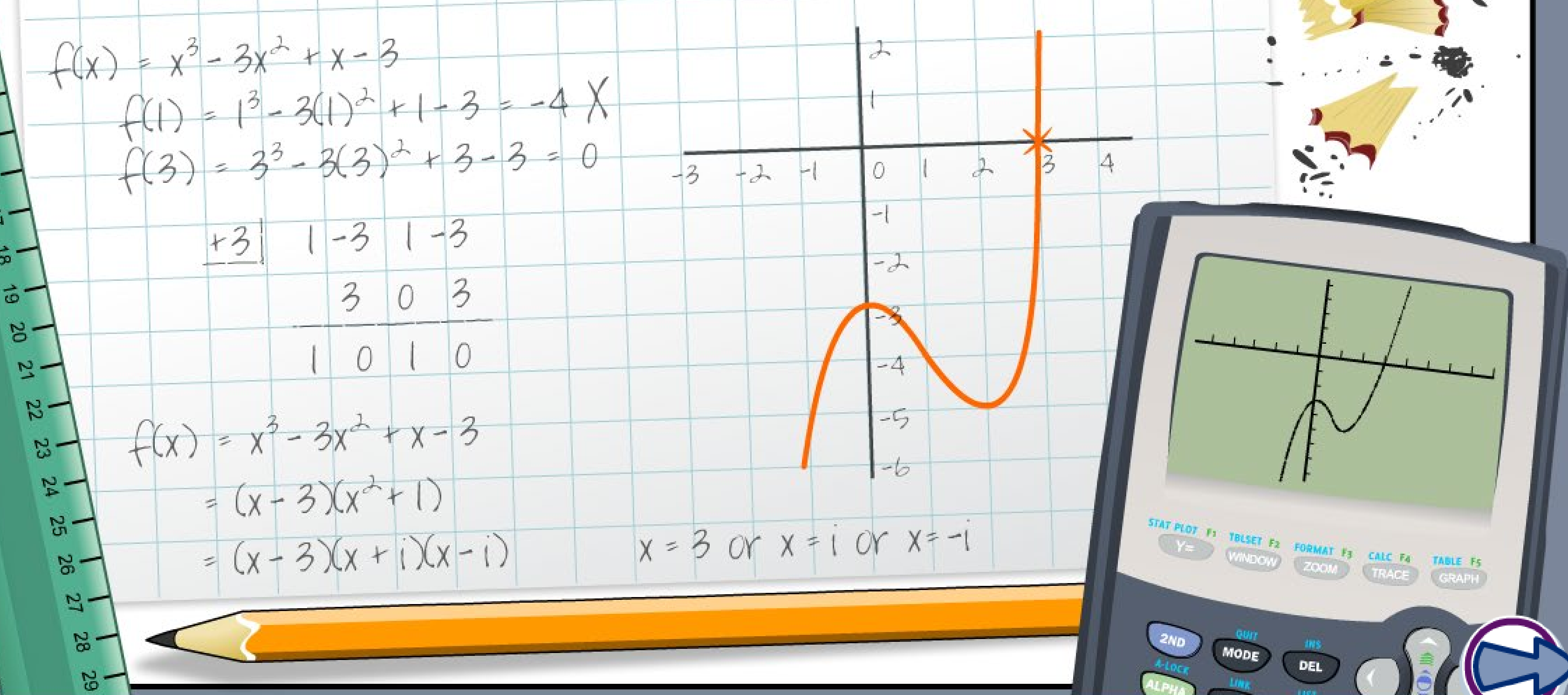
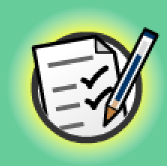
Solving Rational Equations



Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

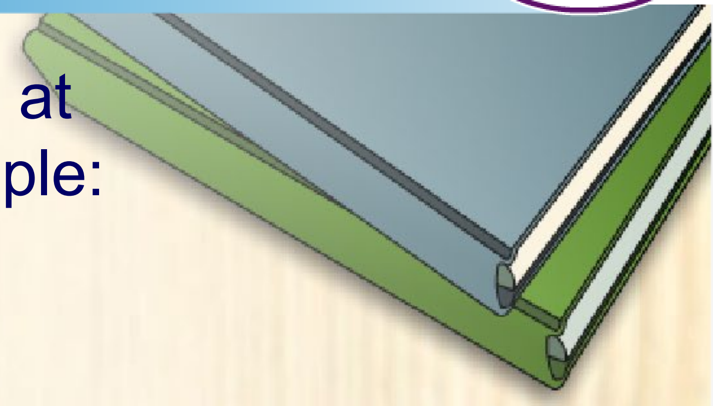
7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

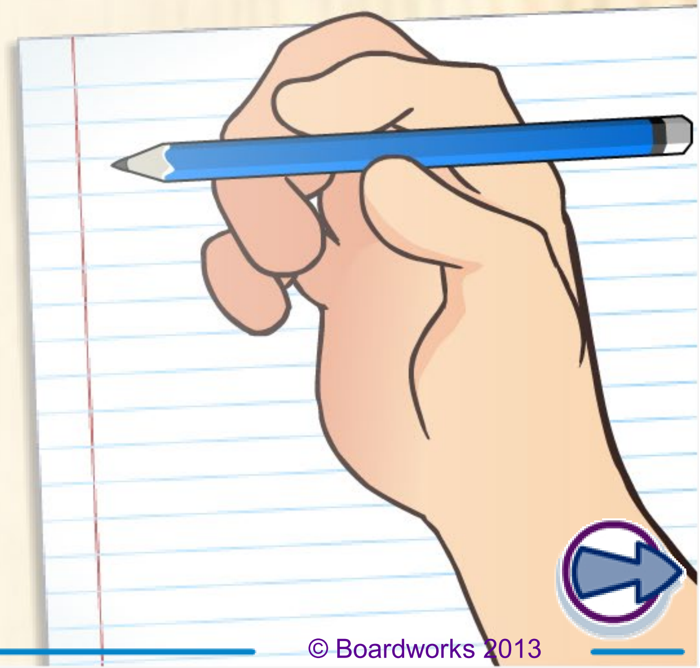
Rational equations



A **rational equation** is an equation with at least one rational expression. For example:

$$\frac{1}{x} + \frac{5}{x+4} = 2$$

The key to solving rational equations is to multiply through by the lowest common denominator (LCD). Then the equation can be solved using techniques for solving polynomial equations.



multiply by $x(x + 4)$: $x + 4 + 5x = 2x(x + 4)$

expand and simplify: $6x + 4 = 2x^2 + 8x$

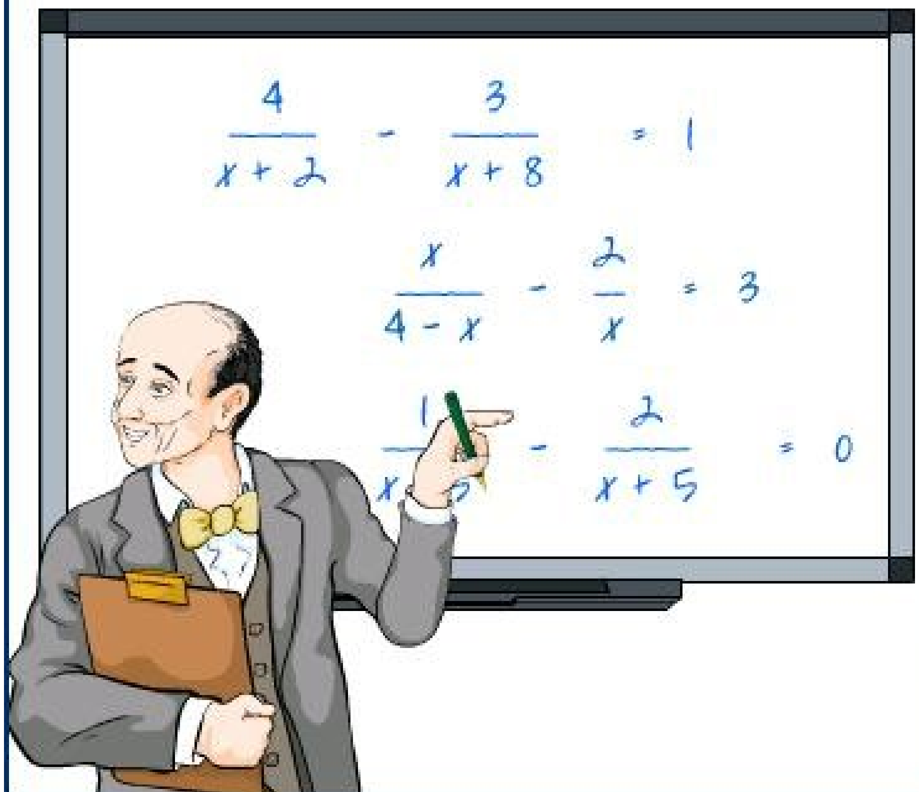
collect all terms: $0 = 2x^2 + 2x - 4$

divide by 2: $0 = x^2 + x - 2$

factor: $0 = (x + 2)(x - 1)$

$\boldsymbol{x = -2}$ or $\boldsymbol{x = 1}$

Solving rational equations



Press on a solving technique to see an example.

factoring

quadratic formula

single solution

Checking solutions graphically

A graphing calculator can be used to plot each side of the equation separately to check the solutions.

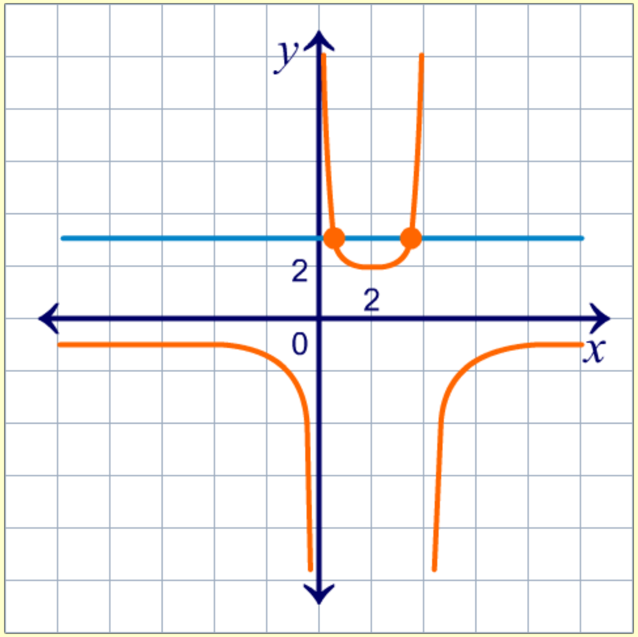
Verify graphically that $\dfrac{x}{4-x} + \dfrac{2}{x} = 3$

has solutions $x = 2.78$ **and** $x = 0.72$**.**

Plot $y_1 = x/(4-x)+(2/x)$ and $y_2 = 3$.

Use the "intersect" function on a graphing calculator to find the intersection of the two graphs.

The graphs intersect at the points where $x = 2.78$ **and** $x = 0.72$, which are the solutions. ✓



Extraneous solutions

Solve the equation $\frac{3}{x-3} = \frac{4x^2}{(x-3)(x+3)} - \frac{2x}{x+3}$.

multiply by LCD:	$3(x + 3) = 4x^2 - 2x(x - 3)$
expand:	$3x + 9 = 4x^2 - (2x^2 - 6x)$
simplify:	$0 = 2x^2 + 3x - 9$
factor:	$0 = (2x - 3)(x + 3)$
solve:	$x = \mathbf{-3}$ or $x = \mathbf{1.5}$

What happens when the solutions are substituted into the original equation?

When $x = -3$, some of the denominators are zero. It is not a solution.

This is called an **extraneous solution**. It is important to check all of the “solutions” in the original equation.

Practice

Drag all of the solutions to the correct boxes

1) $\dfrac{5}{x+7} = \dfrac{3}{x+3}$

2) $\dfrac{1}{x^2-2} = 0$

3) $\dfrac{2}{x-1} = \dfrac{2}{(x-1)^2}$

4) $\dfrac{1}{3x^2} + \dfrac{1}{6} = \dfrac{1}{2x}$

5) $\dfrac{x}{2} - \dfrac{1}{x} = \dfrac{1}{2} - \dfrac{1}{x^2}$

$x = 1$

$x = 2$

$x = 3$

$x = \sqrt{2}$

$x = -\sqrt{2}$

none

In 2 minutes, a large conveyor belt can move one pallet of recycled cans. If this belt and a smaller belt are used, the same number of cans are moved in 1.5 minutes. How long would the small belt take to move the cans on its own?

To solve this problem, find the belts' rates of work using:

rate of work · time spent working = part of job done

The large conveyor belt's rate is ½ pallet per minute.

If the small belt takes t minutes to move the pallet of cans, then in 1 minute it can move $\frac{1}{t}$ the cans so its rate is $\frac{1}{t}$.

Recycling application

If together the belts move one pallet of cans in 1.5 minutes, find t, the time it takes for the small belt to move one pallet of cans.

find the amount of work done by each belt:

belt type	rate of work	time worked	part of job done
large belt	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$
small belt	$\frac{1}{t}$	$\frac{3}{2}$	$\frac{1}{t} \cdot \frac{3}{2} = \frac{3}{2t}$

write an equation relating the parts of the job done: $\frac{3}{4} + \frac{3}{2t} = 1$

multiply the equation by the LCD: $3t + 6 = 4t$

solve: $t = 6$

It takes the small belt **6 minutes** to do the job alone.

Chemistry application

A chemist has 20 ounces of a 30% acid solution, but he wants the concentration to be 60%. How much pure acid must he add to make the new concentration?

use the formula: $\frac{\text{pure acid volume}}{\text{total solution volume}} = \text{final \% concentration}$

find initial volume of pure acid used: $20(0.3) = 6$ ounces

write equation for the new concentration: $\frac{x + 6}{x + 20} = 0.6$

multiply by LCD: $x + 6 = 0.6(x + 20)$

expand: $x + 6 = 0.6x + 12$

solve: $x = 15$

15 oz of pure acid needs to be added.

Solving formulas that are rational expressions

resistors in parallel:

solve for R_2.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

i ?

electrostatic force:

solve for r^2.

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

i ?

surface-area-to-volume ratio for cylinder:

$$\frac{2\pi rh + 2\pi r^2}{\pi r^2 h} > t$$

assume that $th - 2 > 0$ and solve for r.

i ?