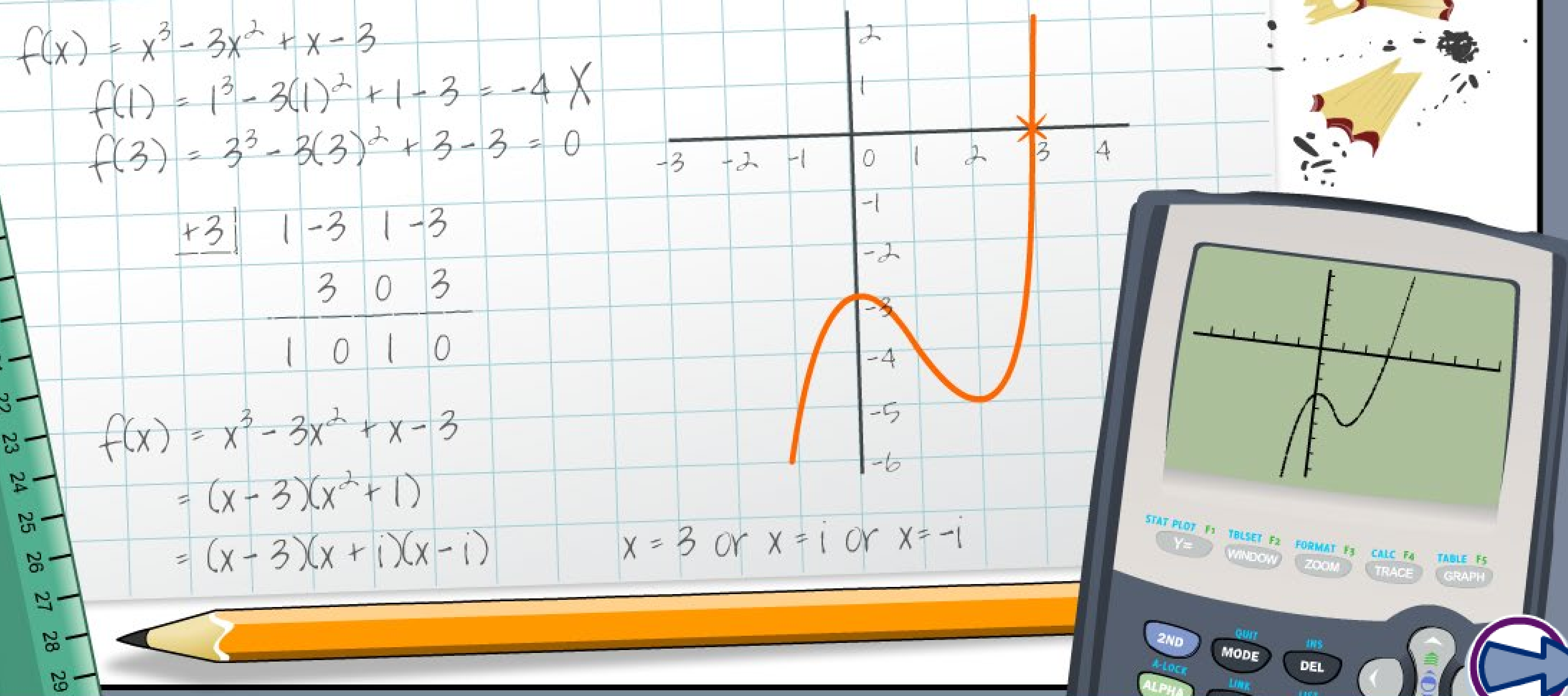
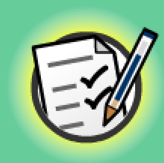
Solving Radical Equations

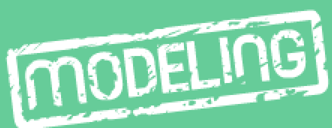


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

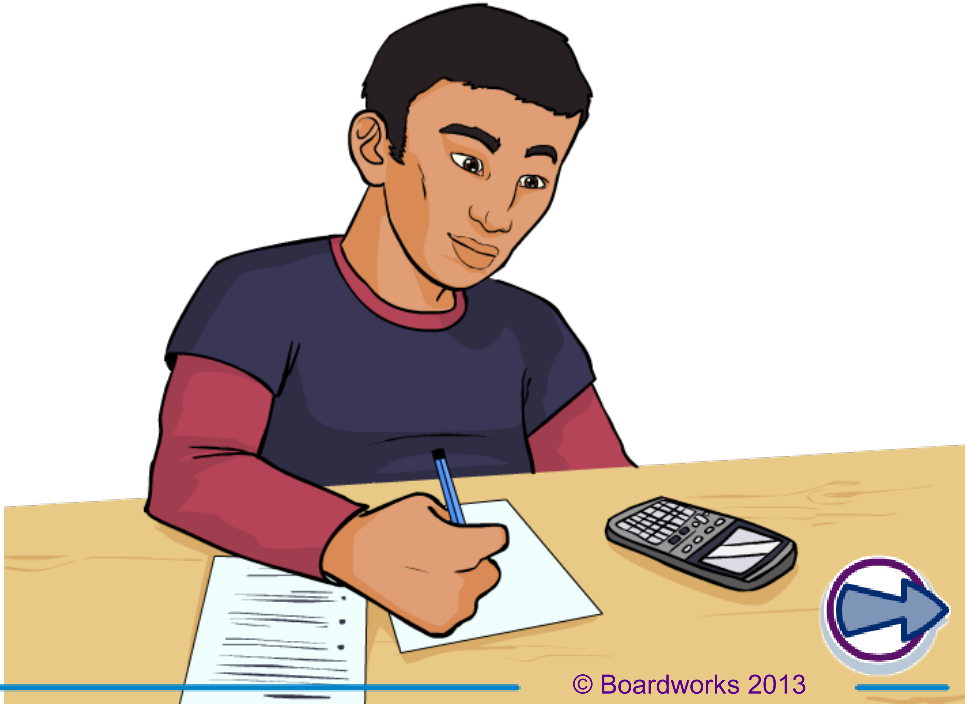
Radical equations

A **radical equation** is an equation that contains a variable in the radicand or an equation with a variable raised to a fractional rational power.

examples: $\sqrt{x+3} = 4$ $\quad x^{2/3} = 16$

non-examples: $\sqrt{54} = x - 6$ $\quad 9^{1/3} = x$

As you will learn, radical equations can have multiple solutions, so it is important to be careful when performing inverse operations.



Sort the equations as radical equations or not

radical equation	not a radical equation

$\sqrt{12} = x - 3^2$

Square root of a square

Complete the chart below. What happens when you take the square root of x^2?

x	x^2	$\sqrt{x^2}$
–2	4	2
–1	1	1
0	0	0
1	1	1
2	4	2

It is not possible to tell if the original number that was squared was positive or negative.

$$\sqrt{x^2} = |x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Solve the equation $x^2 = 25$ for x.

take the square root of both sides: $\sqrt{x^2} = \sqrt{25}$

consider both solutions: $|x| = 5$

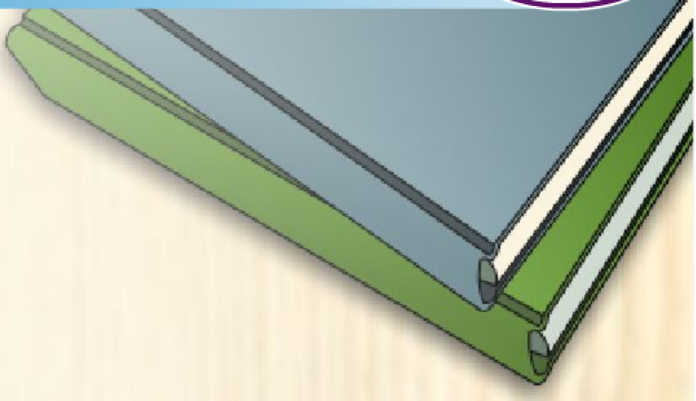
$$x = \pm 5$$

Ambiguity with rational exponents

Compare the solutions to the equations with different types of rational exponents. What do you notice?

odd numerator / even denominator	odd numerator / odd denominator	even numerator / odd denominator	even numerator / even denominator
$x^{3/2} = 1$	$x^{5/3} = 1$	$x^{2/3} = 1$	$x^{4/2} = 1$
$(x^{3/2})^2 = 1^2$	$(x^{5/3})^3 = 1^3$	$(x^{2/3})^3 = 1^3$	$(x^{4/2})^2 = 1^2$
$x^3 = 1$	$x^5 = 1$	$x^2 = 1$	$x^4 = 1$
$x = 1$	$x = 1$	$x = \pm 1$	$x = \pm 1$
one solution	**one solution**	**two solutions**	**two solutions**

When the numerator of the rational exponent is even, there are two real solutions.



The key to solving radical equations is to isolate the radical on one side of the equation.

$$\sqrt{x-5} = 7$$

isolate the radical: $\sqrt{x-5} = 7$

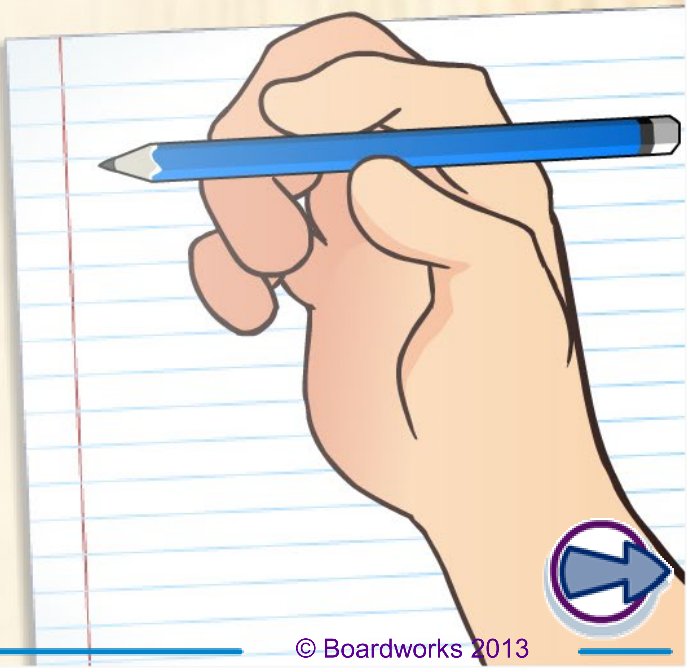
Here, the radical is already isolated on the left hand side.

square both sides of the equation: $x - 5 = 49$

solve for x: $x = 54$

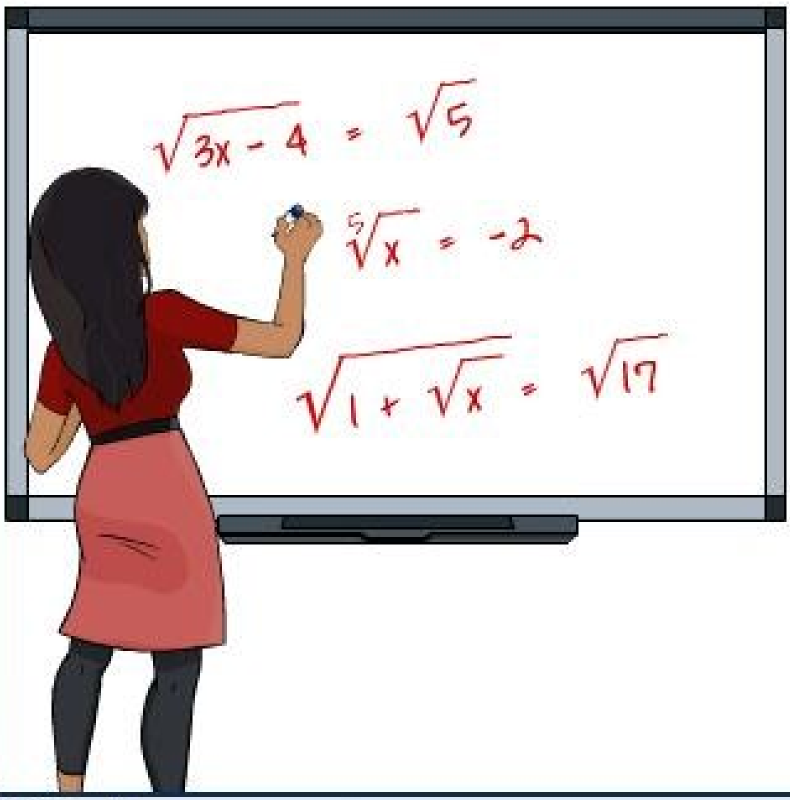
verify the solution by substitution: $\sqrt{x-5} = \sqrt{54-5}$

$= \sqrt{49} = 7$ ✓



Practice solving

Solving radical equations

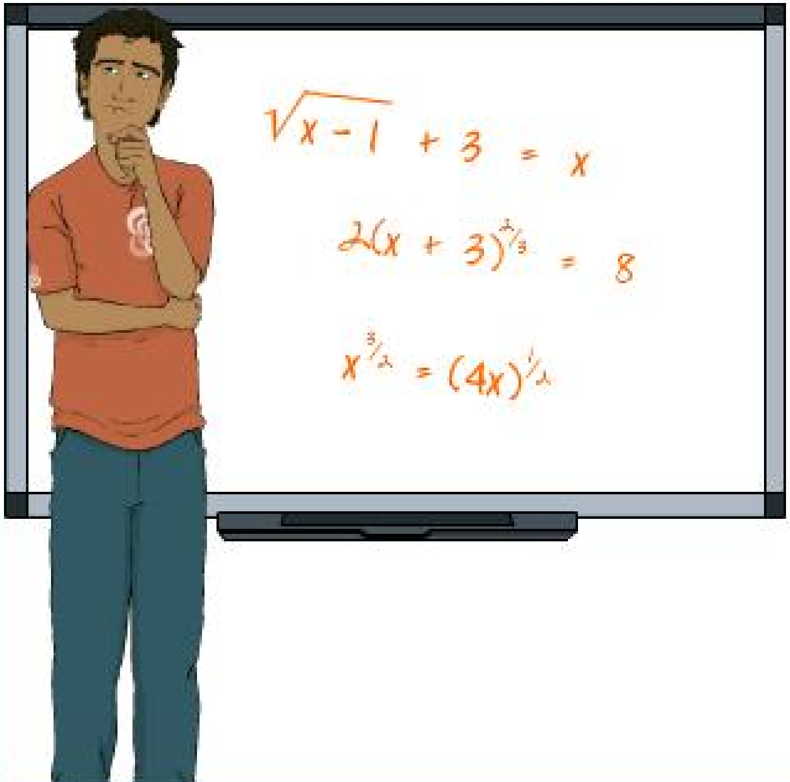


Press on a button to see an example.

- isolated square root
- isolated higher root
- non-isolated root (1)
- non-isolated root (2)
- challenge: nested roots

Multiple solutions

Solving radical equations with multiple solutions



Press on a button to see an example when the possible solutions are found using...

quadratic factoring

single rational exponent

two rational exponents

Match the equations to their solutions

A) $\sqrt{x+4} = -1$	$x = \pm 3$
B) $\sqrt{x} = x - 2$	none
C) $x^2 = 9$	$x = 36$
E) $(x+3)^{2/3} = 9$	$x = 24$, $x = -30$
D) $\sqrt{x} + 4 = 10$	$x = 4$

Graphing radical equations

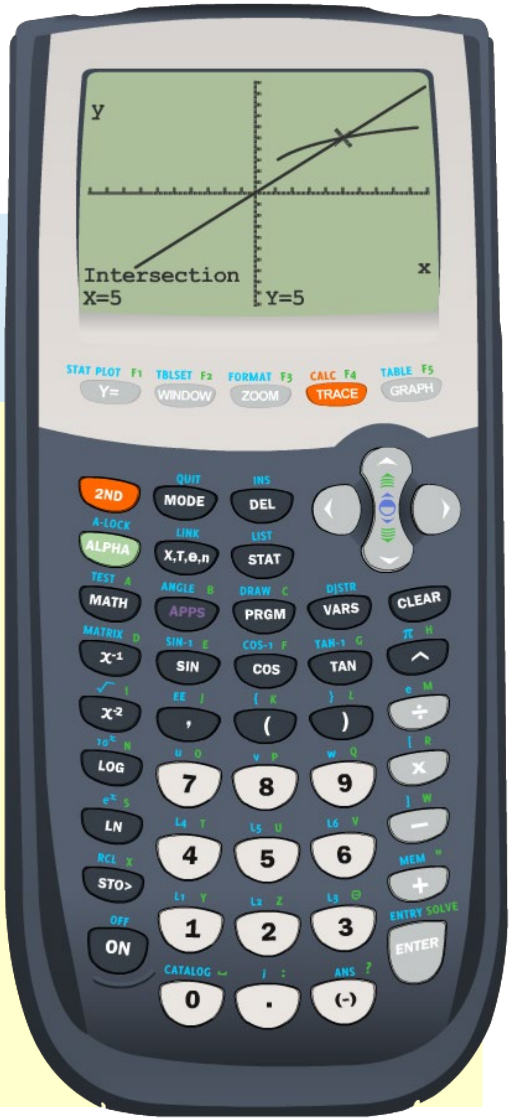
A graphing calculator can be used to check algebraic solutions to equations.

Graph $\sqrt{x - 1} + 3 = x$ to verify the solution $x = 5$ and the extraneous solution $x = 2$.

Enter the left-hand side of the equation into Y1 and the right-hand side of the equation into Y2.

Use the "intersect" feature to find where the graphs intersect.

There is only the single solution $x = 5$ since the graphs only intersect in one place. This means that $x = 2$ is indeed an extraneous solution.



Visibility

Have you ever wondered how far you can see when on the top of a mountain? Visibility (v, in miles) varies with the square root of altitude (a, in feet): $\boldsymbol{v = 1.225\sqrt{a}}$

Madison and Jim hiked Black Dome Mountain in Catskill Park. They estimated they could see 77 miles. What is their altitude, in feet?

$v = 1.225\sqrt{a}$

rearrange for a: $a = v^2/1.225^2$

substitute for v: $a = 77^2/1.225^2$

solve for a: $\boldsymbol{a}$ **= 3,951 feet**

Black Dome is 3,980 feet, so their estimate is fairly accurate.

Tsunami modeling

The speed of a tsunami, in meters per second (m/s), is $s = 3.1\sqrt{d}$, where d is the depth of the ocean in meters (m).

1) Find the speed of a tsunami if the depth of the water is 12 m.

?

2) Find the depth of the water if a tsunami's speed is 325 m/s.

?