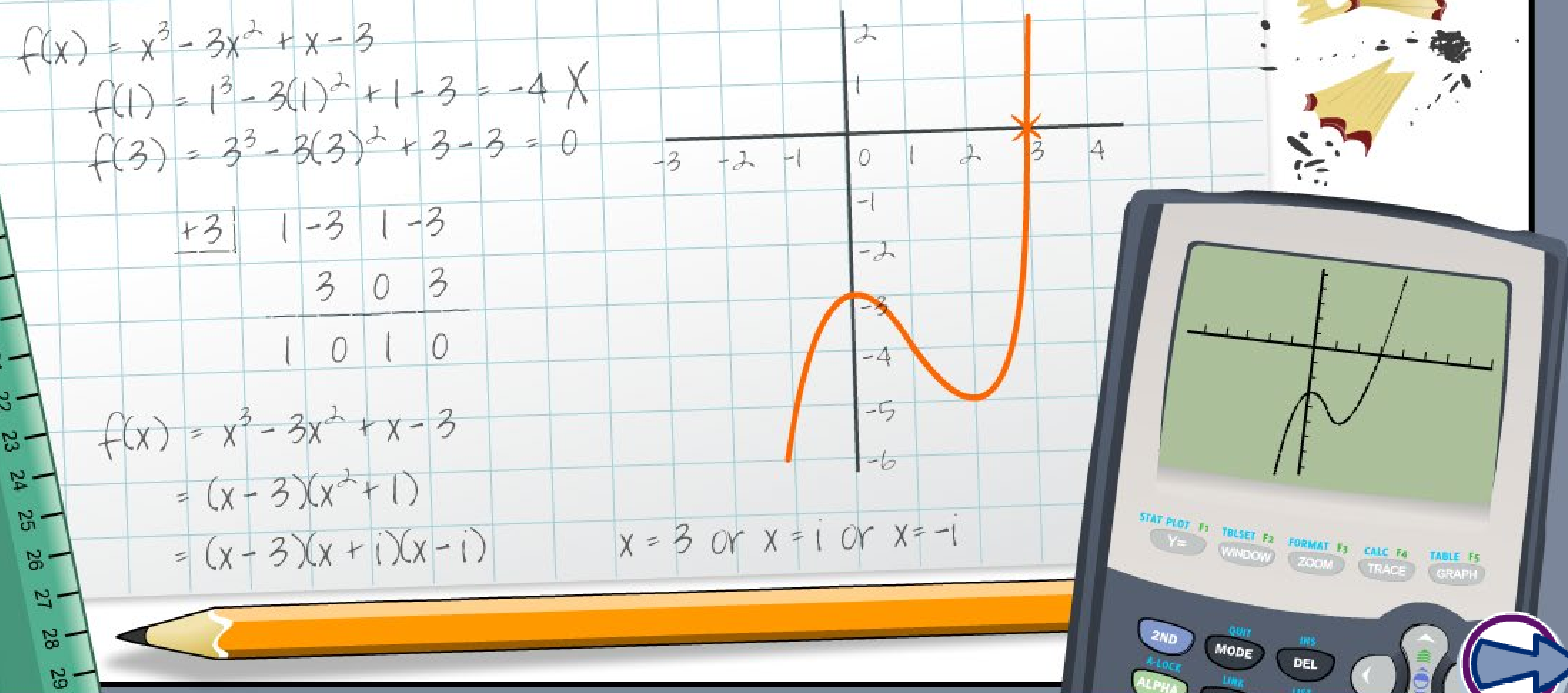
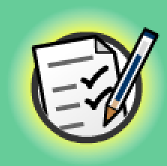
Solving Quadratic Equations

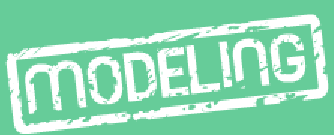


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Methods of solving

How do you solve quadratic equations?

1) Always **factor** if possible. There are several methods:

- taking out a common factor
- factoring into two sets of parentheses.

Once the quadratic is factored and set equal to zero, the roots can be found.

2) **Completing the square** will solve a quadratic that cannot be factored.

3) Use the **quadratic formula**.

Solving quadratics by factoring

Quadratic expressions of the form $x^2 + bx + c = 0$ can be factored if they can be written using parentheses as

$$(x + d)(x + e) = 0 \text{ where } d \text{ and } e \text{ are integers.}$$

How?

Solve the equation $x^2 - 2x - 15 = 0$ by factoring.

Quadratic expressions of the form $ax^2 + bx + c = 0$ can be factored if they can be written using parentheses as

$$(dx + e)(fx + g) = 0 \text{ where } d, e, f \text{ and } g \text{ are integers.}$$

How?

Solve the equation $3x^2 + 11x - 4 = 0$ by factoring.

Find x for $x^2 - 4 = 0$.

$(x + 1)(x + 1) = 0$

Use the scrollers to complete each step of the solution. Once a step is correct, the next step will appear.

Select a difficulty level (1 – 3) below.

1 2 3

Can it be factored?

Sort the quadratics into the correct boxes

can be factored into the form $(ax + b)(cx + d) = 0$	cannot be factored into the form $(ax + b)(cx + d) = 0$

$3x^2 + 2x + 7 = 0$

Recap on completing the square

If a quadratic cannot be factored into two sets of parentheses, solve by completing the square instead.

completing the square: $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

Solve $x^2 + 8x + 9 = 0$ by completing the square.

complete the square: $(x + 8/2)^2 - 4^2 + 9 = 0$

simplify: $(x + 4)^2 - 7 = 0$

add 7: $(x + 4)^2 = 7$

square root: $x + 4 = \pm\sqrt{7}$

subtract 4: $x = \pm\sqrt{7} - 4$

The solution is: $\mathbf{x = \sqrt{7} - 4}$ **or** $\mathbf{x = -\sqrt{7} - 4}$

Completing the square example

Remember, when the coefficient of x^2 is $a \neq 1$, factor out a from the x terms before completing the square

Solve $3x^2 - 6x - 5 = 0$ by completing the square.

take out the factor 3 from x terms: $3(x^2 - 2x) - 5 = 0$

complete the square: $3((x - 1)^2 - 1) - 5 = 0$

distributive property: $3(x - 1)^2 - 3 - 5 = 0$

simplify: $3(x - 1)^2 - 8 = 0$

add 8 to both sides: $3(x - 1)^2 = 8$

divide both sides by 3: $(x - 1)^2 = \frac{8}{3}$

square root both sides: $x - 1 = \pm\sqrt{\frac{8}{3}}$

The solution is: $\mathbf{x = 1 + \sqrt{\frac{8}{3}}}$ or $\mathbf{x = 1 - \sqrt{\frac{8}{3}}}$

Different solving methods

Write the quadratic $2x^2 - 4x - 6 = 0$ in the following different forms, and solve in each form:

1) $d(x + e)(x + f) = 0$, 2) $(gx + h)(j + k) = 0$, 3) $m(x + n)^2 + p = 0$

1) $2(x + 1)(x - 3) = 0$

divide by 2: $(x + 1)(x - 3) = 0$

identify solutions: $\mathbf{x = -1}$ **or** $\mathbf{x = 3}$

2) $(2x + 2)(x - 3) = 0$

identify solutions: $\mathbf{x = -1}$ **or** $\mathbf{x = 3}$

3) $2(x - 1)^2 - 8 = 0$

add 8: $2(x - 1)^2 = 8$

divide by 2 and square root: $x - 1 = \pm\sqrt{4}$

rearrange: $x = \sqrt{4} + 1$ or $x = -\sqrt{4} + 1$

simplify: $x = 2 + 1$ or $x = -2 + 1$

identify solutions: $\mathbf{x = 3}$ **or** $\mathbf{x = -1}$

Each method gives the same solutions, so choose the method that you find the easiest.

Recap on the quadratic formula

Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved by substituting the values of a, b and c into a formula:

quadratic formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression $b^2 - 4ac$ in the quadratic formula (known as the **determinant**) shows how many unique real roots there are.

How many roots does an equation have if $b^2 - 4ac$ is:

i) positive There are **two** unique real roots.

ii) zero There is **one** unique real root.

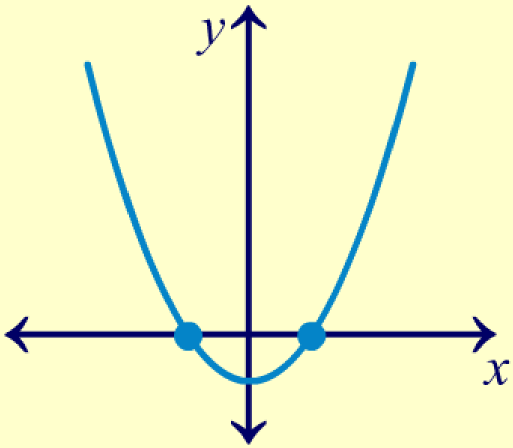
iii) negative? There are **no** unique real roots.

The roots of a graph

We can demonstrate the number of roots using graphs.

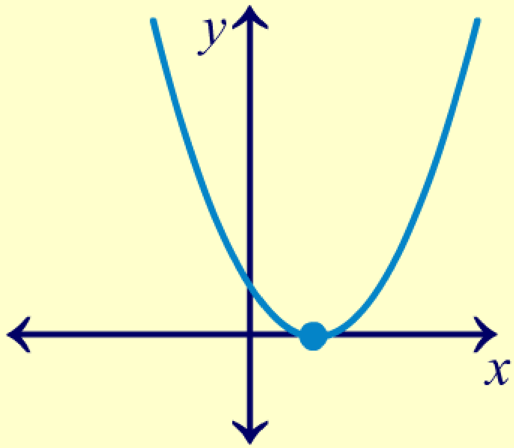
Remember, if we plot the graph of $y = ax^2 + bx + c$, the solutions to the equation $ax^2 + bx + c = 0$ are given by the points where the graph crosses the x-axis.

$b^2 - 4ac$ is positive



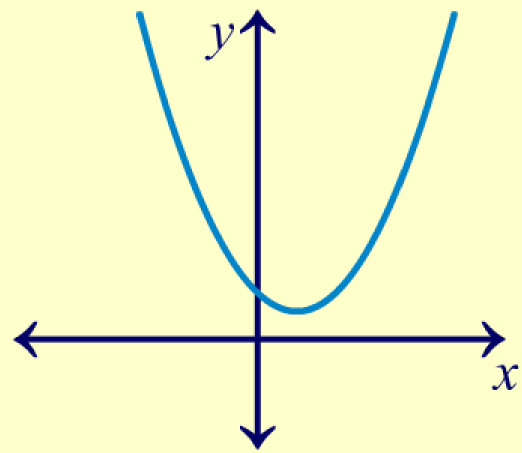
two real roots

$b^2 - 4ac$ is zero



one real root

$b^2 - 4ac$ is negative



no real roots

Solve $x^2 + 9x + 20 = 0$ by factoring.

find two numbers that multiply to give +20 and sum to +9: 5 and 4

factor: $(x + 5)(x + 4) = 0$

zero product rule: $x + 5 = 0$ or $x + 4 = 0$

find solutions: $\boldsymbol{x = -5}$ or $\boldsymbol{x = -4}$

Now solve $x^2 + 9x + 20 = 0$ using the quadratic formula.

recall formula and substitute in values:

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(20)}}{2} = \frac{-9 \pm \sqrt{81 - 80}}{2}$$

$$= \frac{-9 \pm 1}{2} \quad \text{So } x = \frac{-10}{2} = \mathbf{-5} \quad \text{or} \quad x = \frac{-8}{2} = \mathbf{-4}$$

Which solving method do you prefer?

Recall the quadratic formula used to solve quadratic equations:

quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

When a quadratic has no real roots, it has **complex roots**.

Which of the following equations has complex roots:
1) $2x^2 - 5x - 1 = 0$ 2) $3x^2 - 2x + 2 = 0$?

When the determinant, $b^2 - 4ac$ is negative, it results in taking the square root of a negative number, meaning that the roots are complex numbers.

1) $b^2 - 4ac = 25 - 4(2)(-1) = 33$, so it has real roots.

2): $b^2 - 4ac = 4 - 4(3)(2) = -20$, so it has complex roots.

Complex roots?

Do these equations have real or complex roots?

1) $2x^2 + 5x + 2 = 0$?

2) $-x^2 + 3x - 3 = 0$?

3) $x^2 - x + 1 = 0$?

4) $3x^2 - 5x - 1 = 0$?

5) $-x^2 - 8x - 16 = 0$?

real | complex

Finding complex roots

Solve $x^2 + 4x + 10 = 0$ using the quadratic formula.

recall the formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Here, $a = 1$, $b = 4$ and $c = 10$.

substitute a, b, c: $x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(10)}}{2(1)}$

simplify: $x = \dfrac{-4 \pm \sqrt{-24}}{2}$

$\sqrt{-24} = i\sqrt{24}$: $x = \dfrac{-4 \pm 6i\sqrt{2}}{2}$

identify solutions: $\mathbf{x = -2 + 3i\sqrt{2}}$ or $\mathbf{x = -2 - 3i\sqrt{2}}$

What do you notice about the two solutions?

The solutions are a pair of complex conjugates.

Solving quadratics

Question: 1/5

Solve $10x - 34 = x^2$.

Press the "=" button to show the work step-by-step.

$x = \frac{8}{3}$ or $x = -5$

$x = 17$ or $x = -2$

$x = -5 \pm 6i$

$x = 5 \pm 3i$

Window cleaning

Mike is a window cleaner. He and his apprentice Neil can clean all the windows of a certain skyscraper in 4 hours, when working as a team. On his own, Mike can complete the job 6 hours quicker than Neil.

How long will it take Neil to do this job alone?

let N = the number of hours for Neil to do the job alone:

The number of hours it takes Mike to do the job alone is $N - 6$.

In 1 hour, Neil can complete $1/N$ of the job and Mike $1/(N - 6)$ of the job.

Together, they complete $\frac{1}{N} + \frac{1}{N-6}$ of the job each hour.

They can complete the whole job in 4 hours, so $\frac{4}{N} + \frac{4}{N-6} = 1$

This can be rearranged into a quadratic equation and solved for N.

Window cleaning

$$\frac{4}{N} + \frac{4}{N-6} = 1$$

write as one fraction over $N(N-6)$:

$$\frac{4(N-6) + 4N}{N(N-6)} = 1$$

distribute:

$$\frac{4N - 24 + 4N}{N^2 - 6N} = 1$$

simplify:

$$\frac{8N - 24}{N^2 - 6N} = 1$$

multiply by $N^2 - 6N$:

$$8N - 24 = N^2 - 6N$$

rearrange:

$$N^2 - 14N + 24 = 0$$

factor:

$$(N-2)(N-12) = 0$$

solve:

$$N = 2 \text{ or } N = 12$$

Mike completes the job 6 hours quicker than Neil, so $N = 2$ is not possible.

The solution must be that Neil takes **12 hours** to complete the job.