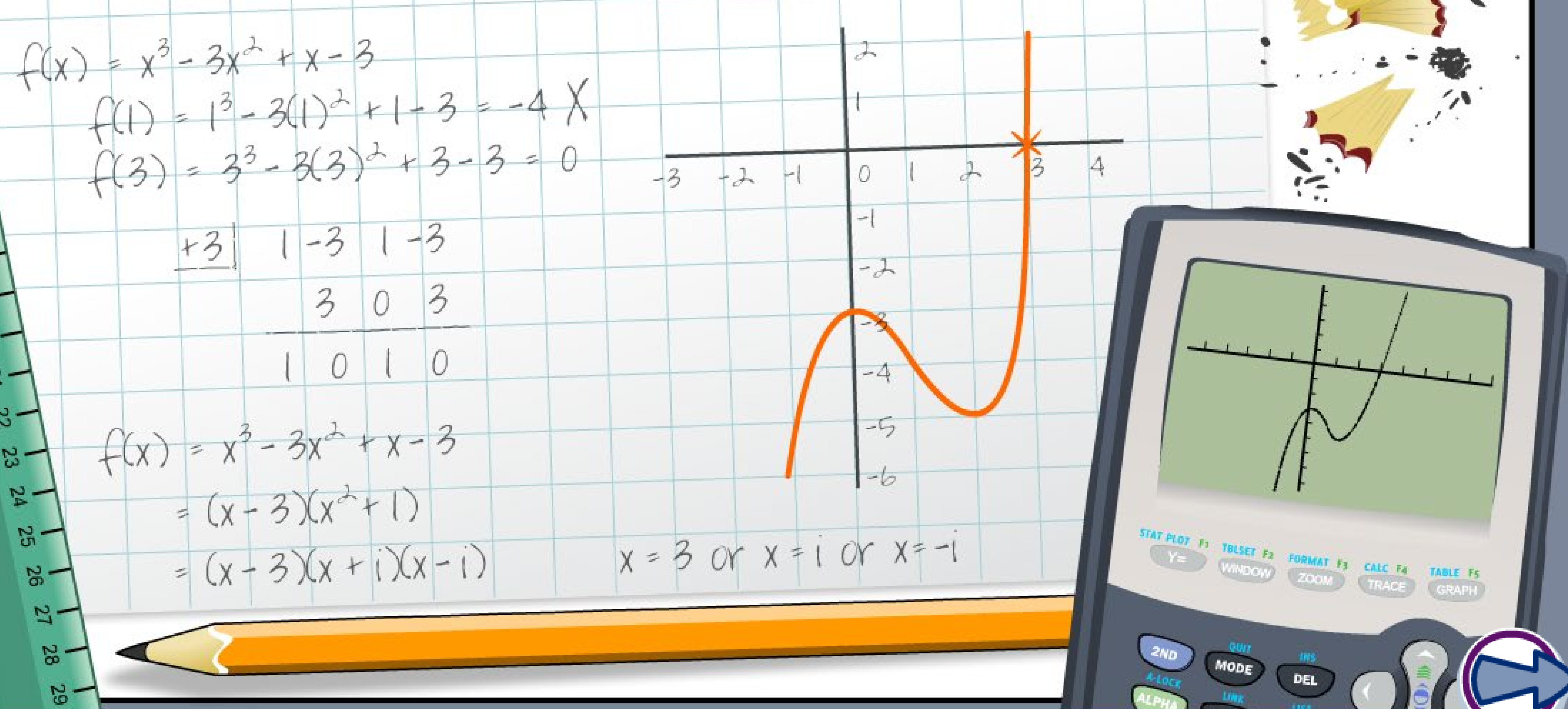
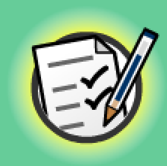
Solving Problems with Graphs



Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

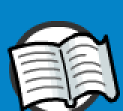
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Solve the equation $x + 1 = -2x + 4$ algebraically.

add $2x$: $3x + 1 = 4$

subtract 1: $3x = 3$

divide by 3: $x = 1$

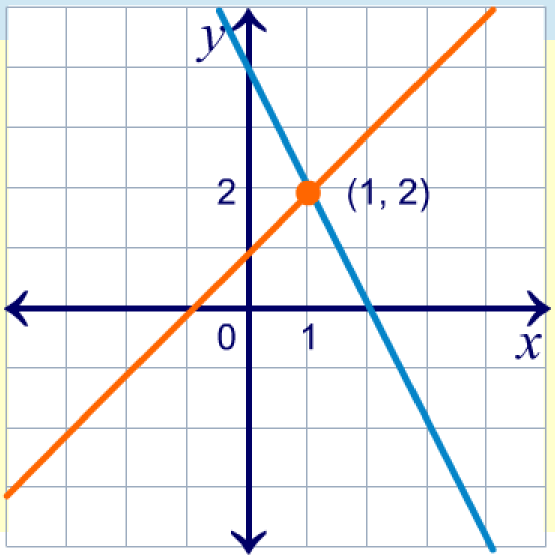
How can the solution of the equation $x + 1 = -2x + 4$ be found graphically?

Look at each side of the equation as a separate equation.

left side: $y = x + 1$

right side: $y = -2x + 4$

The intersection of the two lines gives the solution: $x = 1$.



Solving systems of equations

Solve the following system of linear equations by the substitution method: $y = 2x - 3$ Ⓐ and $y = -x + 6$ Ⓑ.

substitute Ⓐ into Ⓑ: $2x - 3 = -x + 6$

add x: $3x - 3 = 6$

add 3: $3x = 9$

divide by 3: $\mathbf{x = 3}$

substitute $x = 3$ into Ⓐ: $y = 2(3) - 3$

$\mathbf{y = 3}$

The solution to the system is (3, 3).

The point where the graphs of the two equations intersects is (3, 3).

working backwards: given $2x - 3 = -x + 6$ to solve, we can rewrite it as the system $y = 2x - 3$ and $y = -x + 6$ and solve it graphically.

When solving an equation graphically, look at each side of the equation as a separate function to graph.
The solution is the intersection of the functions on the graph.

Determine the possible number of solutions by asking:
"What does the graph of each side of the equation look like and at how many points might their graphs intersect?"

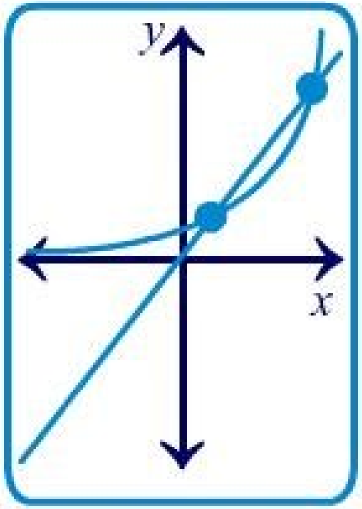
Press **start** to test your knowledge of the number of intersections of different lines and curves.

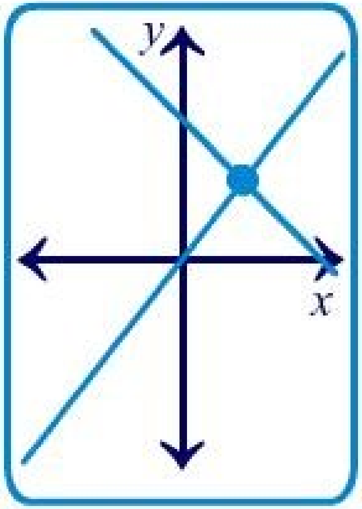
start

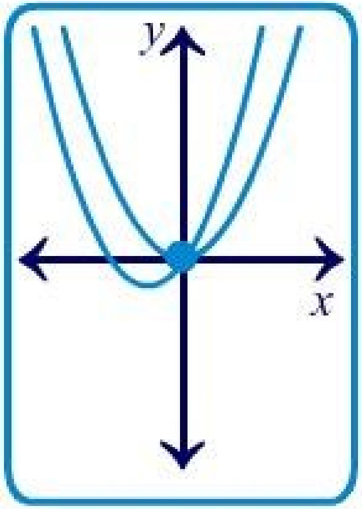
Graphical solutions

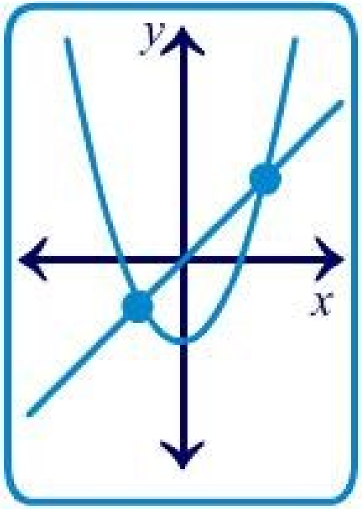
Match each equation to the correct graphical solution

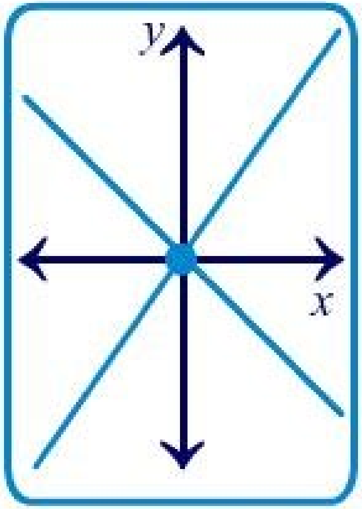
$2x = 6 - x$	$-x = 3x$	$x = x^2 - 5$	$x^2 + x = x^2$	$2^x = 2x$









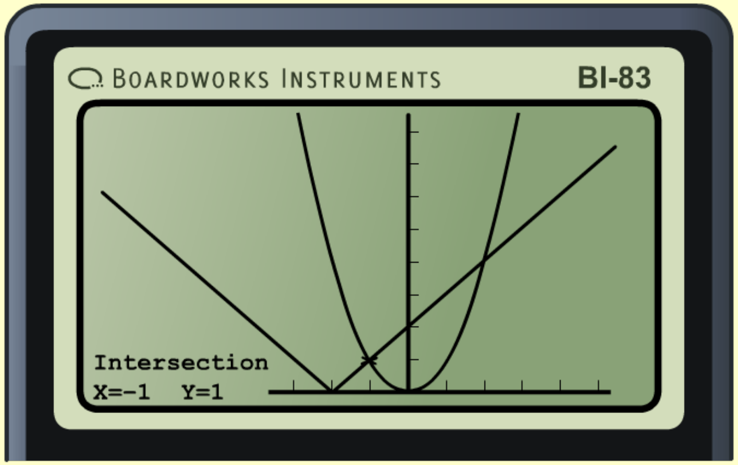


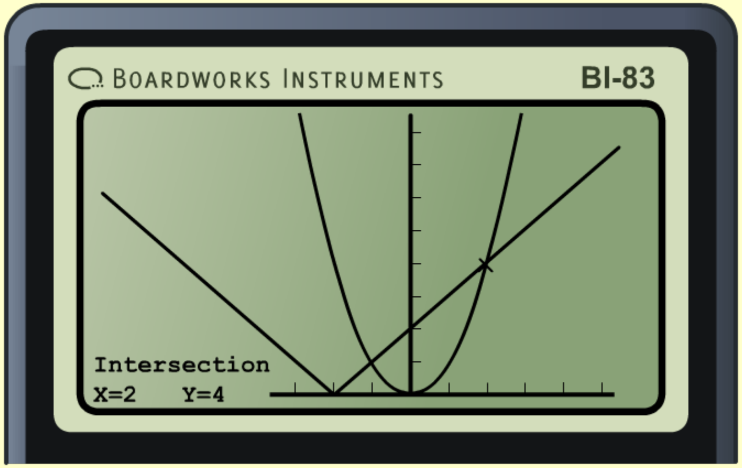
Solving graphically example

Solve $x^2 = |x + 2|$ graphically.

Graph the two equations $y = x^2$ and $y = |x + 2|$.

The graph of $y = x^2$ is a parabola and the graph of $y = |x + 2|$ is V-shaped.





Graphing the two equations shows that the parabola and the absolute value (V-shaped) graph intersect at two points: (2, 4) and (–1, 1).

The solutions to the equation are the x-values: x = **{–1, 2}**.

Extraneous solutions

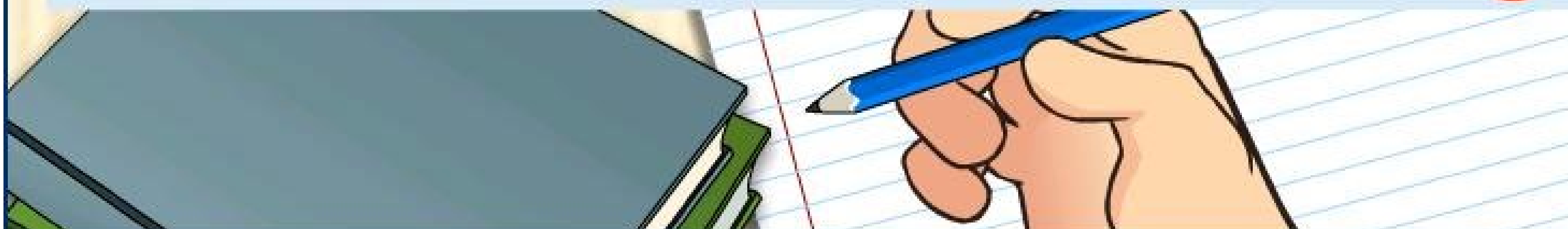
An extraneous solution is a solution obtained from manipulating an equation that is **not** actually a solution of the original equation.

1) Solve $\sqrt{x - 2} = 4 - x$ algebraically.

?

2) Check the solution by substitution.

3) Confirm the solution graphically.



A logarithmic equation (1)

Solve ln $(x + 3) = 2 \ln x$, first algebraically and then graphically. Compare both results. Remember to consider the domain when stating your final answer.

by laws of logarithms: $\ln(x + 3) = \ln(x^2)$

equate arguments: $x + 3 = x^2$

rearrange: $x^2 - x - 3 = 0$

solve using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} = \frac{1 \pm \sqrt{13}}{2}$$

$$= 2.303 \text{ or } -1.303$$

The argument of the logarithms in the equation cannot be negative, so the only possible solution is x **= 2.303**.

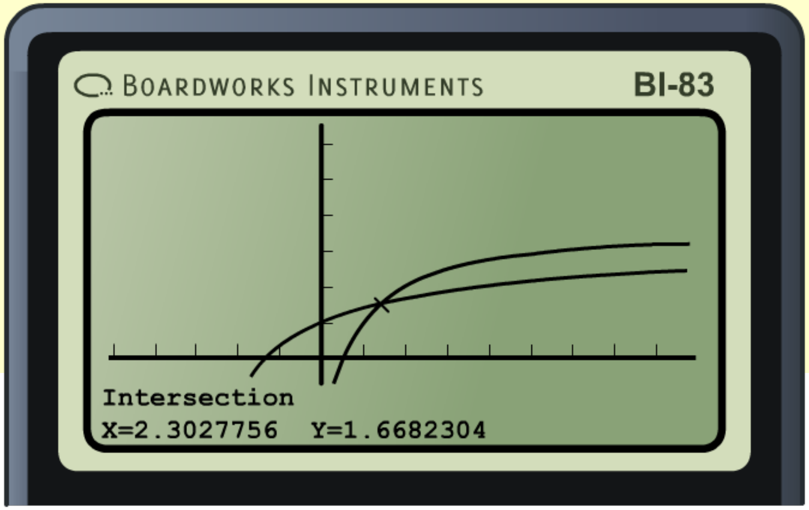
A logarithmic equation (2)

For the equation ln $(x + 3) = 2 \ln x$:

- The right side of the equation is undefined for negative values. The x-values for which this expression is defined are $(0, \infty)$.
- The left side of the equation is defined on $(–3, \infty)$.

Therefore, the whole equation is defined on $(0, \infty)$.

There is one solution: $x = \dfrac{1 + \sqrt{13}}{2}$ = **2.303** (to nearest thousandth)



Notice that the graph shows only one point of intersection. Its x-value is the solution we found algebraically.

Rocket flight

Some physics students were on the edge of a cliff testing a model rocket. They shot it into the air. Its height, h, in feet, as a function of time, t, in seconds, is modeled by the function $h(t) = -16t^2 + 96t + 131$.

Use your graphing calculator to answer the following.

1) How high was the cliff?

2) At what time(s) was the rocket 239 ft above the ground?

3) What is the maximum height that the rocket reached?

4) How many seconds passed after takeoff before the rocket hit the ground?

Mixing chemicals (1)

A chemist has 2 liters of a 40% solution of hydrochloric acid but needs a 70% solution for a particular application.

Write a function $C(x)$ that represents the concentration of the new mixture and use your graphing calculator to determine the amount of pure acid she should add.

x = the amount of pure acid to add.

write the function: $C(x) = \dfrac{x + 0.4(2)}{x + 2} = \dfrac{x + 0.8}{x + 2}$

The numerator of the rational expression is the amount of pure acid you will have: x plus the part that is pure in the 40% mixture.

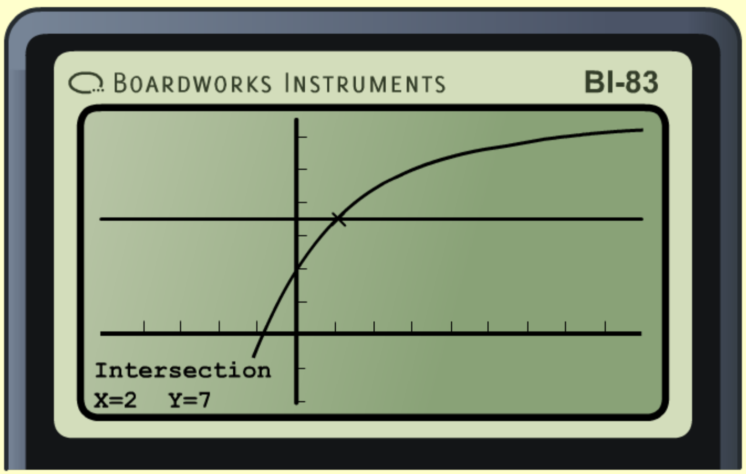
The denominator of the rational function is the total amount of the solution.

Mixing chemicals (2)

Enter $Y_1 = \dfrac{x + 0.8}{x + 2}$ into your calculator.

Then, to find how much pure acid (x) to add to get a 70% concentration, plot the line "$Y_2 = 0.7$".

Use the intersection feature to find x.



The chemist needs to mix **2 liters** of the pure acid with the 2 liters she already has.