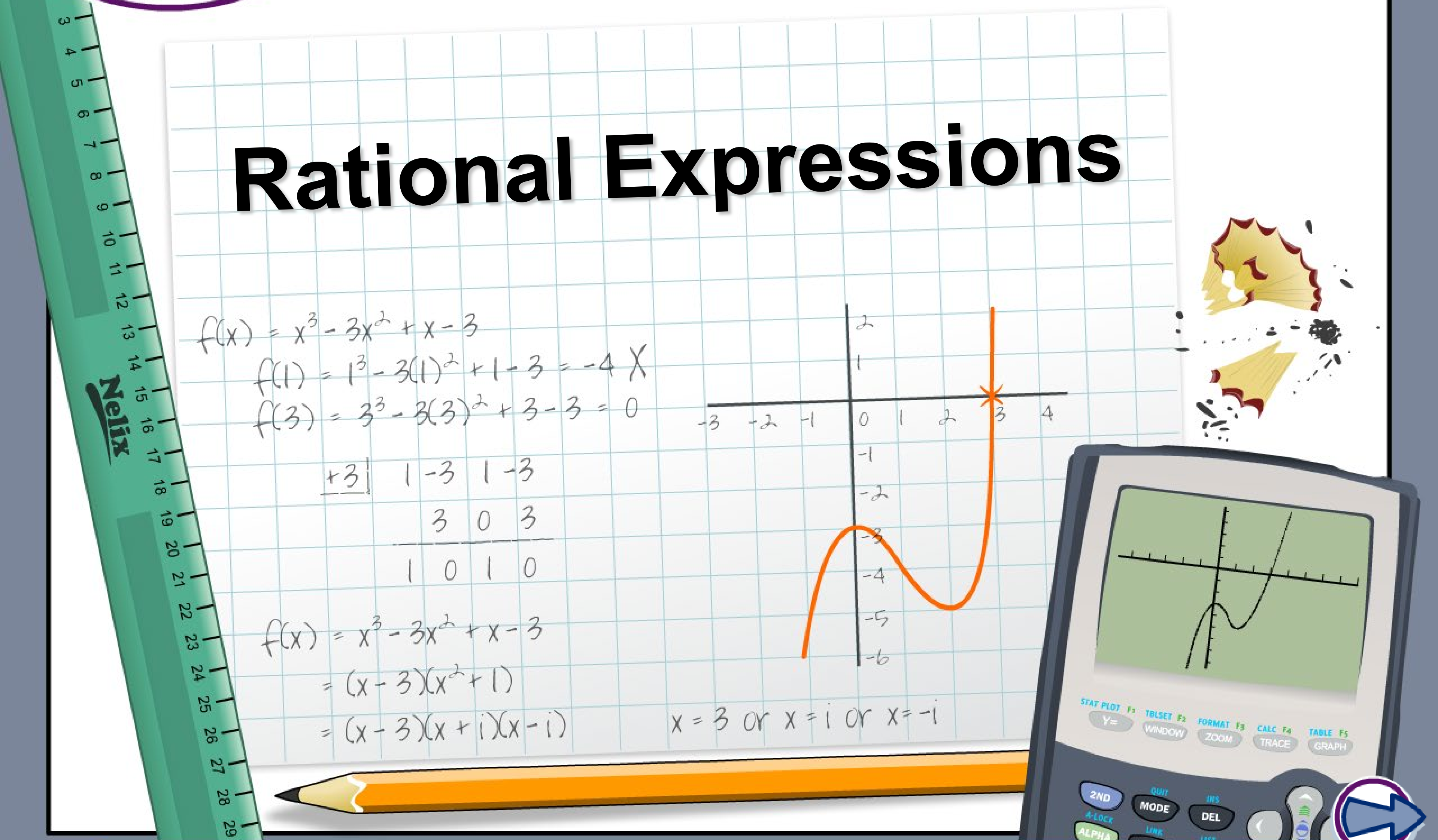
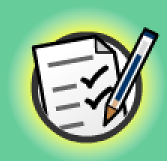
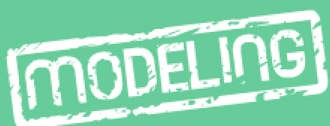
Rational Expressions



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

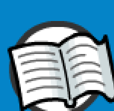
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Rational expressions

Remember, a **rational number** is any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Numbers written in this form are called fractions.

In algebra, a **rational expression** is an **algebraic fraction** of two polynomials.

rational expression: $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.

Can $P(x)$ and $Q(x)$ be any polynomials?

No, $Q(x)$ cannot be 0 because the expression would be undefined.

Sort the expressions as rational expressions or not

rational expression	not a rational expression
$= \frac{P(x)}{Q(x)}$	$\neq \frac{P(x)}{Q(x)}$

x^4

S C ?

Undefined values

Drag the correct undefined values into place

1)	$\dfrac{(x-2)(x+3)}{(x+2)(x+1)}$	
2)	$\dfrac{1}{(x-2)^2}$	
3)	$\dfrac{x^2+1}{x^2-x-2}$	
4)	$\dfrac{x+3}{x(3x^2-3)}$	

$x \neq 3$

$x \neq 2$

$x \neq 1$

$x \neq 0$

$x \neq -1$

$x \neq -2$

$x \neq -3$

S C

When the numerator and the denominator of a numerical fraction contain a common factor, the fraction can be simplified.

A numerical fraction is in its simplest form when the numerator and denominator have no common factors.

Simplify the numerical fraction $\frac{28}{42}$.

find the greatest common factor of the numerator and denominator: $\frac{28}{42} = \frac{2 \cdot \cancel{14}}{3 \cdot \cancel{14}}$

simplify, the numerator and denominator are both divisible by 14: $= \frac{2}{3}$

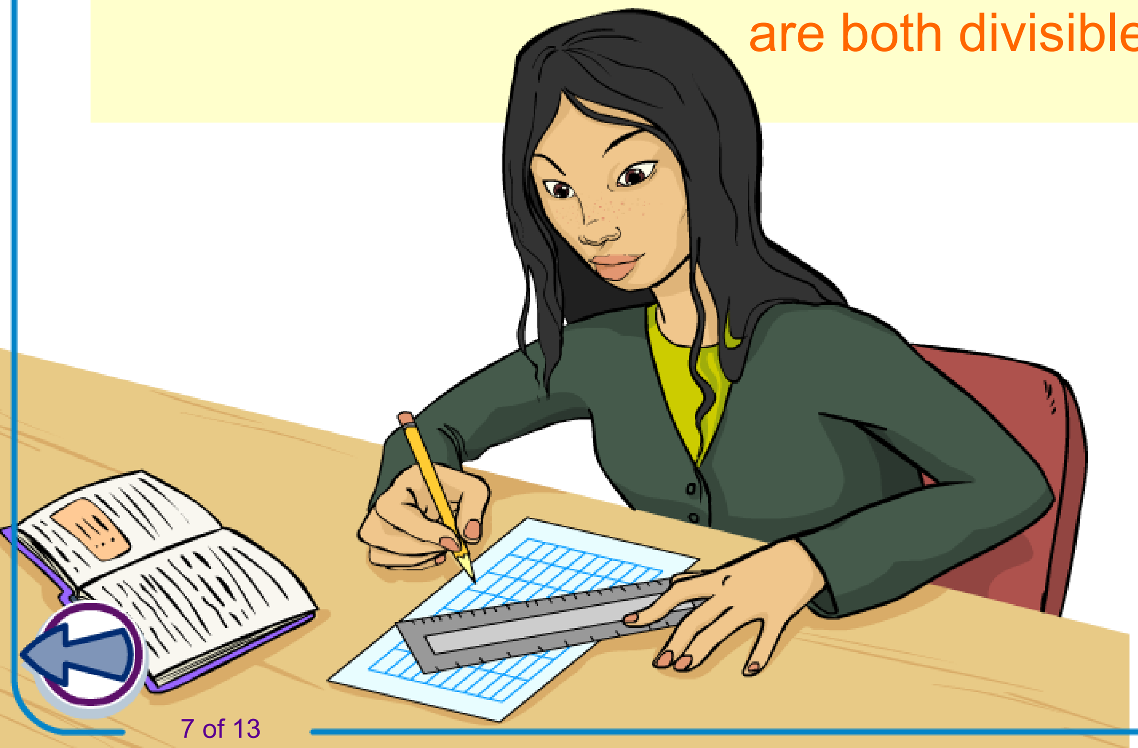
Numerical fractions and algebraic fractions can be manipulated and simplified in similar ways.

Simplifying rational expressions

Simplify the rational expression $\frac{6a^2}{8a^3}$.

find the greatest common factor of the numerator and denominator: $\frac{6a^2}{8a^3} = \frac{3\cdot\cancel{2a^2}}{4a\cdot\cancel{2a^2}}$

simplify, the numerator and denominator are both divisible by $2a^2$: $= \frac{3}{4a}$



When the numerator and denominator of a rational expression have no common divisor, it is in its **simplest form**.

What to look for when simplifying

	common coefficient factors	common polynomial factors	additive inverses
Simplify the expressions.	$\dfrac{3b-6}{3b^2+3}$	$\dfrac{x^2+x-2}{x^2-1}$	$\dfrac{3-y^2}{y^2-3}$
factor the numerator and denominator:	$=\dfrac{3(b-2)}{3(b^2+1)}$	$=\dfrac{(x-1)(x+2)}{(x-1)(x+1)}$	$=\dfrac{-(y^2-3)}{y^2-3}$
divide common factors:	$=\dfrac{\cancel{3}(b-2)}{\cancel{3}(b^2+1)}$	$=\dfrac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)}$	$=\dfrac{\overset{-1}{\cancel{-(y^2-3)}}}{\cancel{y^2-3}}$
simplify:	$=\dfrac{(b-2)}{(b^2+1)}$	$=\dfrac{(x+2)}{(x+1)}$	$=-1$

Simplifying practice

Simplifying rational expressions

Question 1/4: Simplify $\frac{7x^2y^2}{21xy^3}$

$\frac{x}{3y}$

$\frac{y}{3x}$

$\frac{x}{7y}$

$\frac{y}{7x}$

Press the "=" button to show the work step by step.

Multiplying rational expressions

Multiplying rational expressions is similar to multiplying numerical fractions, but with polynomials instead of integers.

Multiply the rational expressions $\frac{1-x}{x+1}$ and $\frac{x+1}{x-3}$.

multiply the two numerators:
multiply the two denominators:

$$\frac{1-x}{x+1} \cdot \frac{x+1}{x-3} = \frac{(1-x)(x+1)}{(x+1)(x-3)}$$

the numerator and denominator have a common factor, divide each by $(x + 1)$:

$$= \frac{1-x}{x-3}$$

identify restrictions on denominator: $x \neq 3,\ x \neq -1$

The restrictions must be identified from the original expressions.

Dividing rational expressions

Like with numerical fractions, to divide two rational expressions, multiply the dividend by the reciprocal of the divisor.

Divide the rational expression $\frac{1-x}{x+1}$ by $\frac{x+1}{x-3}$.

multiply the dividend by the reciprocal of the divisor: $\frac{1-x}{x+1} \cdot \frac{x-3}{x+1} = \frac{(1-x)(x-3)}{(x+1)(x+1)}$

expand the numerator and denominator: $= \frac{-x^2+4x-3}{x^2+2x+1}$

identify restricted values: $x \neq -1, x \neq 3$

Why is $x \neq 3$ a restricted value?

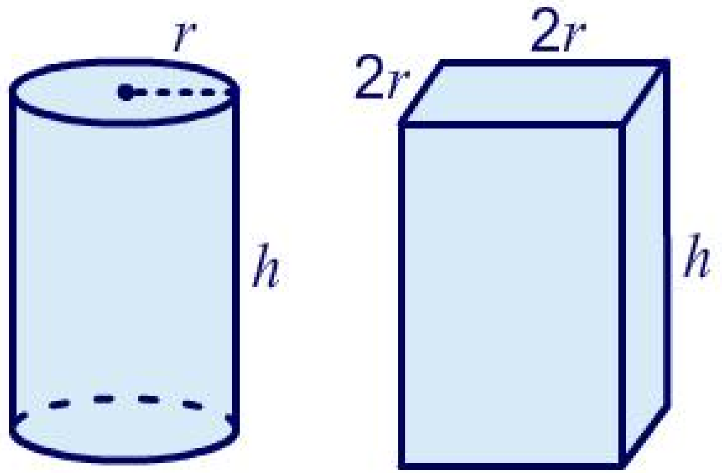
Match the expressions to their simplified versions

C) $\dfrac{3x}{7xy} \div \dfrac{6y}{5x^3}$	$\dfrac{5}{14x^2y}$
A) $\dfrac{3x}{7y^2} \cdot \dfrac{5y}{6x^3}$	$\dfrac{5x^4y}{14(x+1)}$
D) $\dfrac{9-3x}{x-1} \div \dfrac{3x}{x+1}$	$\dfrac{5x^3}{14y^2}$
E) $\dfrac{3y^2}{7x} \cdot \dfrac{x^2}{(x+1)} \div \dfrac{6y}{5x^3}$	$\dfrac{3+2x-x^2}{x^2-x}$
B) $\dfrac{9-3x}{x-1} \cdot \dfrac{x^2}{3(x+1)}$	$\dfrac{3x^2-x^3}{x^2-1}$

Heat loss problem

You are charged with investigating the best shapes for a container to hold hot liquid. The container must be either a rectangular block with dimensions $2r \times 2r \times h$, or a circular cylinder with radius r and height h.

Heat loss from a volume of liquid is proportional to its surface area, so the heat loss from the container depends on its **surface-area-to-volume ratio**.



Press **start** to begin investigating the shapes.

start