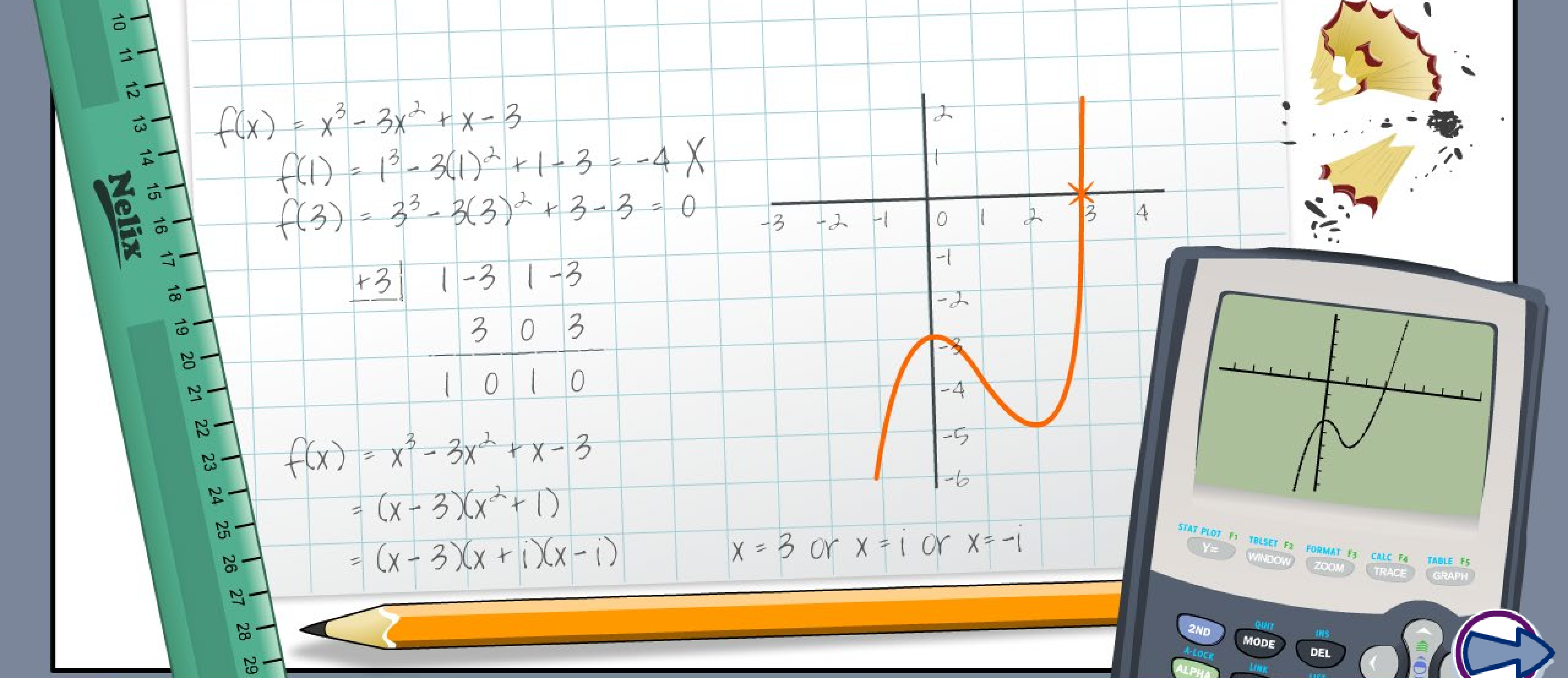
# Rational Exponents



## Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

**1) Make sense of problems and persevere in solving them.**

**2) Reason abstractly and quantitatively.**

**3) Construct viable arguments and critique the reasoning of others.**

**4) Model with mathematics.**

**5) Use appropriate tools strategically.**

**6) Attend to precision.**

**7) Look for and make use of structure.**

**8) Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

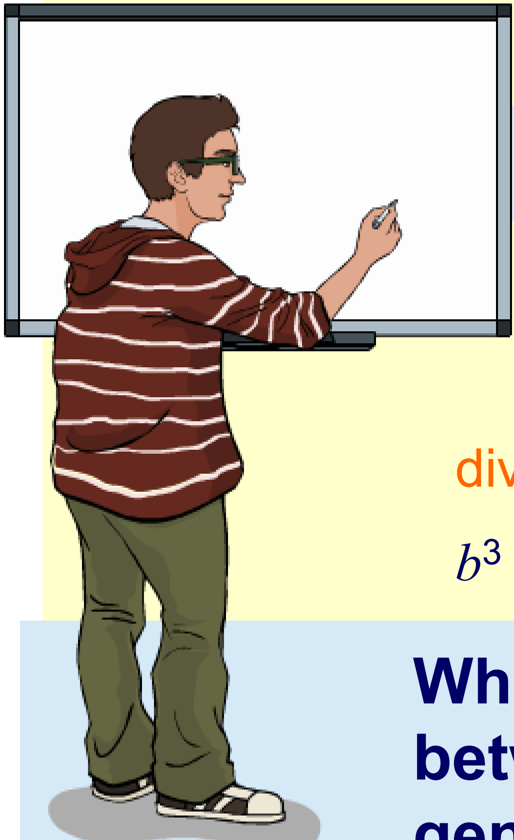
This icon indicates teacher's notes in the Notes field.

# Exploring rational exponents

**If $b^3 = a^6$, write $b$ in terms of $a$.**

using properties of exponents, divide the exponent on both sides by 3:

$b^3 = a^6$ $\quad b^{3/3} = a^{6/3}$ $\quad b^1 = a^2$ $\quad \boldsymbol{b = a^2}$



**If $b^3 = a$, write $b$ in terms of $a$.**

$b$ is a radical expression: $\boldsymbol{b = \sqrt[3]{a}}$

but $b$ can also be found using exponents:

divide the exponent on both sides by 3:

$b^3 = a$ $\quad b^3 = a^1$ $\quad b^{3/3} = a^{1/3}$ $\quad b^1 = a^{1/3}$ $\quad \boldsymbol{b = a^{1/3}}$

**What is the relationship between $\sqrt[3]{a}$ and $a^{1/3}$?, or more generally $\sqrt[n]{a}$ and $a^{1/n}$?**

$$\sqrt[n]{a} = a^{1/n}$$

**Using what you know about exponents and radicals, write $\sqrt[3]{a^2}$ as a rational exponent.**

$a^{2/3}$

The power on the radicand becomes the numerator of the rational exponent, and the index of the radical becomes the denominator of the rational exponent.

**What does it mean if the rational exponent is negative, e.g., $a^{-2/3}$?**

Remember that $x^{-1} = \dfrac{1}{x}$, so $a^{-2/3} = \dfrac{1}{a^{2/3}}$.

A negative exponent makes the number its reciprocal.

# Rational exponent summary

***n*^th^ root of a real number *a*:**

$$\sqrt[n]{a} = a^{1/n}$$

$n$ is a whole number.

***n*^th^ root of a real number *a* raised to the *m*^th^ power:**

$$\sqrt[n]{a^m} = a^{m/n}$$

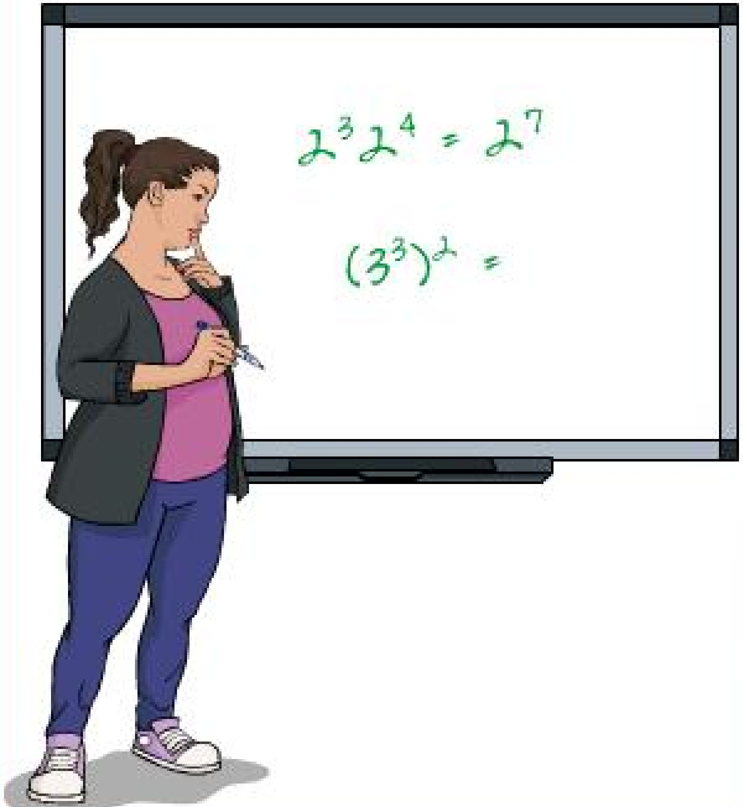
$n$ is a whole number

$m$ is an integer, and if $m < 0$, $a \neq 0$.

## Match the equivalent expressions

| | |
|---|---|
| $64^{-2/3}$ | $100$ |
| $8^{-1/3}$ | $\frac{1}{2}$ |
| $1000^{2/3}$ | $7$ |
| $4^{5/2}$ | $32$ |
| $49^{1/2}$ | $\frac{1}{16}$ |

# Properties of rational exponents




*Rational exponents have similar properties to integer exponents. Press to see an example.*

*$a$ and $b$ are real, and $x$ and $y$ are rational.*

$$a^x a^y = a^{x+y}$$

$$a^{-x} = \frac{1}{a^x}$$

$$(a^x)^y = a^{xy}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

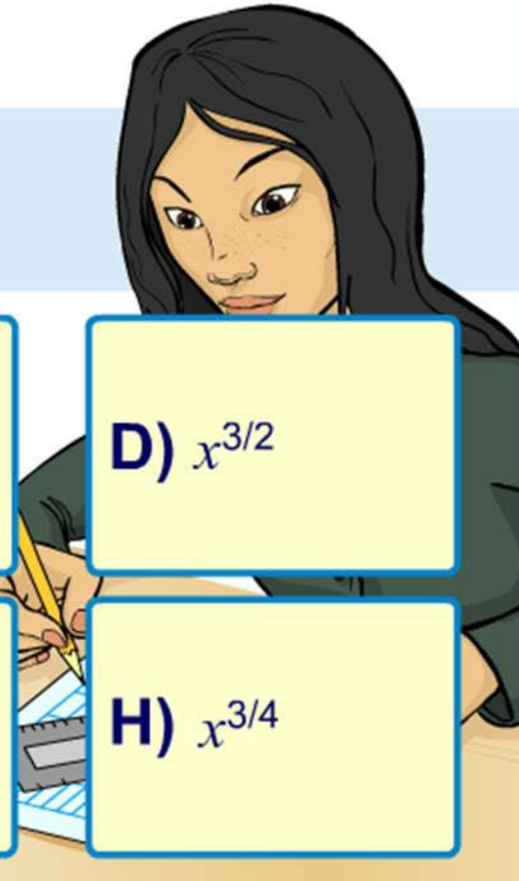
$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \left(\frac{a^x}{b^x}\right)$$

## Converting between rational exponent and radical form

Question: 7/7

**Select the two equivalent forms of $\sqrt{x^{3/4}}$.**



A) $\sqrt{x^3}$

B) $\sqrt[4]{x^3}$

C) $\sqrt[8]{x^3}$

D) $x^{3/2}$

E) $\sqrt{\frac{x^3}{x^4}}$

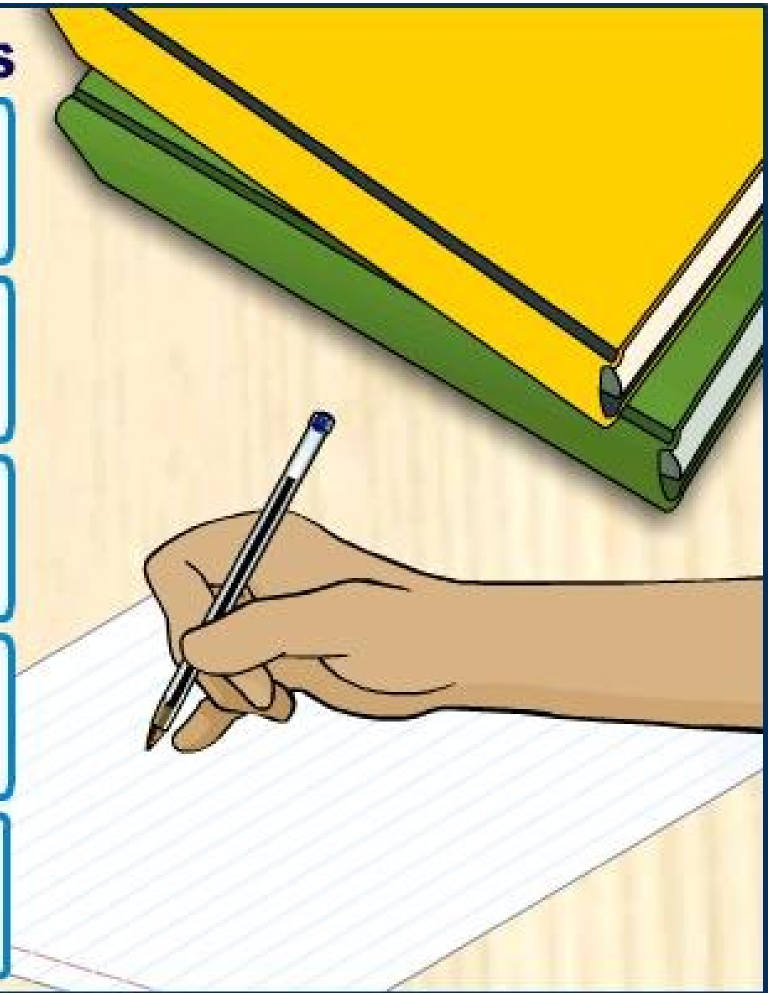
F) $\frac{\sqrt{x^3}}{x^2}$

G) $x^{3/8}$

H) $x^{3/4}$

## Match the equivalent expressions

| | |
|---|---|
| $(ab)^{-1/3}$ | $\dfrac{a^{7/6}}{b^{3/2}}$ |
| $\left(\dfrac{a^{3/5}}{b^{9/4}}\right)\left(\dfrac{b}{a}\right)^{-1/3}$ | $a^{8/3}$ |
| $(a^6a^{2/3})^{1/2}$ | $a^{10/3}$ |
| $\dfrac{(a^{5/3})^2}{a^{2/3}}$ | $\dfrac{1}{a^{1/3}b^{1/3}}$ |
| $\left(\dfrac{1}{b^{9/4}a^{-7/4}}\right)^{2/3}$ | $\dfrac{a^{14/15}}{b^{31/12}}$ |



# Practice with variables

## Simplify the radical expressions as specified

| | | |
|---|---|---|
| 1. $(x^4y^2)^{1/2}$ | ? | without using any fractional powers |
| 2. $x^{1/2}(x^{3/2} + x^{-1/2})$ | ? | as a polynomial |
| 3. $x^{-1/3}(x^{4/3} - x^{-2/3})$ | ? | as a ratio of two polynomials |
| 4. $x^{5/2} + 2x^{3/2} + x^{1/2}$ | ? | as a product of something and a quadratic |