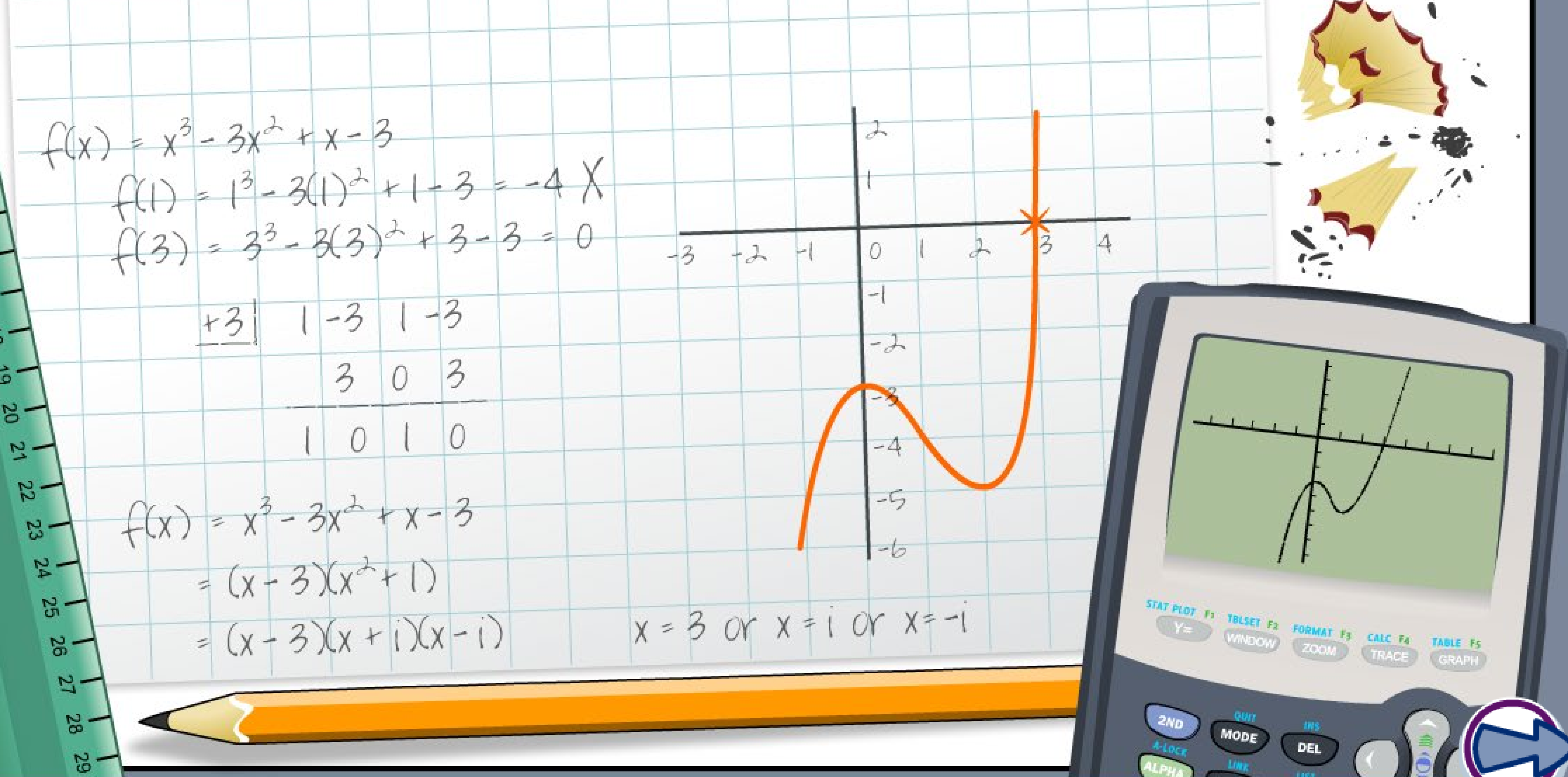
Radical Expressions



Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Roots and radicals

***n*^*th* root**: x is an n^{th} root of b if $x^n = b$.

How many roots does $x^n = b$ have?
hint: recall the fundamental theorem of algebra.

The fundamental theorem of algebra says that the equation has n real roots.

Not all of these roots are real:

- If n is odd, there is only one real root, and it is denoted $\sqrt[n]{b}$.
- If n is even and x is positive, there are two real roots. One is positive and the other is negative: $\sqrt[n]{b}$ and $-\sqrt[n]{b}$.
- If n is even and x is negative, there are no real roots.

Radical terminology

Radicals follow the general form:

$$\sqrt[n]{a}$$

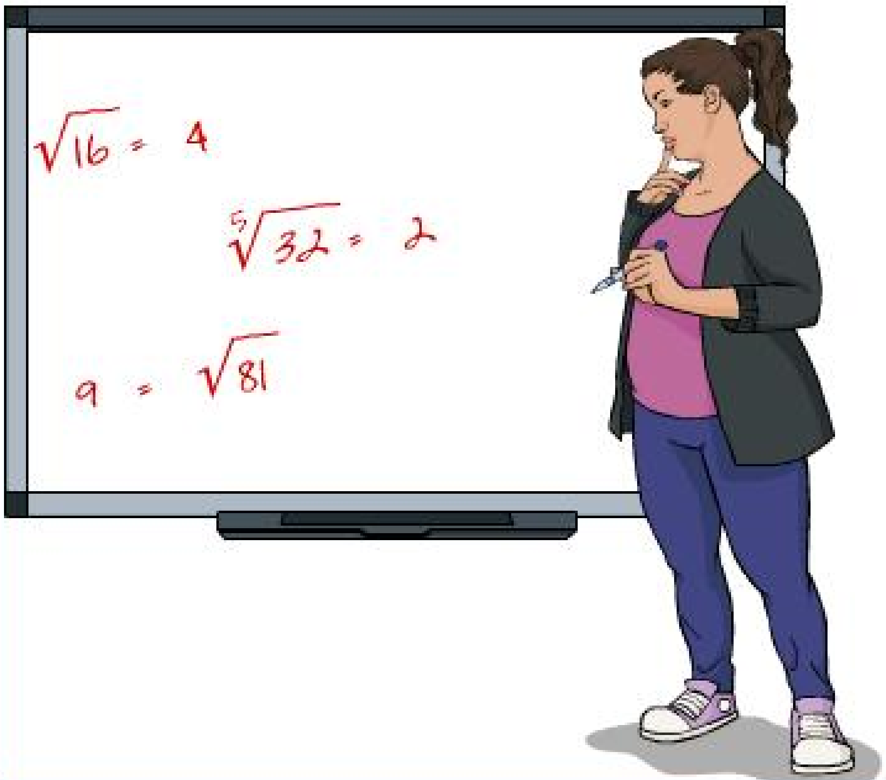
$\sqrt{}$

a

n

Press the yellow boxes to reveal the name and description of each component of the radical.

Examples of radical expressions



Press on a button to see how to find the real number root of each radical.

$\sqrt[3]{27}$

$\sqrt{(-2)^2}$

$\sqrt[4]{-16}$

Finding real roots

Match the radicals to their roots

$\sqrt[3]{27}$	4
$\sqrt{169}$	10
$3 + \sqrt{81}$	12
$\sqrt{16}$	13
$2\sqrt{25}$	3

Combining radical expressions

Products of expressions under radicals with the same index can be multiplied.

products of radicals:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

Similarly, quotients of expressions under radicals with the same index can be divided.

quotients of radicals:

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{a/b}\,,\quad b \neq 0$$

Evaluate the expressions:

a) $\sqrt{5} \cdot \sqrt{5}$ **b)** $\sqrt{30} \div \sqrt{5}$

a) $\sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{25} = \mathbf{5}$

b) $\sqrt{30} \div \sqrt{5} = \sqrt{30 \div 5} = \sqrt{\mathbf{6}}$

Simplifying radical expressions

A radical expression is in its simplest form when the quantity under the radical sign does not contain any integers or variables to the power n.

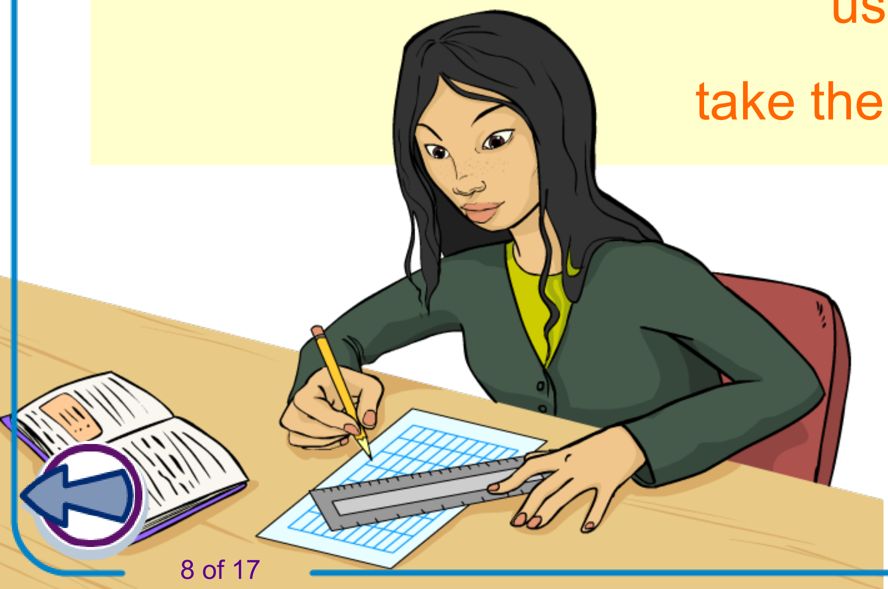
Simplify $\sqrt{50}$ by writing it in the form $a\sqrt{b}$.

find the largest square number that divides into 50: $\sqrt{25} = 5$

rewrite the radicand: $\sqrt{50} = \sqrt{25 \times 2}$

use the product rule: $= \sqrt{25} \times \sqrt{2}$

take the square root of 25: $\mathbf{= 5\sqrt{2}}$



Simplifying radicals

Simplify the radicals by writing them in the form $a\sqrt{b}$.

1) Simplify $\sqrt{45}$.

2) Simplify $\sqrt{98}$.

3) Simplify $\sqrt[3]{40}$.

4) Simplify $\sqrt{245}$.

5) Simplify $\sqrt[3]{54}$.

Adding and subtracting radicals

Radical expressions can only be added or subtracted if they have the same index and the same radicand.

sums of radicals: $a\sqrt[n]{x} + b\sqrt[n]{x} = (a+b)\sqrt[n]{x}$

differences of radicals: $a\sqrt[n]{x} - b\sqrt[n]{x} = (a-b)\sqrt[n]{x}$

It helps to simplify radicals before adding or subtracting.

Put $\sqrt{45} + \sqrt{80}$ into simplest form.

write each term in simplest form: $\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

$\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$

substitute simplified form: $\sqrt{45} + \sqrt{80} = 3\sqrt{5} + 4\sqrt{5}$

addition property: $= (3+4)\sqrt{5} = \mathbf{7\sqrt{5}}$

Adding and subtracting radicals

Question: 1/5

What is the simplest form of the expression $\sqrt{3} + 2\sqrt{3}$?

A) $\sqrt{12}$

B) $3\sqrt{3}$

C) $\sqrt{3} - \sqrt{6}$

D) cannot be simplified

Rationalizing the denominator

Typically, there should be no radical in the denominator of a fraction.

How can you rewrite the fraction $\frac{5}{\sqrt{2}}$ so that there is no radical in the denominator?

multiply the numerator and denominator by $\sqrt{2}$ so that the value of the fraction remains the same:

$$\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}^2}$$

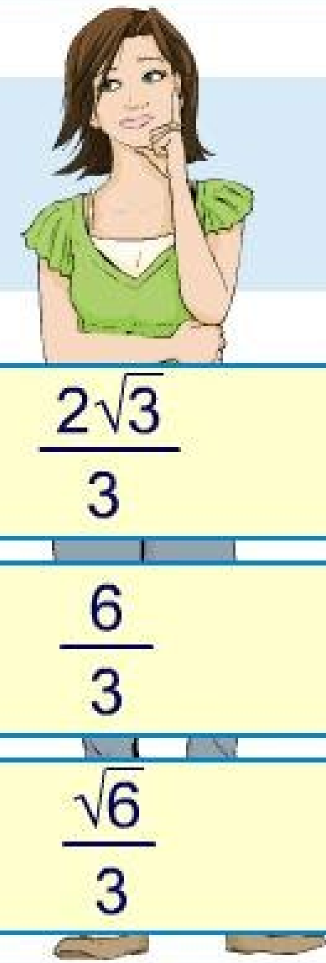
$\sqrt{a}^2 = a$:

$$\frac{5\sqrt{2}}{2}$$

Turning a radical in the denominator into a rational number is called **rationalizing the denominator**.

Rationalizing the denominator

Question 1/3: Simplify $\frac{2}{\sqrt{3}}$ by rationalizing the denominator.



Press the "=" button to show the work step by step.

$$\frac{2\sqrt{3}}{3}$$

$$\frac{6}{3}$$

$$\frac{\sqrt{6}}{3}$$

Conjugates

Conjugates are expressions that are the same except for the sign of the second term.

For example, $(\sqrt{5} - 2)$ and $(\sqrt{5} + 2)$ are conjugates.

If a and b are rational numbers, show that the product of the conjugates $(\sqrt{a} - \sqrt{b})$ and $(\sqrt{a} + \sqrt{b})$ is a rational number.

distribute: $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = \sqrt{a}^2 + \cancel{\sqrt{ab}} - \cancel{\sqrt{ab}} - \sqrt{b}^2$

simplify: $= \sqrt{a}^2 - \sqrt{b}^2$

$\sqrt{a^2} = a$: $\boldsymbol{= a - b}$

This is the radical conjugate property and is the same as the difference of squares: $(a + b)(a - b) = a^2 - b^2$.

radical conjugate property:

$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$$

Rationalizing using conjugates

Can you apply the distributive property to rationalize the expressions containing radicals?

$(8+\sqrt{5})(8-\sqrt{5}) =$

$=$

$=$

Conjugates can be used to rationalize the denominator using the difference of squares.

Simplify the fraction $\frac{1}{\sqrt{5} - 2}$ by rationalizing the denominator.

find the conjugate of the denominator: $\sqrt{5} + 2$

multiply the numerator and denominator by the conjugate: $\frac{1}{\sqrt{5} - 2} \times \frac{\sqrt{5} + 2}{\sqrt{5} + 2} = \frac{\sqrt{5} + 2}{(\sqrt{5})^2 - 4}$

simplify: $\frac{\sqrt{5} + 2}{(\sqrt{5})^2 - 4} = \frac{\sqrt{5} + 2}{5 - 4} = \mathbf{\sqrt{5} + 2}$

Rationalizing the denominator

Question 1/5: Simplify $\dfrac{12}{4-\sqrt{2}}$ by rationalizing the denominator.

Press the "=" button to show the work step by step.

$$\frac{24+12\sqrt{2}}{7}$$

$$\frac{24-6\sqrt{2}}{14}$$

$$\frac{24+6\sqrt{2}}{7}$$