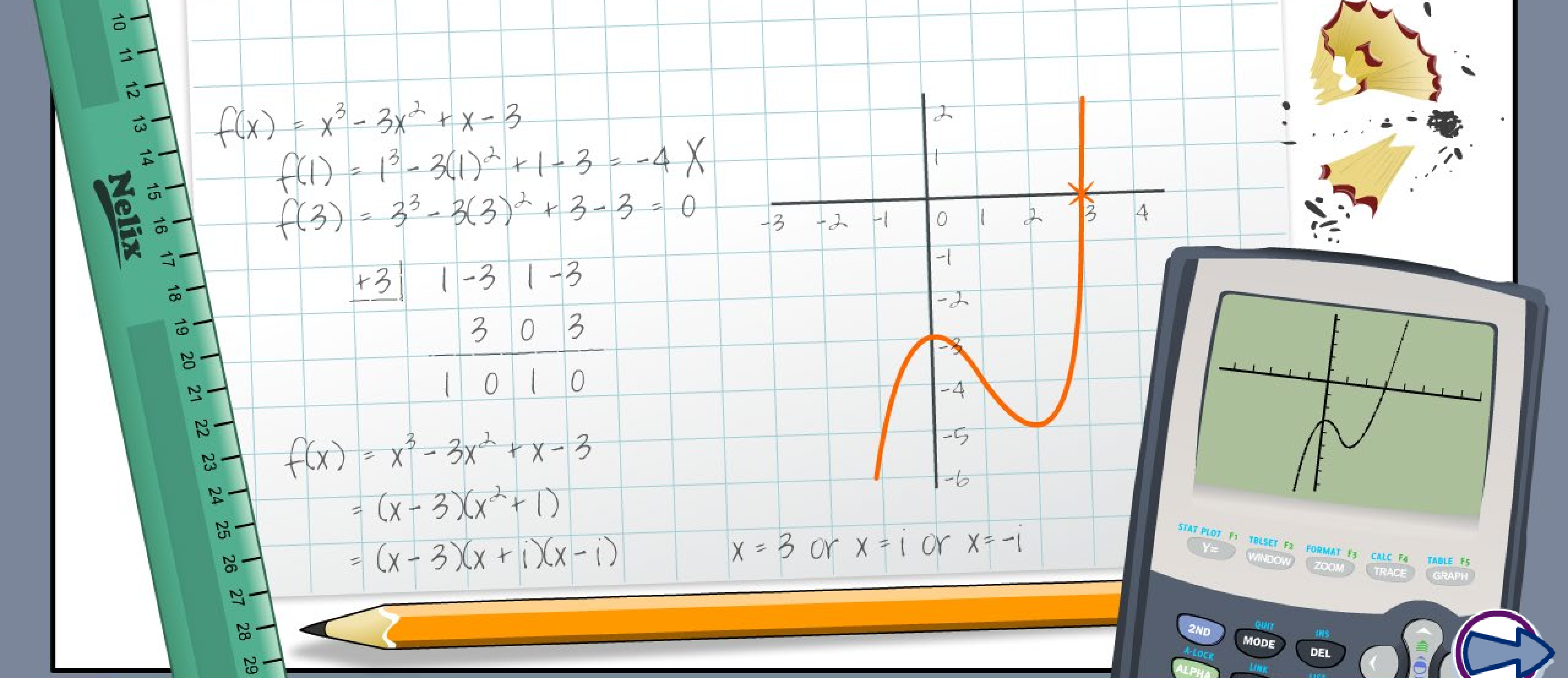
Quadratic Inequalities



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Quadratic inequalities on number lines

Quadratic inequalities are inequalities where the highest power of the variable is 2. For example:

$$x^2 + x - 6 \geq 0$$

How can this inequality be represented on a number line?

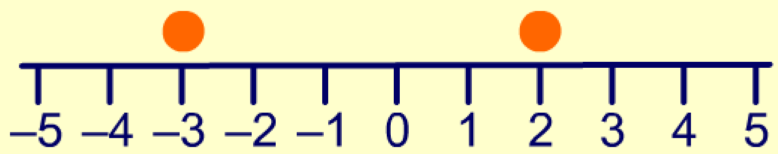
The equation $x^2 + x - 6 = 0$ needs to be solved first and then the correct regions determined.

factor: $(x + 3)(x - 2) \geq 0$

solve: $x + 3 = 0$ and $x - 2 = 0$

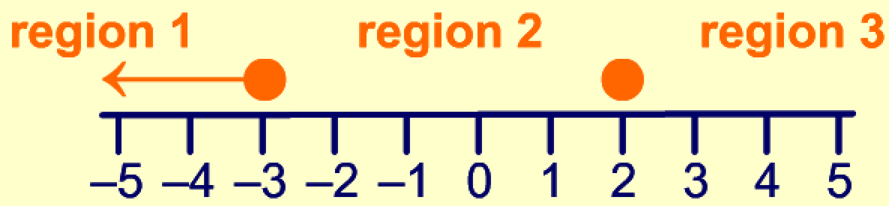
$x = -3$ $x = 2$

These values give the end points of the solution set.



Quadratic inequalities on number lines

To find the solution set, test a value from each of the three regions around the known endpoints:



Substitute each value into the original inequality $x^2 + x – 6 \geq 0$.

substitute $x = –4$: $(–4)^2 + (–4) – 6 \geq 0$

$6 \geq 0$ ✓

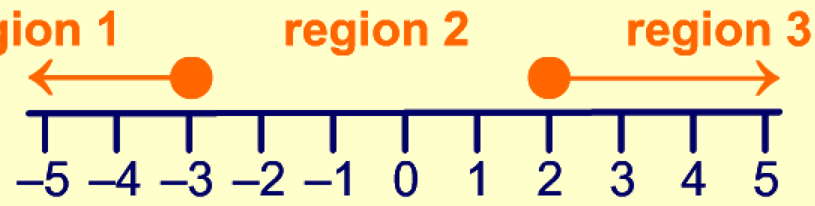
Values in region 1 therefore satisfy the inequality.

substitute $x = 0$: $0^2 + 0 – 6 \geq 0$

$–6 \geq 0$ ✗

Values in region 2 therefore do not satisfy the inequality.

Quadratic inequalities on number lines

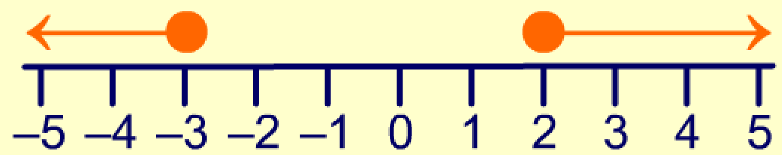


substitute $x = 3$: $3^2 + 3 - 6 \geq 0$

$6 \geq 0$ ✓

Values in region 3 therefore satisfy the inequality.

The completed solution set graphed on a number line looks like this:



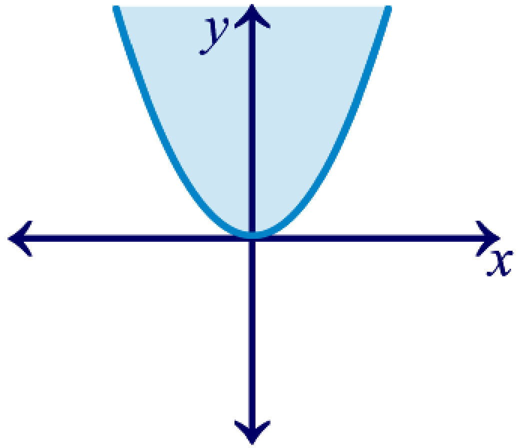
So the solution to the inequality $x^2 + x - 6 \geq 0$ is:

$\boldsymbol{x \leq -3}$ or $\boldsymbol{x \geq 2}$

Graphing quadratic inequalities

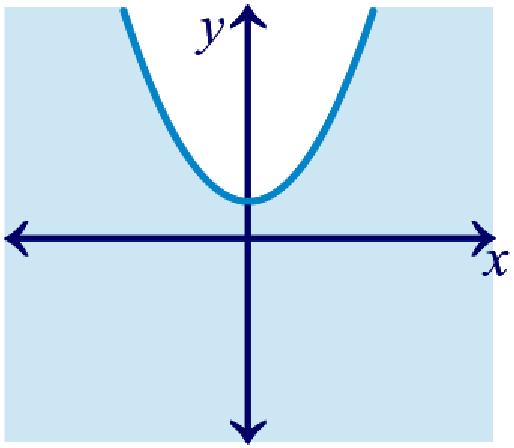
A quadratic inequality can be shown on a graph as a region.

To graph $y \geq x^2$, first draw the graph of $y = x^2$



then shade the region **above** this curve.

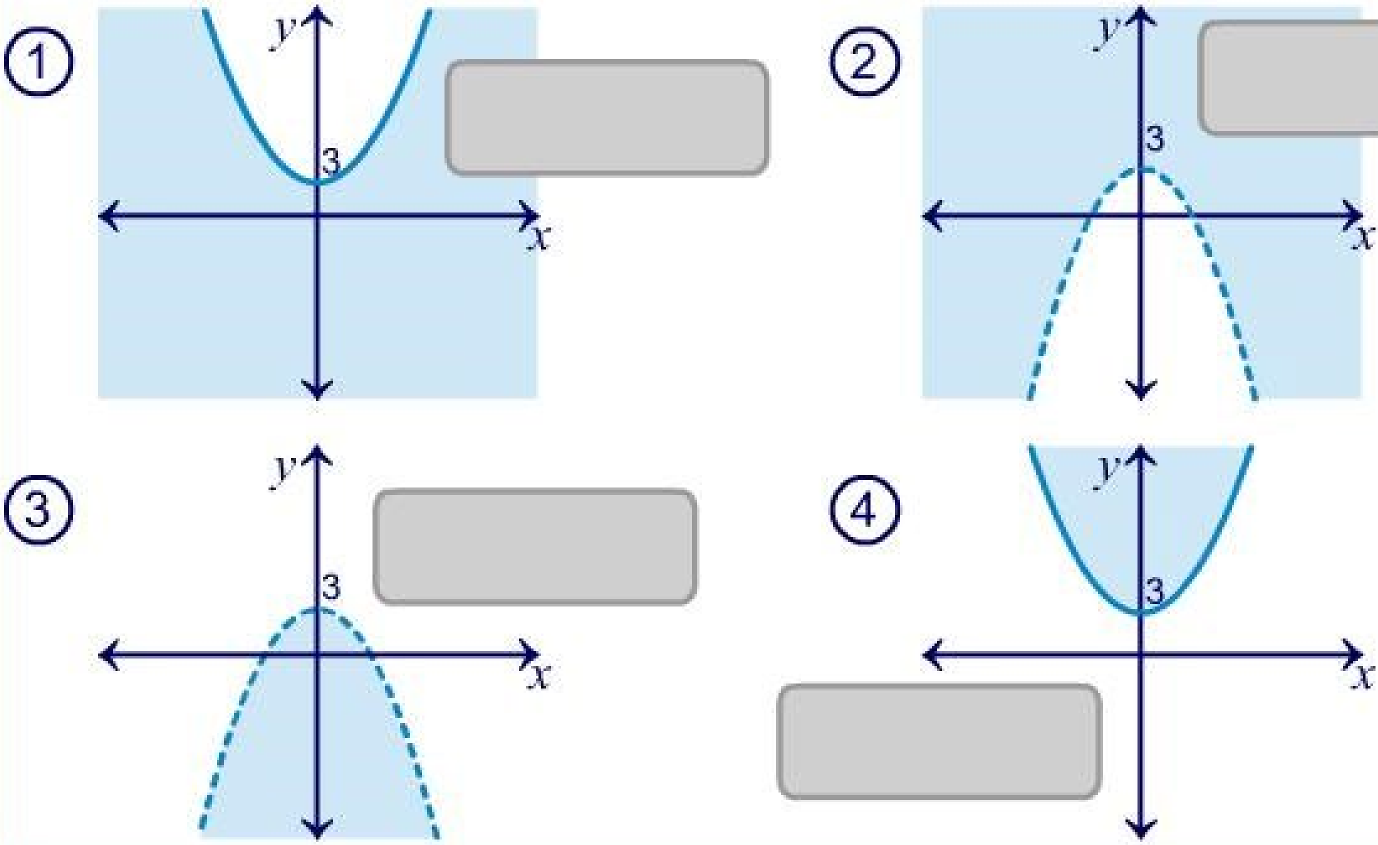
To graph $y \leq x^2 + 1$, first draw the graph of $y = x^2 + 1$.



then shade the region **below** this curve.

Note: if the inequality symbols are < or >, the curves are drawn as **dotted** lines.

Matching quadratic inequalities to their graphs



$3 - x^2 > y$

$y - 3 \leq x^2$

$y > -x^2 + 3$

$3 \leq y - x^2$

Solving quadratic inequalities

The solution to a quadratic inequality can be found graphically.

Solve the inequality $x^2 < 4$ using a graph.

Sketch the graph of $y = x^2$.

Draw the line $y = 4$.

Find the x-values of the intersections.

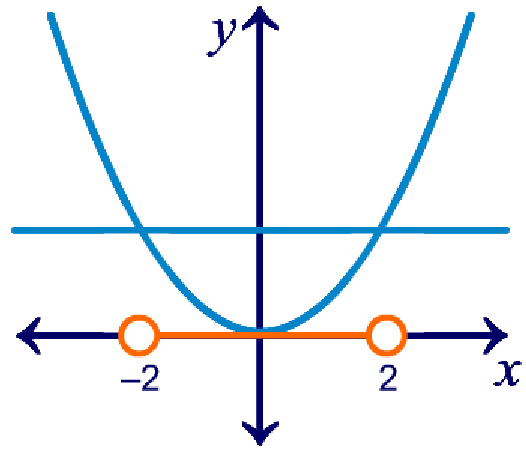
equate the functions: $x^2 = 4$

take square root: $x = 2$ or $x = -2$

The inequality is true for all values of x where the curve is below the line $y = 4$.

Therefore the solution region is:

$$-2 < x < 2$$



Open circles must be used on the graph.

Solve the inequality $x^2 + x - 6 \geq 0$.

Sketch the graph of $y = x^2 + x - 6$ by identifying the roots.

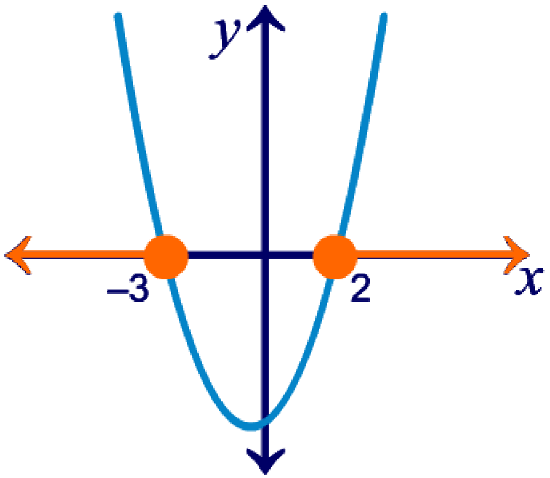
factor: $(x + 3)(x - 2) = 0$

The roots are at $x = -3$ and $x = 2$.

The inequality is true for the region where the graph is greater than 0, i.e. where the curve is above the x-axis.

Therefore the solution region is:

$$x \leq -3 \text{ or } x \geq 2$$



Closed circles must be used on the graph.

Solving quadratic inequalities

Solve the inequality $x^2 + x - 3 > 4x + 1$ graphically.

rearrange into the form $ax^2 + bx + c > 0$:

$$x^2 - 3x - 4 > 0$$

Sketch the graph of $y = x^2 - 3x - 4$ by finding the roots.

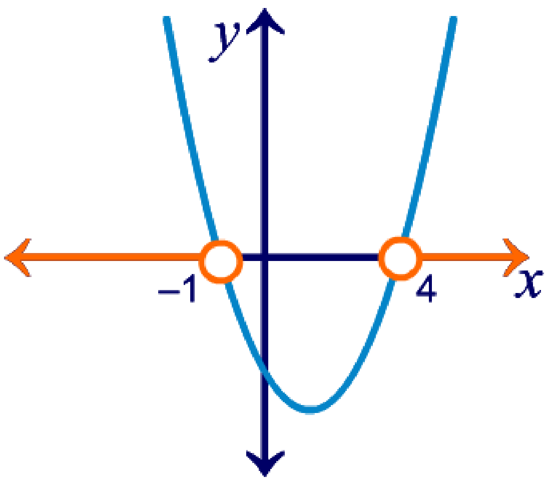
factor: $(x + 1)(x - 4) = 0$

The roots are at $x = -1$ and $x = 4$.

The inequality is true for the region where the graph is greater than 0.

Therefore the solution region is:

$$x < -1 \text{ or } x > 4$$



Open circles must be used on the graph.

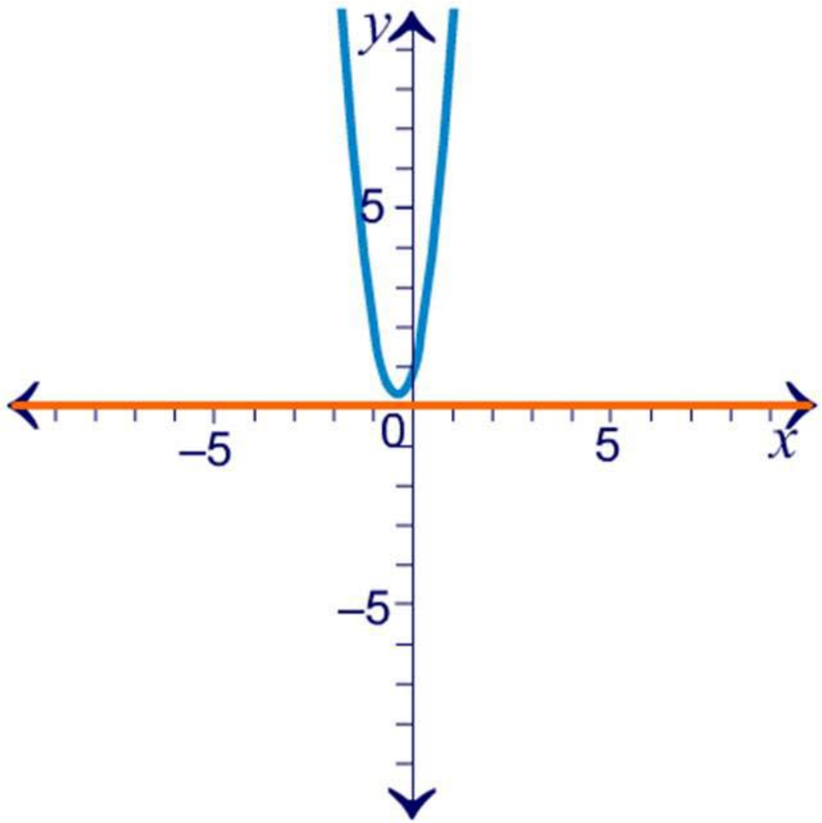
Quadratic inequalities

Question: 1/5

Solve the inequality $x^2 + 2x - 3 \geq 0$.

Press the "=" button to show the work step-by-step.

- $-3 \leq x \leq 1$
- no solutions
- $x \leq -3$ or $x \geq 1$
- $x < -3$ or $x > 1$



Solving quadratic inequalities graphically

$5x^2 + 4x + 1 \geq 0$

Solution set:

$-\infty < x < \infty$

Poster area

a) Write an expression for the area, A, of this poster:

b) If the area satisfies the inequality $5 < A < 12$, find the range of possible values for x.

$(x - 2)$ ft

$(x + 2)$ ft

a) area = width × length: $A = (x + 2)(x - 2)$ $= \mathbf{x^2 - 4}$

b) The range of possible values for x is given by:

$$5 < x^2 - 4 < 12$$

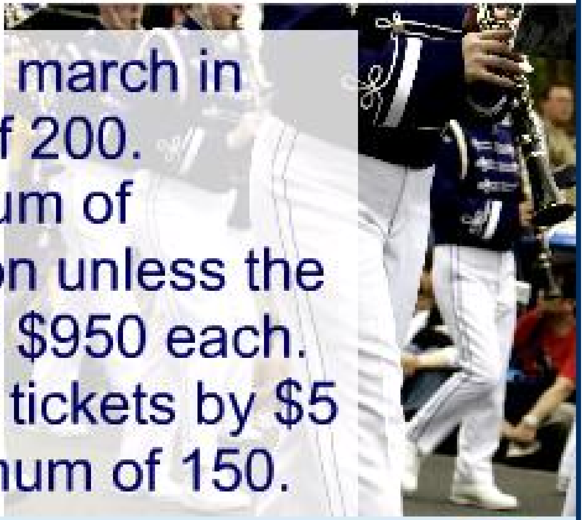
add 4: $9 < x^2 < 16$

square root: $3 < x < 4$ or $-3 > x > -4$

The negative solutions imply negative lengths, so disregard these.

The solution is therefore $\mathbf{3 < x < 4}$.

Plane ticket costs



Some student bands are flying to California to march in a parade, chartering a plane with a capacity of 200. The charter company wants to make a minimum of $142,500 and so will not accept the reservation unless the group guarantee buying at least 150 tickets at $950 each. They then agree to reduce the cost of all their tickets by $5 for every **additional** ticket sold over the minimum of 150.

1) What is the maximum amount the company can make?

2) Write an inequality to determine the range of additional students that the company can accept to maintain their desired minimum income.

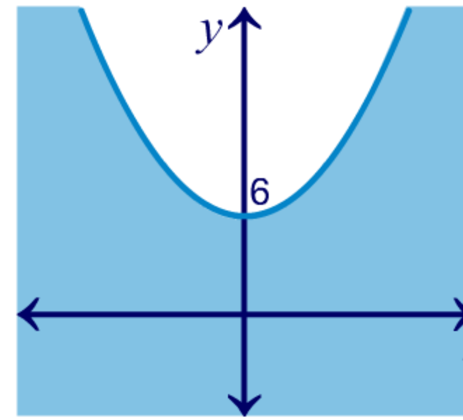
3) Confirm your findings graphically.

Graph the inequality $y - 1 < x^2 + 5$.

$$y - 1 < x^2 + 5$$

add 1: $y < x^2 + 6$

Graph the line $y < x^2 + 6$. Shade the correct region.



Graph the inequality $2 - x^2 \geq 6 + x^2 - y$.

$$2 - x^2 \geq 6 + x^2 - y$$

add y: $y + 2 - x^2 \geq 6 + x^2$

add x^2: $y + 2 \geq 6 + 2x^2$

subtract 2: $y \geq 4 + 2x^2$

Graph the line $y \geq 2x^2 + 4$. Shade the correct region.