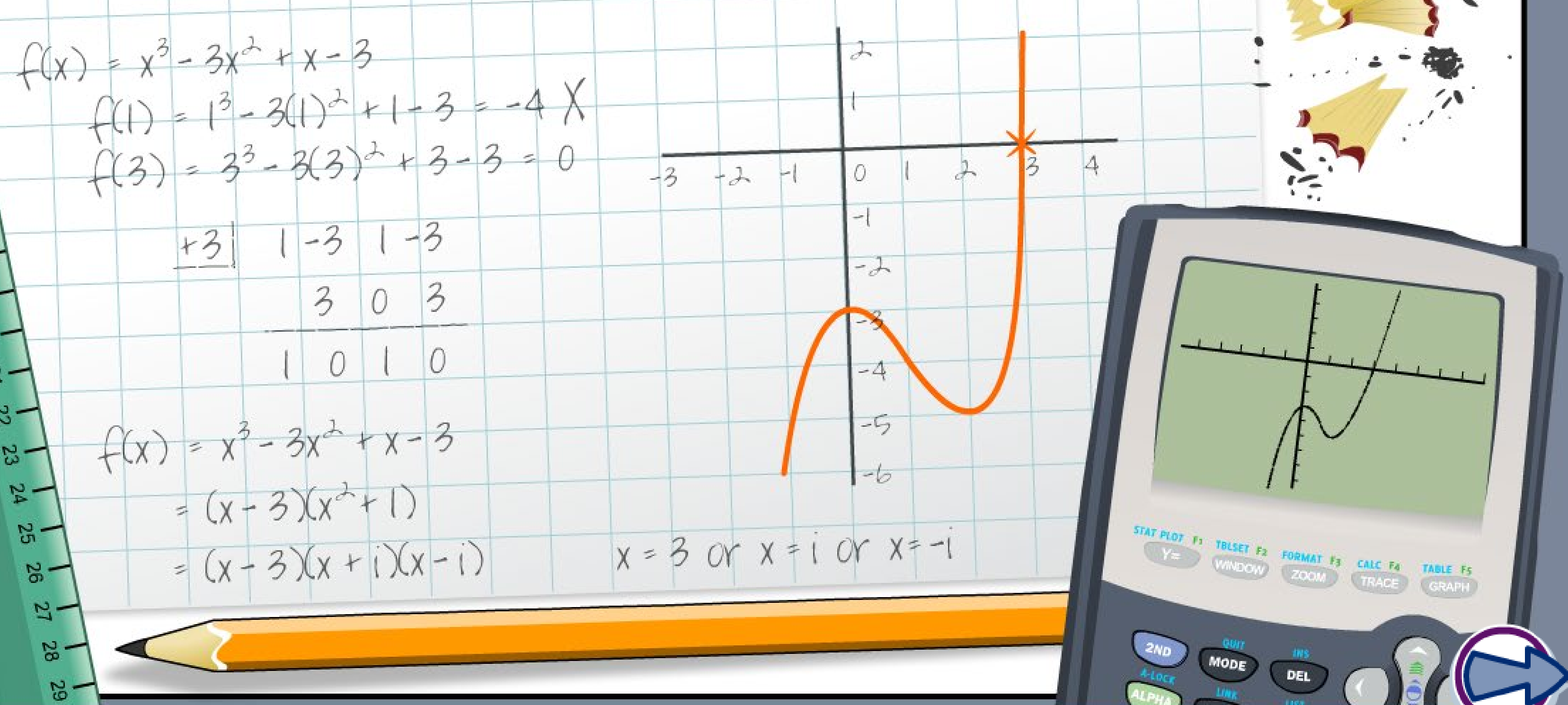
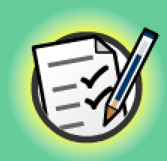
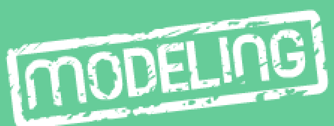
Properties of Logarithms



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

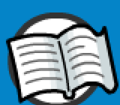
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

The inverse of an exponential is called a **logarithm** (log).

logarithm:
if $x = a^y$, then $y = \log_a x$

for a positive value of x
and for $a > 0$, $a \neq 1$

"$\log_a x$" is read as "log base a of x."

The logarithm base 10 of x can be written as just "$\log x$".

The logarithm base e (approximately 2.718) of x is called the natural logarithm and is denoted "$\ln x$".

Rewrite the following as logarithmic equations:

a) $p = k^m$ **b)** $5 = e^x$ **c)** $81 = 3^4$

a) $m = \log_k p$ b) $\ln 5 = x$ c) $4 = \log_3 81$

Calculating logarithms

A scientific calculator can be used to evaluate logarithms base 10 (using the "log" button) and logarithms base e (using the "ln" button).

For example, to calculate log 4, press "log" followed by "4".

Find the following logarithms to the nearest thousandth:

a) log 8	= 0.903	**d) 2 ln 4**	= 2.773
b) ln 16	= 2.773	**e) ln 12 – ln 4**	= 1.099
c) ln 3	= 1.099	**f) log 2 + log 4**	= 0.903

What do you notice about the results? Can you use these results to establish any rules?

Basic rules of logarithms

What is the link connecting each pair of equations below?

Press the the "connection" box to see the link between the expressions.

Press the "rule" box to see the logarithmic rule that the result demonstrates.

a) $\log 8 = 0.903$ and $\log 2 + \log 4 = 0.903$

connection

rule

b) $\ln 12 - \ln 4 = 1.099$ and $\ln 3 = 1.099$

connection

rule

c) $\ln 16 = 2.773$ and $2 \ln 4 = 2.773$

connection

rule

Taking logarithms

Logarithms are the inverse of exponentials.

For example, to solve $e^x = 10$, take natural logarithms (base e) of both sides to get: $x = \ln 10$. This can now be solved for x.

Solve $10^x = 15$ to the nearest thousandth.

$$10^x = 15$$

take logs (base 10) of both sides: $x = \log 15$

evaluate using a calculator: $\mathbf{x = 1.176}$

Expressions with bases other than 10 or e cannot be evaluated directly using a calculator.

To calculate these, the base must be changed to 10 or e.

The change of base rule

Expressions with bases other than 10 or e can be rewritten in base 10 or e to be evaluated directly using a calculator.

How can $\log_b x = m$ be rewritten using a logarithm base a?

Hint: rewrite it in exponent form and then take logarithms base a.

In exponent form, this expression is: $b^m = x$

take logs (base a) of both sides: $\log_a (b^m) = \log_a x$

exponent rule: $m \log_a b = \log_a x$

rearrange: $m = \log_a x / \log_a b$

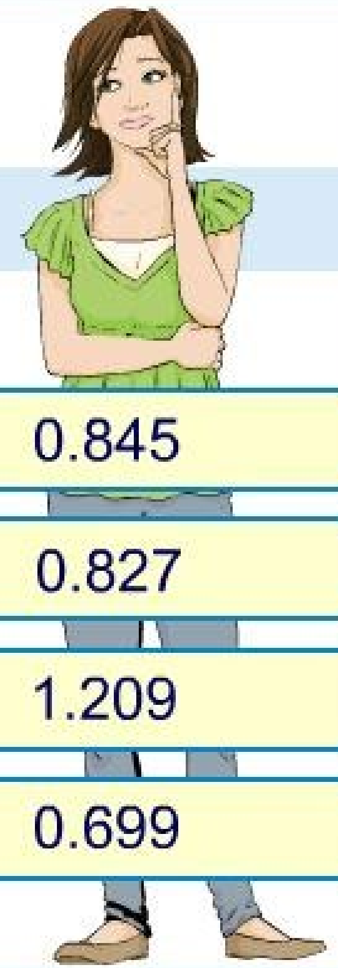
change of base rule: $\log_b x = \log_a x / \log_a b$

Changing logarithmic bases

Question: 1/3

Calculate $\log_5 7$ to the nearest thousandth.

Press the "=" button to show the work step by step.



- 0.845
- 0.827
- 1.209
- 0.699

Logarithmic properties

Establishing some properties of logarithms

There are several basic properties of logarithms.

Complete the properties below.
Write each of the properties as a question to help you, for example, complete this property: ln e = ?
This asks "To what power do we raise e to get e?".
The answer is one, so the property is: **ln e = 1**.

1) $\log_b b = ?$

2) $\log_b 1 = ?$

3) $\log_b b^x = ?$

4) $b^{\log_b x} = ?$

Proving the logarithmic rules

Question: 1/3

Prove the exponent rule: $\log_a(x^n) = n\log_a x$.

Hint: let $m = n\log_a x$.

Press the "=" button to show the calculations step by step.

Logarithmic scales

pH scale | decibel scale | Richter scale

A real-life phenomenon that covers a wide range of values, e.g. 1 to 10,000,000,000, can be made simpler and easier to manage if measured on a **logarithmic scale** instead, e.g. 0 (log 1) to 10 (log 10,000,000,000).

Press on each tab to learn about three examples of logarithmic scales.