Polynomial Identities

$f(x) = x^3 - 3x^2 + x - 3$

$f(1) = 1^3 - 3(1)^2 + 1 - 3 = -4$ X

$f(3) = 3^3 - 3(3)^2 + 3 - 3 = 0$

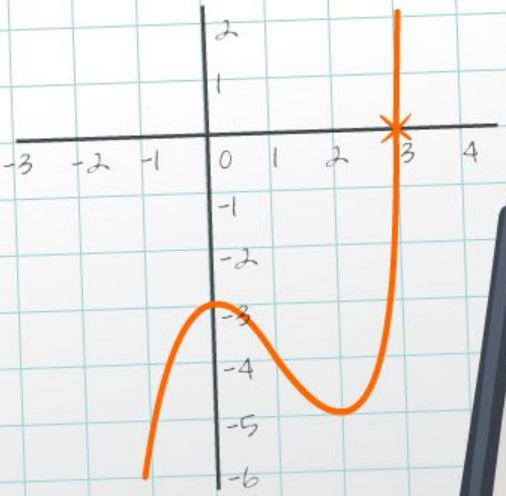
+3	1	-3	1	-3
		3	0	3
	1	0	1	0

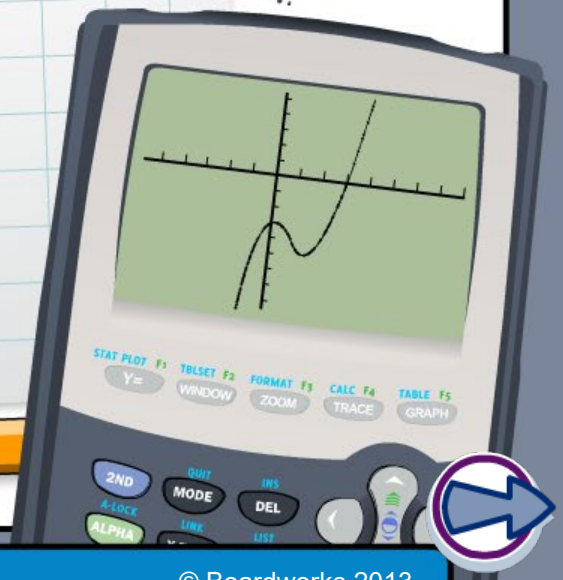
$f(x) = x^3 - 3x^2 + x - 3$

$= (x - 3)(x^2 + 1)$

$= (x - 3)(x + i)(x - i)$

$x = 3$ or $x = i$ or $x = -i$





Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

What is a mathematical identity?

$1 = 1$ $\quad 2 \times 3 = 6$ $\quad 7^2 = 49$ $\quad 0x = 0$

$a + b = b + a$ $\quad a(b + c) = ab + ac$

An identity holds true for every value of the variables.

Mathematical relationships between specific numbers are identities.

Laws like the distributive property are also identities.

An identity is an equation with infinitely many solutions.

You cannot solve an identity.

For example, $x^2 + 2x + 1 = 0$ is not an identity, but $x^2 + 2x + 1 = (x + 1)^2$ is.

A **polynomial identity** is an identity between polynomial expressions.

Identity or equation?

Are these identities or equations with finite solutions?

identity	equation

$(5x)^2 - 5 - 25x^2 = -5$

Polynomial identities

Prove that the equations below are identities:

- **difference of squares** $a^2 - b^2 = (a - b)(a + b)$ W
- **sum of cubes** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ W
- **difference of cubes** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ W

*Press **W** to see the work.*

Using polynomial identities

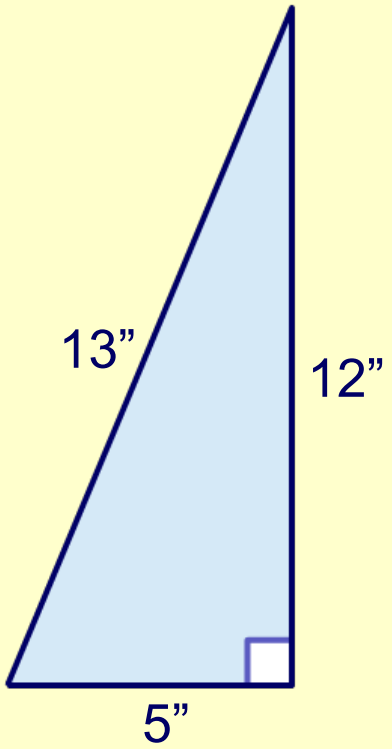
How can the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ be used to construct right triangles?

To construct a right triangle we need lengths a, b, c that satisfy the Pythagorean theorem, $a^2 + b^2 = c^2$.

Choose any two integers with $x \geq y$: e.g. $x = 3$, $y = 2$

Evaluate:

$a = x^2 - y^2$	$b = 2xy$	$c = x^2 + y^2$
$a = 3^2 - 2^2$	$b = 2(3)(2)$	$c = 3^2 + 2^2$
$a = 9 - 4$	$\boldsymbol{b = 12}$	$c = 9 + 4$
$\boldsymbol{a = 5}$		$\boldsymbol{c = 13}$



By the identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ we know that $c^2 = a^2 + b^2$ so a, b, c is a Pythagorean triple and we know that a triangle with these side lengths is a right triangle.

Extending polynomial identities

Polynomial identities are true for every number, including radicals and complex numbers.

Factor $f(x) = x^2 - 3$.

write as difference of squares: $f(x) = x^2 - (\sqrt{3})^2$

factor using rule: $f(x) = \mathbf{(x - \sqrt{3})(x + \sqrt{3})}$

This can be used to find irrational roots to help sketch graphs.

Factor $f(x) = x^2 + 4$.

write as difference of squares: $f(x) = x^2 - (2i)^2$

factor using rule: $f(x) = \mathbf{(x - 2i)(x + 2i)}$

Sum of squares: $a^2 + b^2 = (a + bi)(a - bi)$

Sum of squares

Match the equivalent expressions

$x^4 + 25$	$(x + 4i)(x - 4i)$
$-x^4 - 4x^2$	$(2x^2 + 3i)(2x^2 - 3i)$
$4x^4 + 9$	$(6x + i)(6x - i)$
$x^2 + 16$	$-x^2(x + 2i)(x - 2i)$
$36x^2 + 1$	$(x^2 + 5i)(x^2 - 5i)$