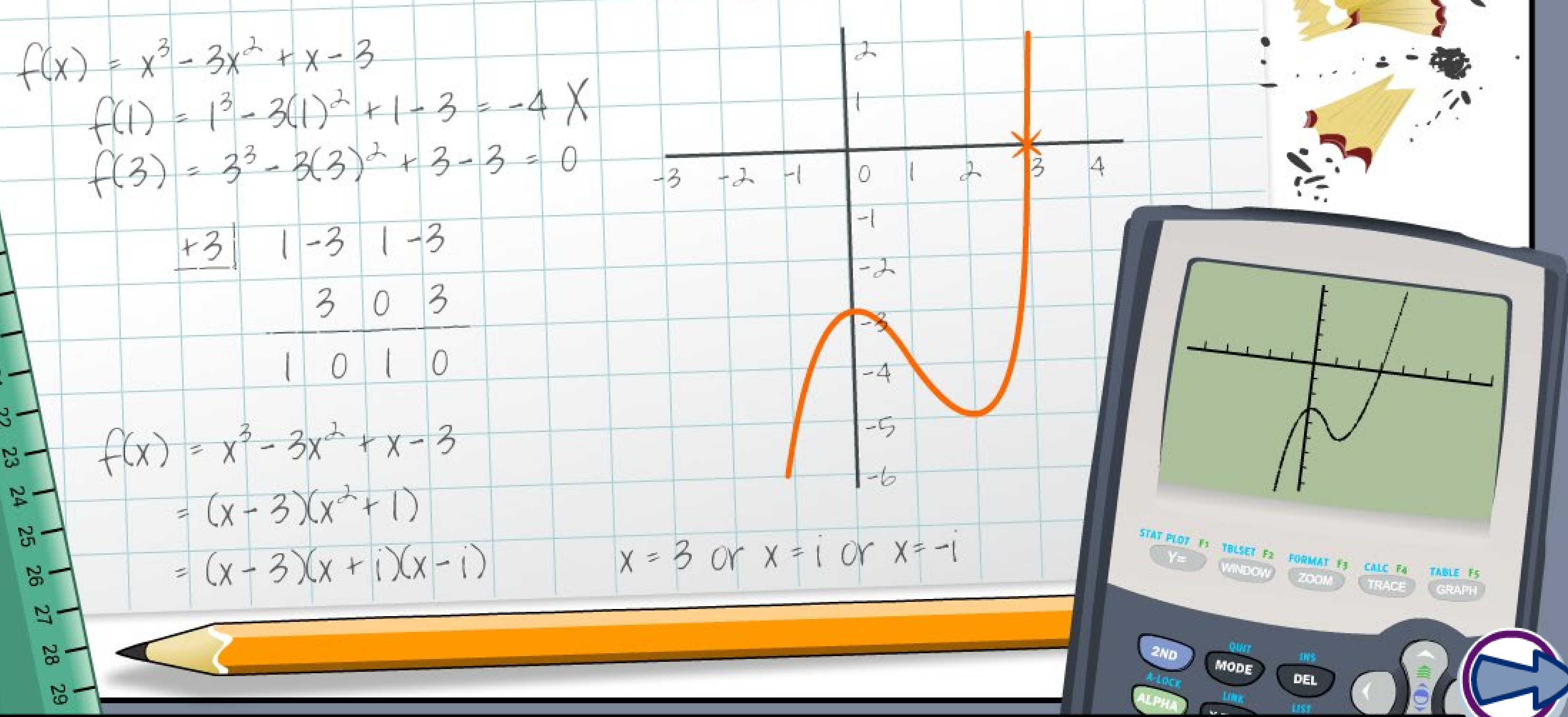
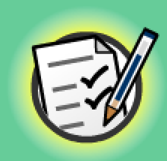
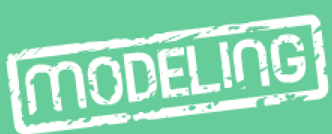
Polynomial Functions



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

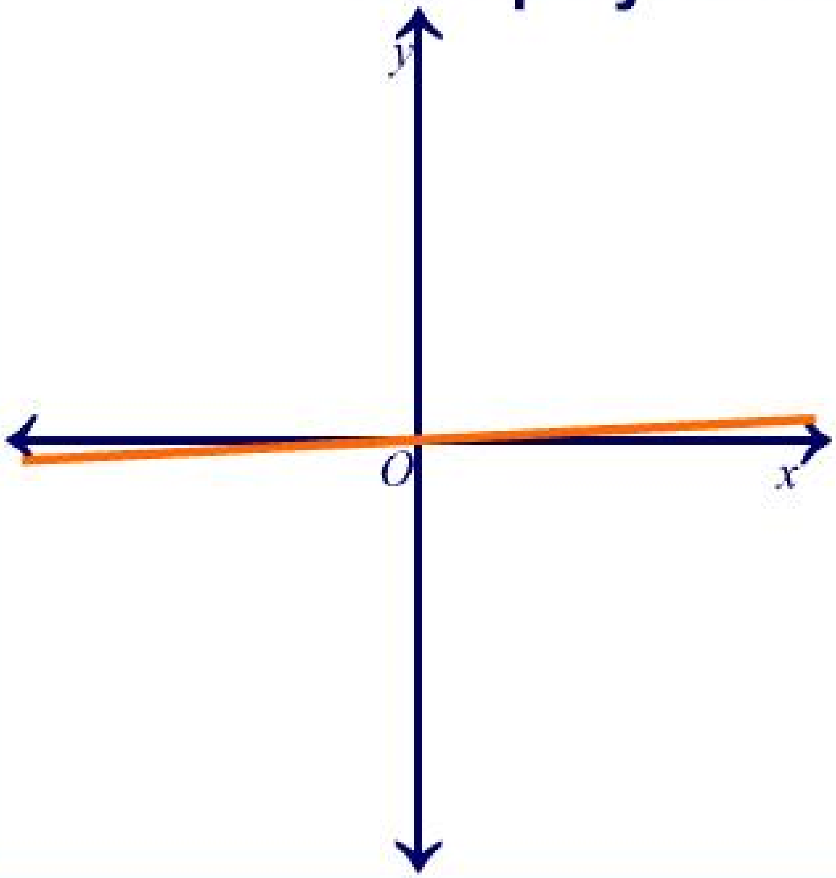
These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

How polynomial order affect graphs

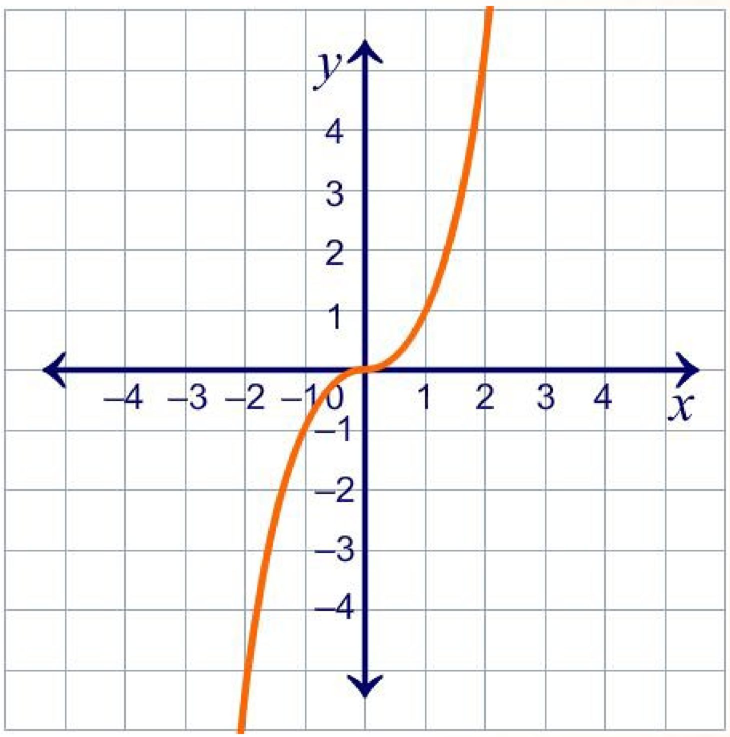


Press the arrows to change the coefficient and power.

$y = 0x^1$

Cubic functions

parent function: $f(x) = x^3$



vertical asymptote:	none
horizontal asymptote:	none
domain:	$(-\infty, \infty)$
range:	$(-\infty, \infty)$
roots:	$x = 0$

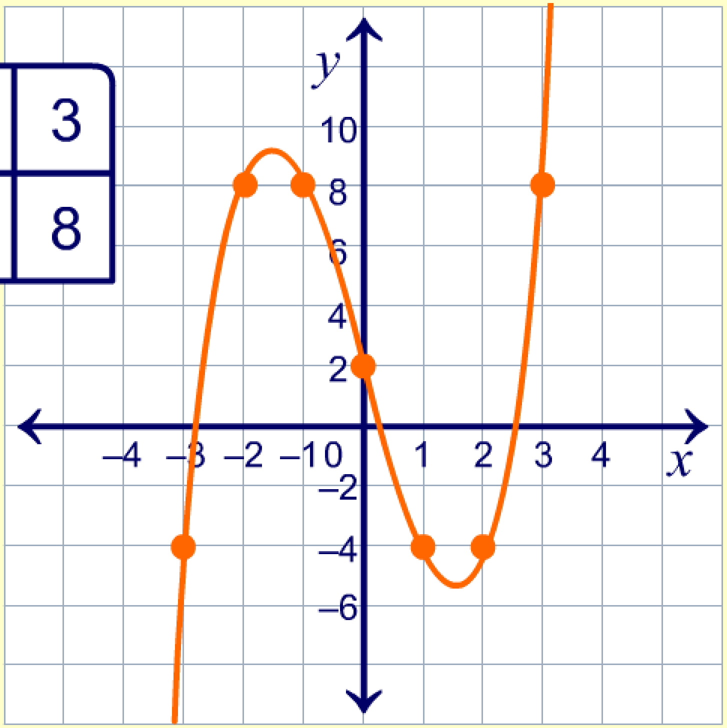
Plot the graph of $y = x^3 - 7x + 2$ for values of x between –3 and 3.

Complete the table of values.

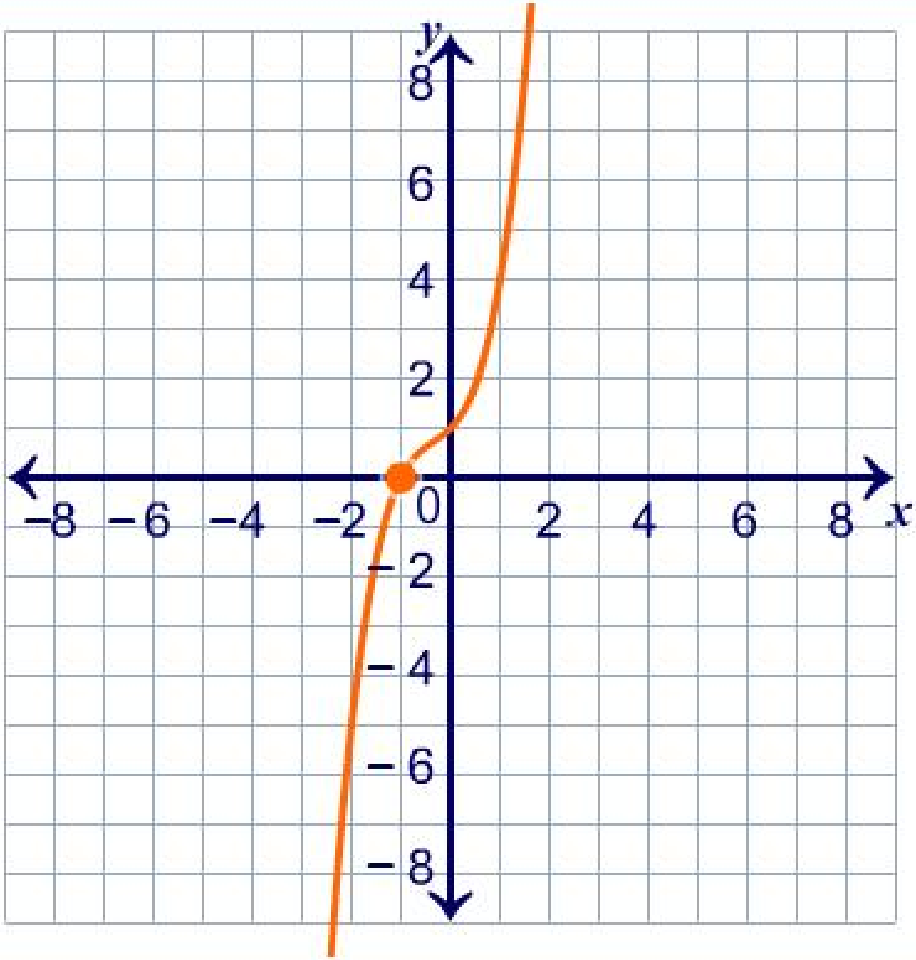
x	–3	–2	–1	0	1	2	3
$x^3 - 7x + 2$	–4	8	8	2	–4	–4	8

Plot the points from the table on the graph paper.

Join the points together with a smooth curve.



Exploring cubic graphs

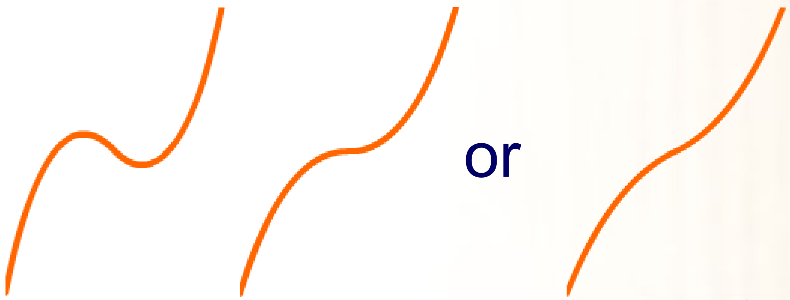


x	y
-3	-20
-2	-5
-1	0
0	1
1	4
2	15

$y = x^3 + x^2 + x + 1$

general cubic function: $f(x) = ax^3 + bx^2 + cx + d$
where $a \neq 0$

When the coefficient of x^3 is **positive** the shape is:



When the coefficient of x^3 is **negative** the shape is:

vertical asymptote:	none
horizontal asymptote:	none
domain:	$(-\infty, \infty)$
range:	$(-\infty, \infty)$

Sketching graphs

What can we find to help us sketch the graph of a function?

1) find the point where the curve intersects the y-axis by evaluating $f(0)$
2) find any points where the curve intersects the x-axis by solving $f(x) = 0$
3) predict the behavior of y when x is very large and positive
4) predict the behavior of y when x is very large and negative
5) estimate turning points.

When a cubic function is written in factored form $y = a(x - p)(x - q)(x - r)$, where does it intersect the x-axis?

The graph intersects the x-axis at the points $(p, 0)$, $(q, 0)$ and $(r, 0)$.
p, q and r are the **roots** of the cubic function.

Sketching cubic graphs (1)

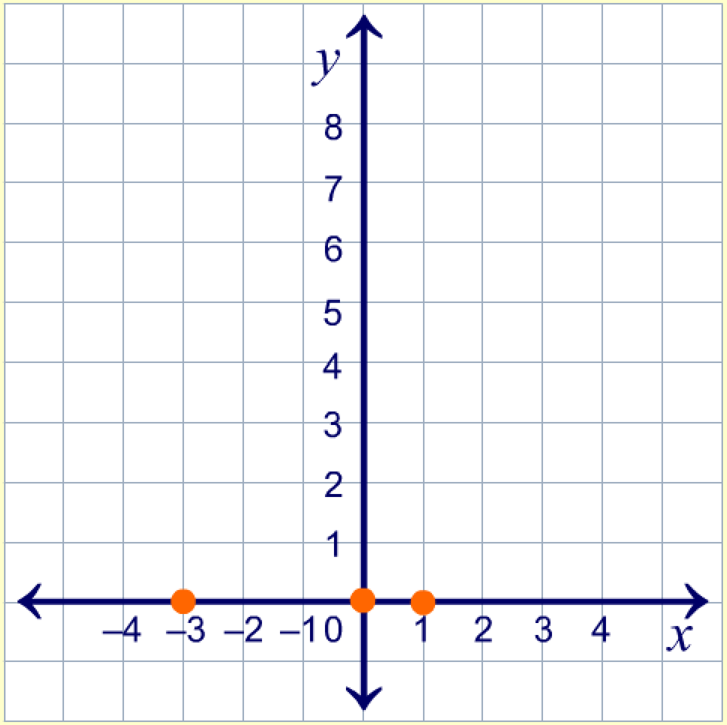
Sketch the graph of $y = x^3 + 2x^2 - 3x$.

1) find y-intercept:

evaluate $f(0)$: $f(0) = 0^3 + 2(0)^2 - 3(0)$

$f(0) = 0$

The curve passes through the point (0, 0).



2) find x-intercepts:

set $f(x) = 0$: $x^3 + 2x^2 - 3x = 0$

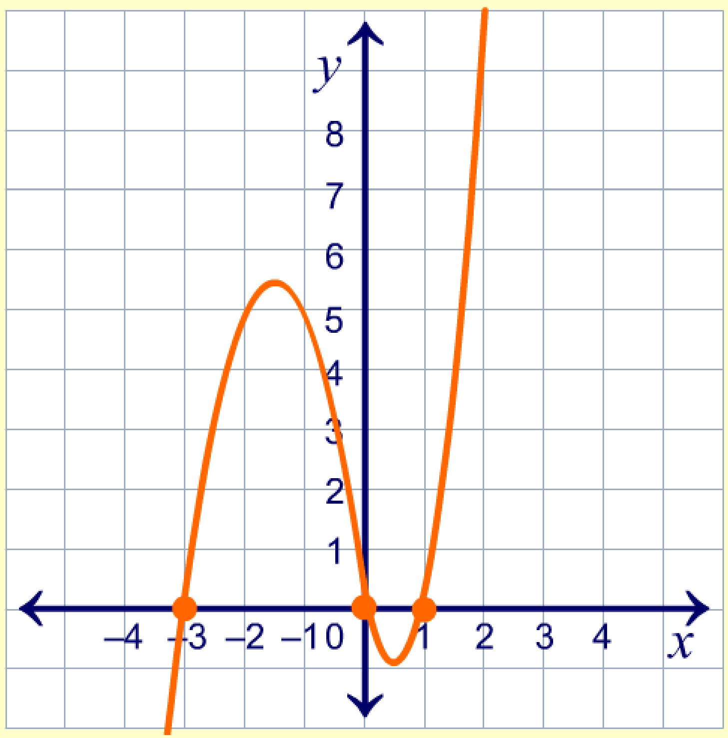
GCF: $x(x^2 + 2x - 3) = 0$

by grouping: $x(x + 3)(x - 1) = 0$

zero product: $x = 0$, $x = -3$ or $x = 1$

The curve also passes through the points (–3, 0) and (1, 0).

Sketch the graph of $y = x^3 + 2x^2 - 3x$.



3) as $x \to \infty$, $y \to \infty$

4) as $x \to -\infty$, $y \to -\infty$

5) estimate turning points:

since cubic: at most 2 turning points

x-value: between –3 and 0 and 0 and 1

estimate y: $f(-1.5) = 5.625$

$f(0.5) = -0.875$

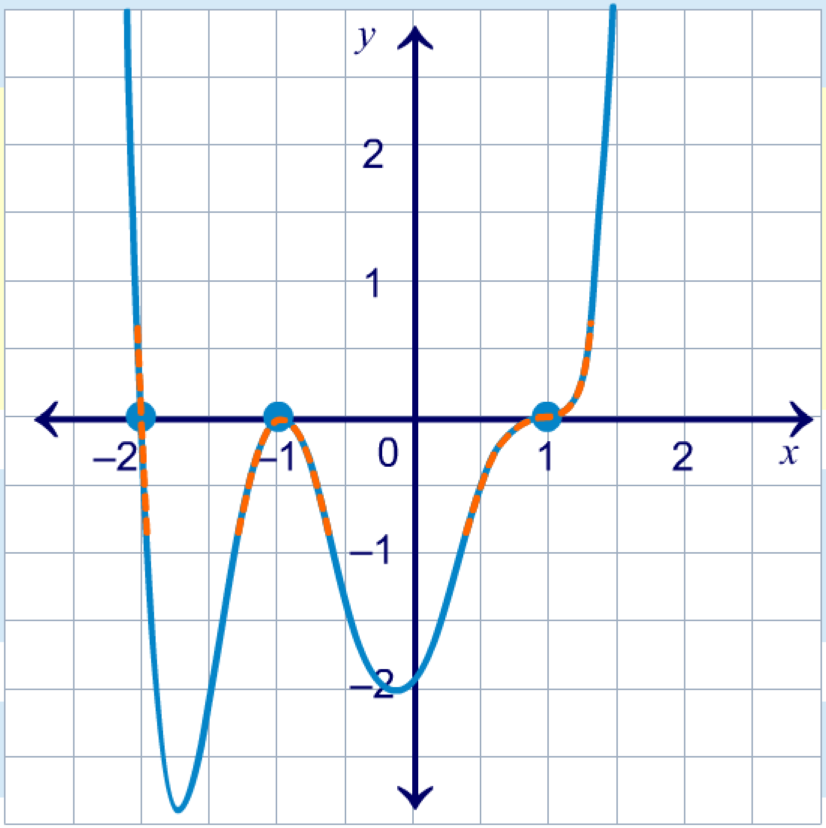
Multiple zeros

Give the multiplicity of each zero in the function $y = (x + 2)(x + 1)^2(x - 1)^3$.

$x = -2$ has multiplicity 1

$x = -1$ has multiplicity 2

$x = 1$ has multiplicity 3.



Now describe the behavior of the graph at each zero.

Do you notice a pattern?

The behavior of a graph at a zero of multiplicity n resembles the behavior of $y = x^n$ at zero.

Multiple zeros example

Sketch the graph of $y = (x + 1)^2(x - 1)$.

1) find y-intercept:

evaluate $f(0)$: $f(0) = -1$

The curve passes through (0, –1).

2) find x-intercepts:

set $f(x) = 0$: $(x + 1)^2(x - 1) = 0$

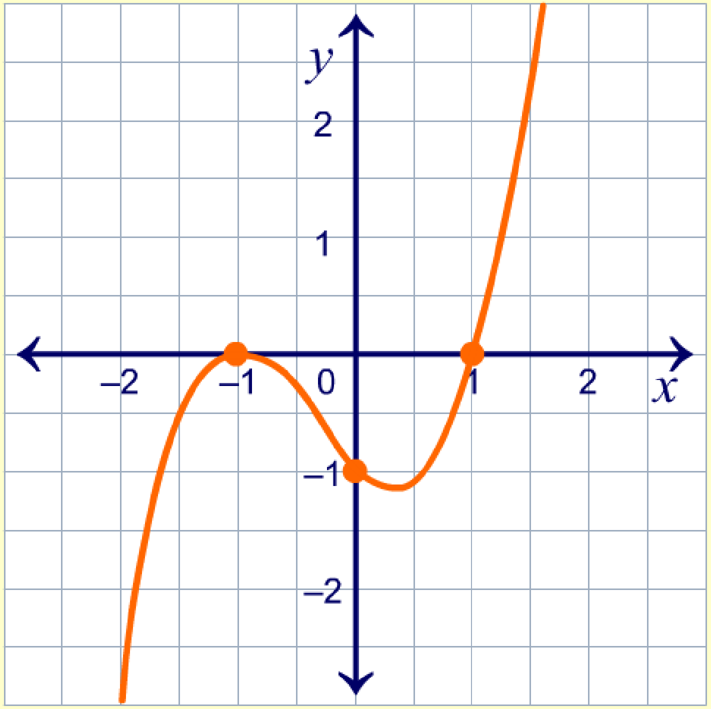
zero product property: $x = -1$ or $x = 1$

The curve passes through (–1, 0) and (1, 0).

3) as $x \rightarrow \infty$, $y \rightarrow \infty$

4) as $x \rightarrow -\infty$, $y \rightarrow -\infty$

5) estimate turning points:



Since $x = -1$ is a zero of multiplicity 2, the behavior of the graph at –1 is similar to quadratic, so that is a relative maxima.

Sketching quartic graphs (1)

Sketch the graph of $y = x^4 - 5x^2 + 4$.

1) find y-intercept:

evaluate $f(0)$: $f(0) = 0^4 - 5(0)^2 + 4$

$f(0) = 4$

The curve passes through the point (0, 4).

2) find x-intercepts:

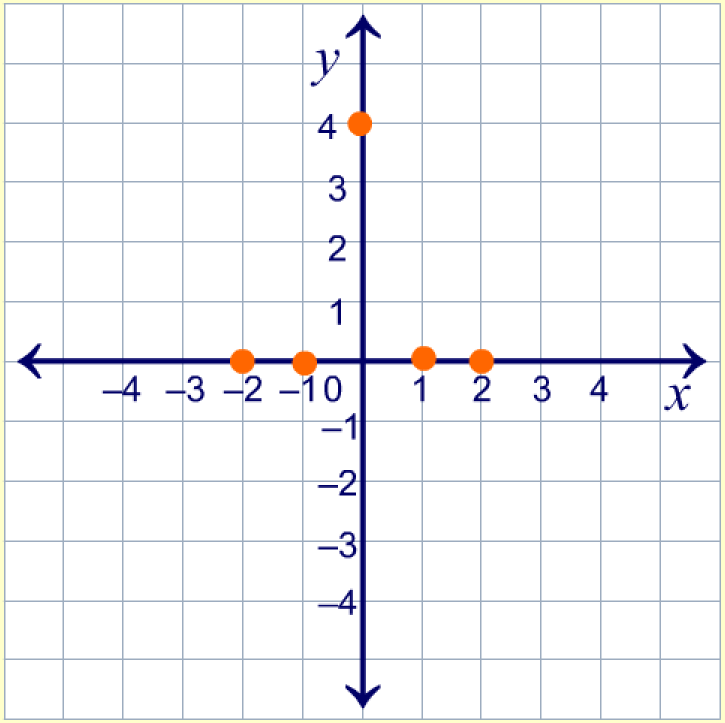
set $f(x) = 0$: $x^4 - 5x^2 + 4 = 0$

$(x^2)^2 - 5(x^2) + 4 = 0$

$(x^2 - 1)(x^2 - 4) = 0$

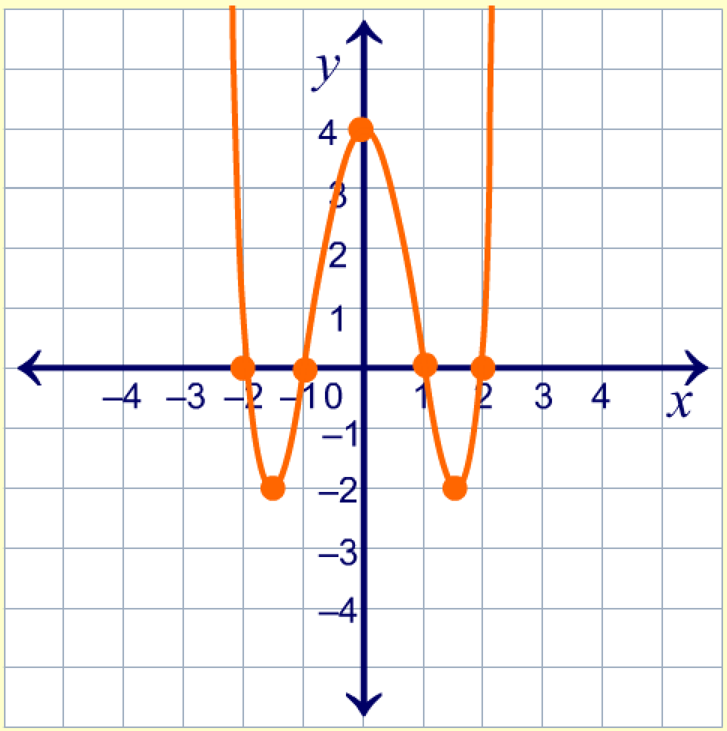
difference of squares: $(x + 1)(x - 1)(x + 2)(x - 2) = 0$

zero product property: $x = \pm 1$ and $x = \pm 2$



The curve also passes through the points (–2, 0), (–1, 0), (1, 0) and (2, 0).

Sketch the graph of $y = x^4 - 5x^2 + 4$.



3) as $x \to \infty$, $y \to \infty$

4) as $x \to -\infty$, $y \to \infty$

5) estimate turning points:

since quartic: at most 3 turning points

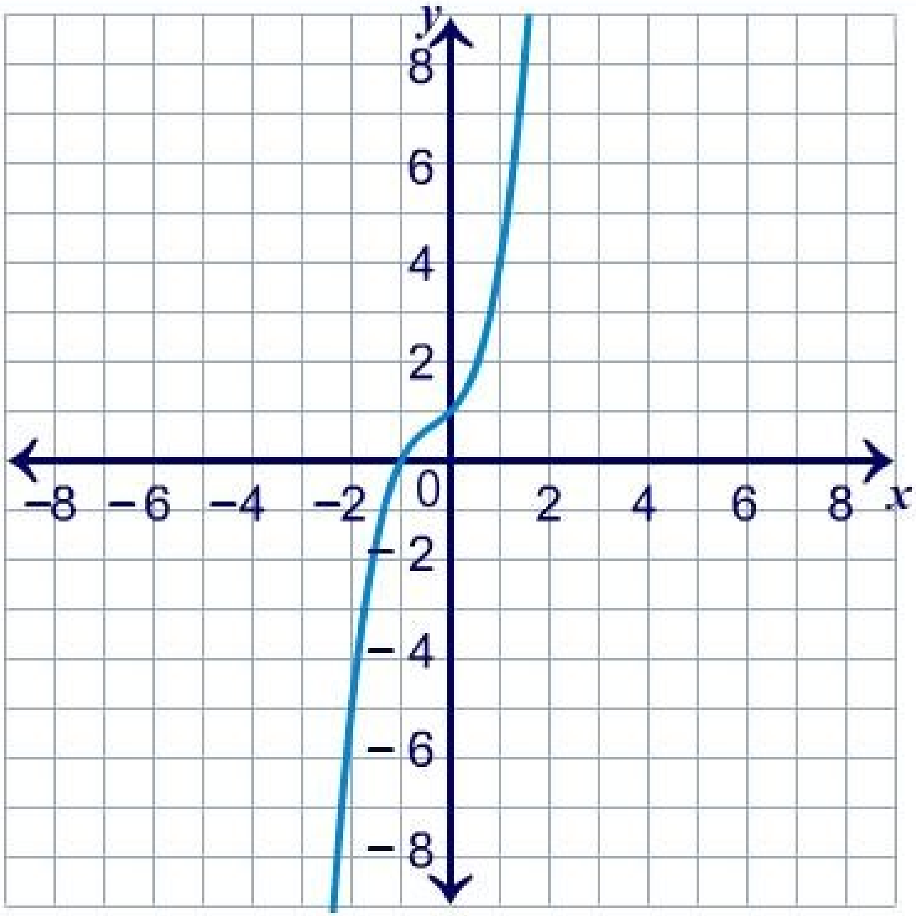
x-value: between –1 and 1, between –2 and –1, and between 1 and 2

estimate y: $f(0) = 4$

$f(-1.5) = -2.1875$

$f(1.5) = -2.1875$

Transforming cubic functions



$y = x^3 + x^2 + x + 1$

1. $f(x) + a$
2. $f(x + a)$
3. $af(x)$
4. $f(ax)$

$a = 0$

$y' = f(x)$

Types of transformation

Match the description of each transformation with the function formed from $y = f(x)$.

Press the info buttons to see an example of each transformation.

Press **start** to begin.

start

A box problem

A box is made from a sheet of paper, 29.7 cm × 21 cm.