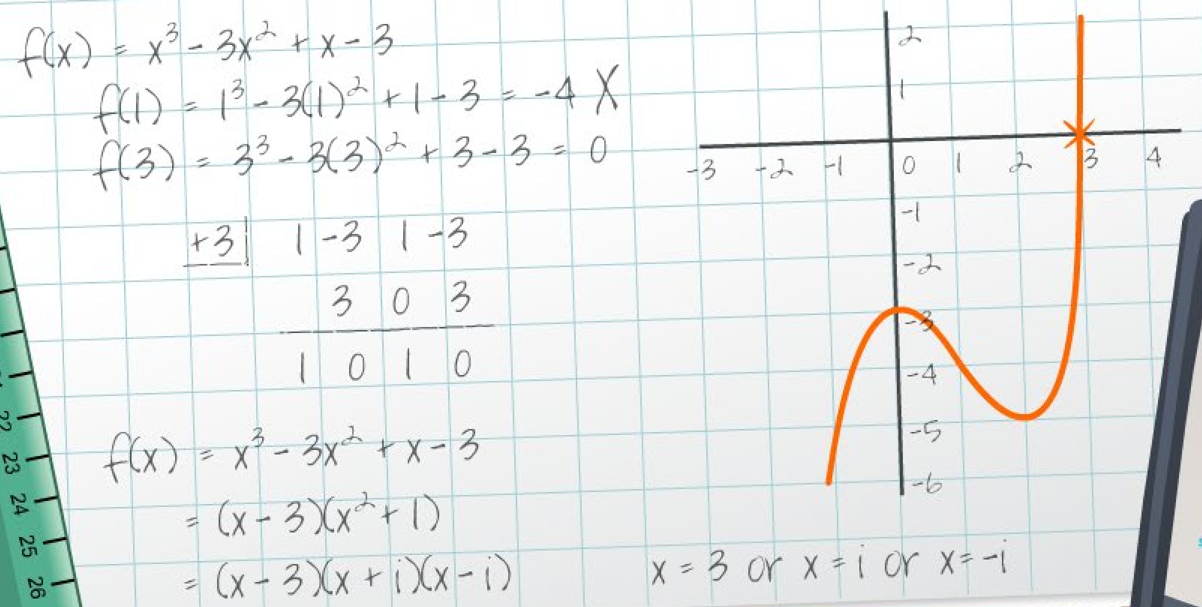
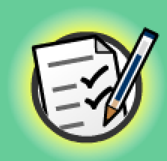
# Polynomial Expressions



# Information

## Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.


Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

# Polynomials

**polynomial in standard form:** $ax^n + bx^{n-1} + cx^{n-2} + \ldots + px^2 + qx + r$

where $x$ is a variable, $a$, $b$, $c$, … are constant coefficients and $n$ is a nonnegative integer.

$a$ is called the **leading coefficient**.
$n$ gives the **degree**, or **order**, of the polynomial.

- A polynomial of degree 0 is a constant — $P(x) = 5$
- A polynomial of degree 1 is **linear** — $P(x) = 3x - 7$
- A polynomial of degree 2 is **quadratic** — $P(x) = 5x^2 + 20$
- A polynomial of degree 3 is **cubic** — $P(x) = x^3 - 6x^2 + 2x - 1$
- A polynomial of degree 4 is **quartic** — $P(x) = 12x^4 + x^2 + 5$

# Evaluating polynomials

**Find $P(x) = 3x^7 + x^6 - x^4 + 2x^2 - 7$ when $x = 5$.**

substitute $x = 5$: $P(5) = 3(5)^7 + (5)^6 - (5)^4 + 2(5)^2 - 7$

evaluate: $P(5) = \mathbf{249,418}$

**Find $P(x) = -x^4 - x^3 - x^2 - x - 1$ when $x = -1$.**

substitute $x = -1$: $P(-1) = -(-1)^4 - (-1)^3 - (-1)^2 - (-1) - 1$

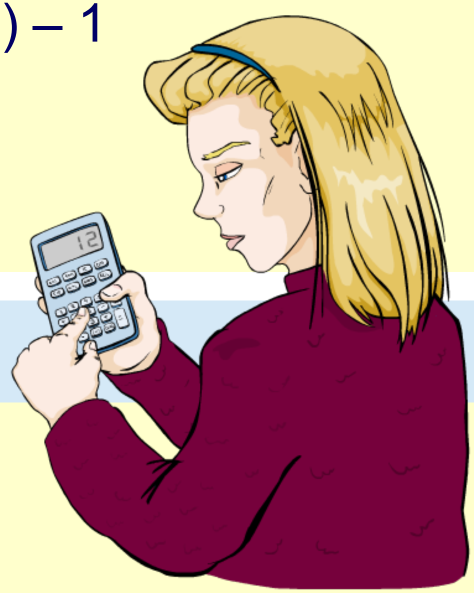
evaluate: $P(-1) = -1 + 1 - 1 + 1 - 1$

$P(-1) = \mathbf{-1}$

**Find $P(x) = 15x^6 - 120x^3$ when $x = 2$.**

substitute $x = 2$: $P(2) = 15(2)^6 - 120(2)^3$

evaluate: $P(2) = \mathbf{0}$



# Adding polynomials

When two or more polynomials are added together, the result is another polynomial.

**$P(x) = 3x^4 + 6x^3 + x^2 – 3x – 5$ and $Q(x) = 7x^3 – 7x^2 – x – 1$.**
**How can we add $P(x)$ and $Q(x)$? Find $R(x) = P(x) + Q(x)$.**

Polynomials are added by **combining like terms**.

$R(x) = (3x^4 + 6x^3 + x^2 – 3x – 5) + (7x^3 – 7x^2 – x – 1)$

order terms: $R(x) = 3x^4 + 6x^3 + 7x^3 + x^2 – 7x^2 – 3x – x – 5 – 1$

combine like terms: $R(x) = 3x^4 + (6 + 7)x^3 + (1 – 7)x^2 + (–3 – 1)x – 5 – 1$

simplify coefficients: $\mathbf{R(x) = 3x^4 + 13x^3 – 6x^2 – 4x – 6}$

When a polynomial is subtracted from another polynomial, the result is a polynomial.

**$P(x) = -4x^3 + 6x^2 + 9x$ and $Q(x) = -2x^5 + 2x^3 + 2x^2 + 6x + 5$.**
**Find $R(x) = P(x) - Q(x)$.**

Polynomials are subtracted by **combining like terms**.

$R(x) = (-4x^3 + 6x^2 + 9x) - (-2x^5 + 2x^3 + 2x^2 + 6x + 5)$

distributive property: $R(x) = -4x^3 + 6x^2 + 9x + 2x^5 - 2x^3 - 2x^2 - 6x - 5$

order terms: $R(x) = 2x^5 - 4x^3 - 2x^3 + 6x^2 - 2x^2 + 9x - 6x - 5$

combine like terms: $R(x) = 2x^5 + (-4 - 2)x^3 + (6 - 2)x^2 + (9 - 6)x - 5$

simplify coefficients: $\mathbf{R(x) = 2x^5 - 6x^3 + 4x^2 + 3x - 5}$

## Adding and subtracting polynomials

Find $P(x) + Q(x)$ and $P(x) - Q(x)$ for the pairs of polynomials below.

1. $P(x) = -2x^5 + 2x^2 + 3x - 3$
   $Q(x) = 2x^5 + 2x^4 + 3x^3 + 4x^2 - 2x - 5$

2. $P(x) = x^4 + 5x^3 + 2x^2 - 2x + 1$
   $Q(x) = 3x^4 - 4x^3 + x^2 - 3x - 1$

3. $P(x) = x^3 + x^2 - 2x - 1$
   $Q(x) = 6x^5 - 2x^4 - x^2 + 2x - 3$

4. $P(x) = 5x^4 - x^3 + x^2 - 1$
   $Q(x) = x^6 - x^4 - x^3 + 3x + 3$

When two polynomials are multiplied together, the result is another polynomial.

Polynomials are multiplied using the **distributive property**.

**$P(x) = -x^4 + x^3 + 2x^2 - 3x - 1$ and $Q(x) = 6x^3 + x^2 - 5x + 4$.**
**Find $R(x) = P(x)\,Q(x)$.**

$R(x) = (-x^4 + x^3 + 2x^2 - 3x - 1)(6x^3 + x^2 - 5x + 4)$

distribute: $R(x) = -6x^7 - x^6 + 5x^5 - 4x^4 + 6x^6 + x^5 - 5x^4 + 4x^3 + 12x^5 + 2x^4 - 10x^3 + 8x^2 - 18x^4 - 3x^3 + 15x^2 - 12x - 6x^3 - x^2 + 5x - 4$

group like terms: $R(x) = -6x^7 + (-1 + 6)x^6 + (5 + 1 + 12)x^5 + (-4 - 5 + 2 - 18)x^4 + (4 - 10 - 3 - 6)x^3 + (8 + 15 - 1)x^2 + (-12 + 5)x - 4$

simplify: $\mathbf{R(x) = -6x^7 + 5x^6 + 18x^5 - 25x^4 - 15x^3 + 22x^2 - 7x - 4}$

## Multiplying polynomials

$P(x) = x^3 + 4x^2 + x - 6$ and $Q(x) = 2x^3 + x^2 - 5x - 2$.

$P(x)Q(x) = (x^3 + 4x^2 + x - 6)(2x^3 + x^2 - 5x - 2)$

$= 2x^6 + 9x^5 + x^4 - 33x^3 - 19x^2 + 28x + 12$

**Question: 1/7**

**Which terms contribute to give the term in $x^6$ in $P(x)Q(x)$?**

$x^3$ | $4x^2$ | $x$ | $6$ | $2x^3$ | $x^2$ | $5x$ | $2$

*press any correct terms*

# Polynomials and integers

**What type of number is produced when two integers are added together?** e.g. $5 + 250 = 255$

**What type of number is produced when an integer is subtracted from another integer?** e.g. $3 - 7 = -4$

**What type of number is produced when two integers are multiplied together?** e.g. $-12 \times 4 = -48$

The number resulting from any of these operations is **always** another integer.

The integers are said to be **closed** under addition, subtraction and multiplication.

Polynomials are also closed under the same operations.

# Marking homework

**Kat had the wrong answers in her homework.**
**Where did she go wrong?** *Press on the mistakes to correct them.*

$P(x) = 3x^3 + x^2 - 4, \quad Q(x) = x^4 + x^3 + 5x^2 - 7x - 9$

1. $P(x) + Q(x) = 3x^3 + x^2 - 4 + x^4 + x^3 + 5x^2 - 7x - 9$

   $P(x) + Q(x) = x^4 + 3x^3 + 5x^2 - 7x - 5$

2. $P(x) - Q(x) = 3x^3 + x^2 - 4 - x^4 - x^3 - 5x^2 - 7x - 9$

   $P(x) - Q(x) = -x^4 + 2x^3 - 4x^2 - 7x - 13$

3. $P(x)\,Q(x) = (3x^3 + x^2 - 4)(x^4 + x^3 + 5x^2 - 7x - 9)$

   $P(x)\,Q(x) = 3x^7 + 3x^6 + 8x^5 - 10x^4 - 12x^3 + x^6 + x^5 + 5x^4 - 7x^3 - 9x^2 - 4x^4 - 4x^3 - 20x^2 + 28x + 36$

   $P(x)\,Q(x) = 3x^7 + 4x^6 + 9x^5 + x^4 - 23x^3 - 29x^2 + 28x + 36$