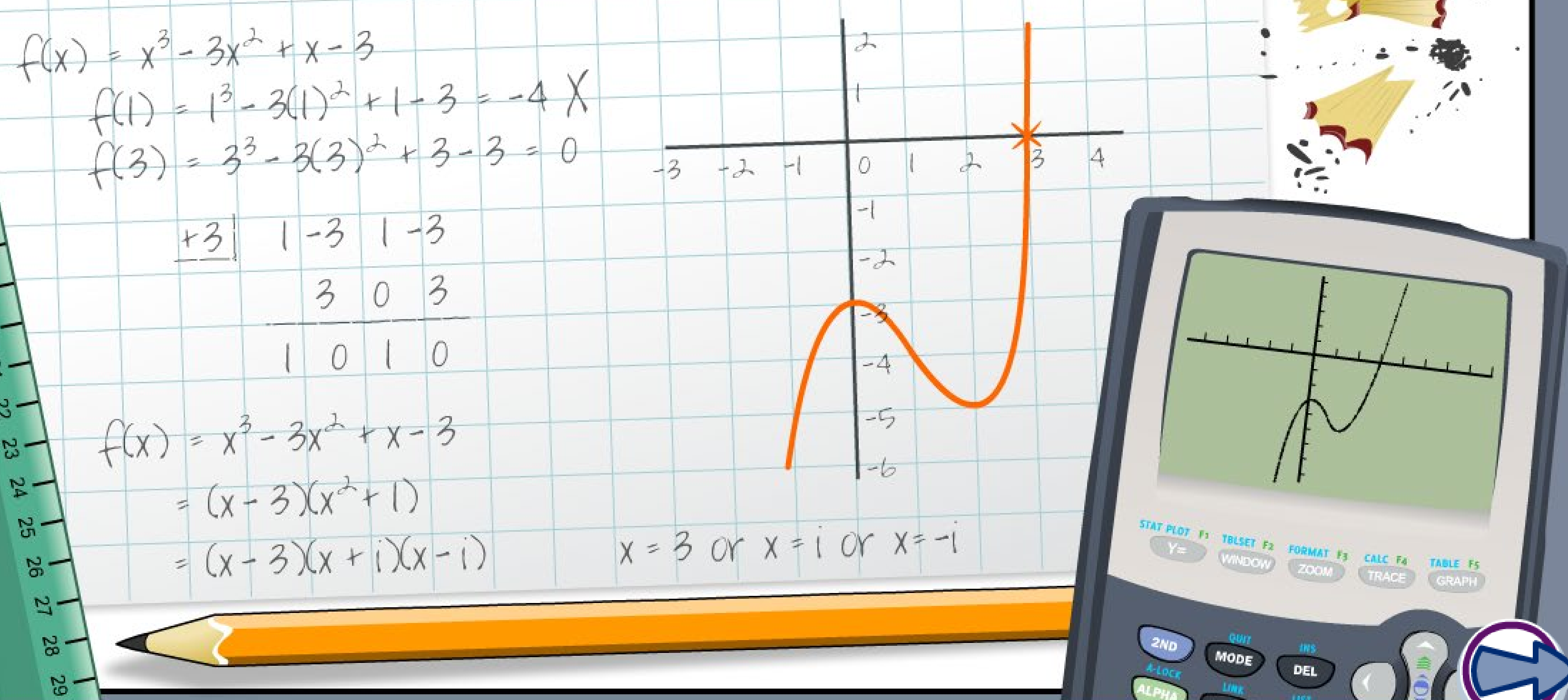
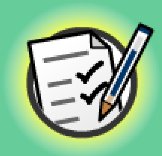
Natural Exponential and Logarithm



Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Why is e important?

A general form of an exponential function is $y = b^x$.
The **natural exponential function** is $\boldsymbol{y = e^x}$.

e (≈ 2.718) is an irrational number, but
why is this specific number so important?

Suppose you have \$1 and you earn 100% compound interest on this amount each year. After 1 year you will have \$2.

What happens if you compound more often, say, every month? How about every day? Every minute?

Realistically, exponential growth involves compounding continuously, effectively like compounding **infinite** times.

Compounding interest

If an investment earns more money the more often interest is compounded, I must be able to make the amount as large as I like by compounding sufficiently often!

Press "**play**" to test this claim.

The exponential function

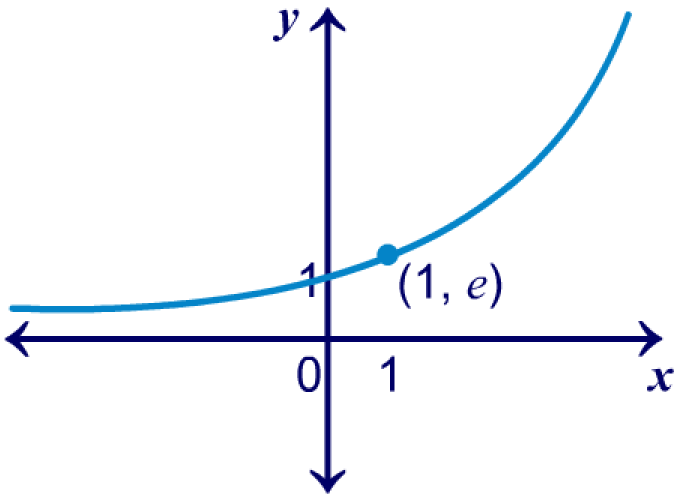
Exponential functions (base e) are used to describe continuous growth.

the exponential function (base e): $y = e^x$

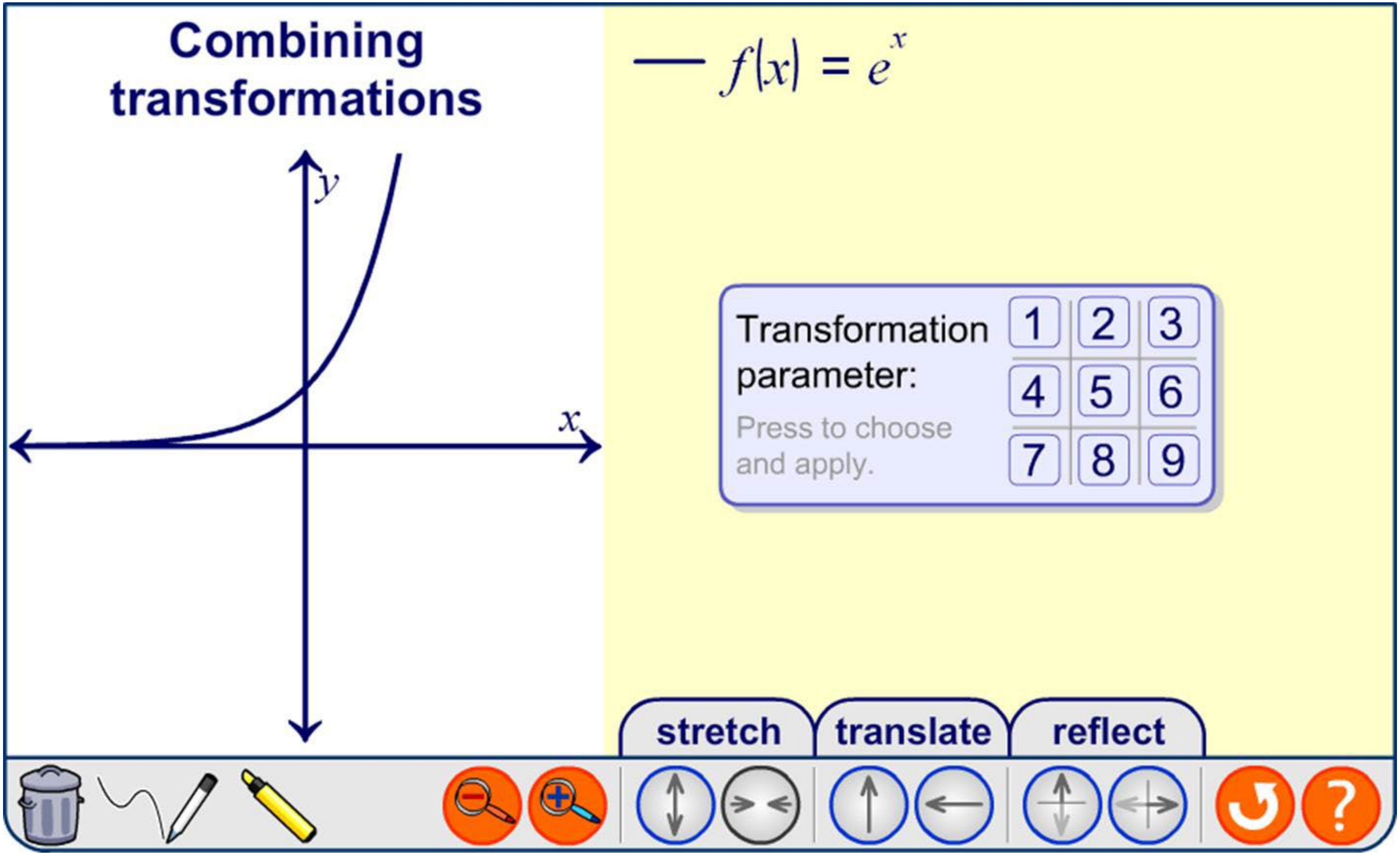
where e is the irrational constant, approximately 2.718

The graph of $y = e^x$ has the following shape:

The curve passes through (0, 1) and (1, e).



Transformations of $f(x) = e^x$



Evaluating natural exponents

Natural exponents can be evaluated using a graphing calculator.

Press “2$^{\text{nd}}$” followed by the “LN” button, then enter the value of x and press “ENTER”.

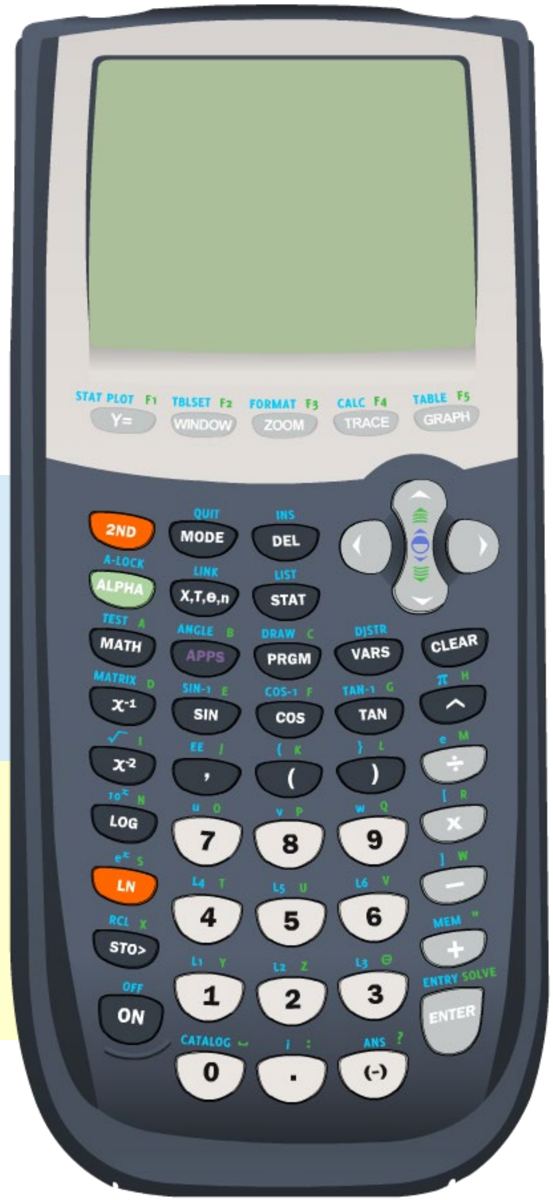
Evaluate the following exponential values to the nearest hundredth:

a) e^2 b) $e^5/2$ c) $3e^3$

a) e^2 = **7.39**

b) $e^5/2$ = **74.21**

c) $3e^3$ = **60.26**



The inverse of e

For the function $y = e^x$, find the value of y when $x = 4$.

$y = e^x$

substitute $x = 4$: $y = e^4$

evaluate: **y = 54.60 to nearest hundredth**

Now can you find the value of x for the function $f(x) = e^x$ when $f(x) = 50$, without using a graph?

$y = e^x$

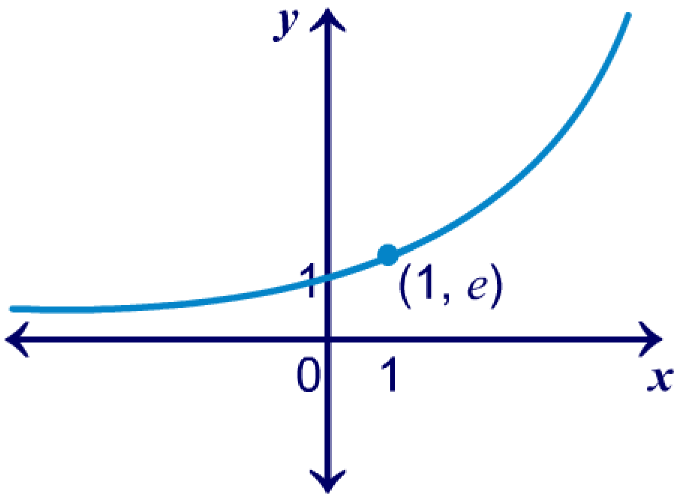
substitute $f(x) = 50$: $50 = e^x$

solve: $x = ?$

In order to find the value of x, it is necessary to be able to find the **inverse of** e.

The inverse of the exponential function

The graph of $y = e^x$ shows that it is a one-to-one function, meaning it will have an inverse.



Start with the equation

$$y = e^x$$

We can find the inverse by interchanging the x and the y and making y the subject of the equation:

$$x = e^y$$

Remember that if $x = e^y$ then $\log_e x = y$.

So, we can write this using logarithms as: $y = \log_e x$

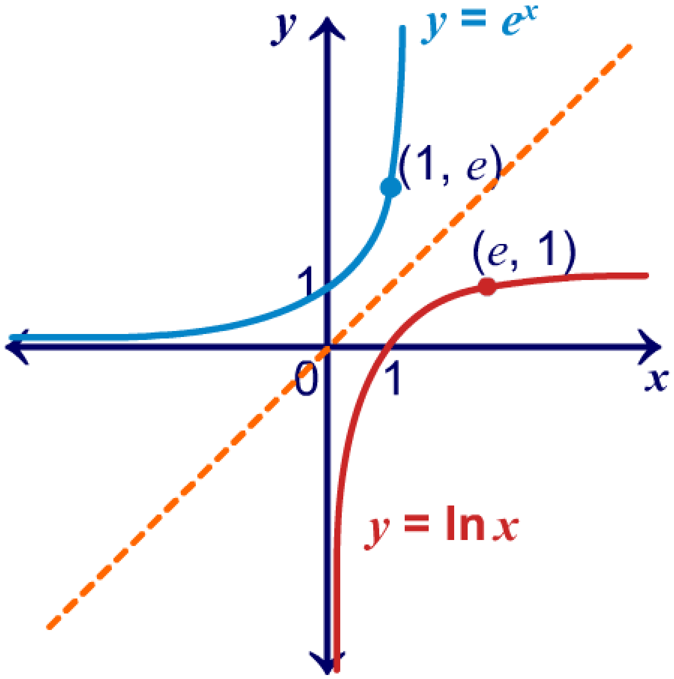
The natural logarithm

The inverse of e is called the **natural logarithm**, denoted **ln**.

natural logarithmic function: $y = \ln x$ (or $y = \log_e x$)

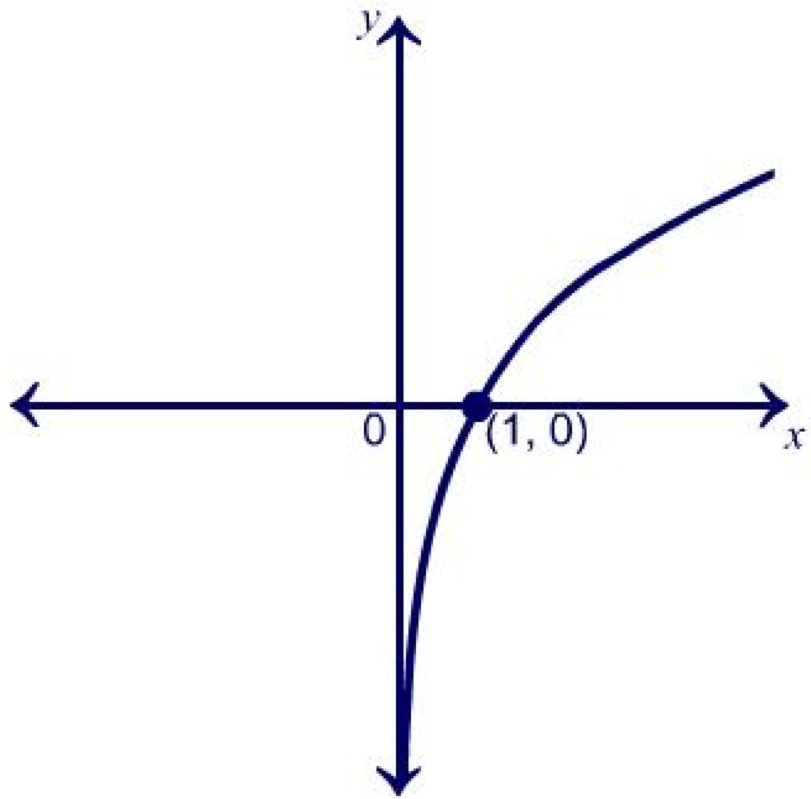
What do you think the graph of the natural logarithmic function looks like?

The graph of $y = \ln x$ is a reflection of $y = e^x$ in the line $y = x$.



Translating $f(x) = \ln x$

Translating the natural logarithmic function



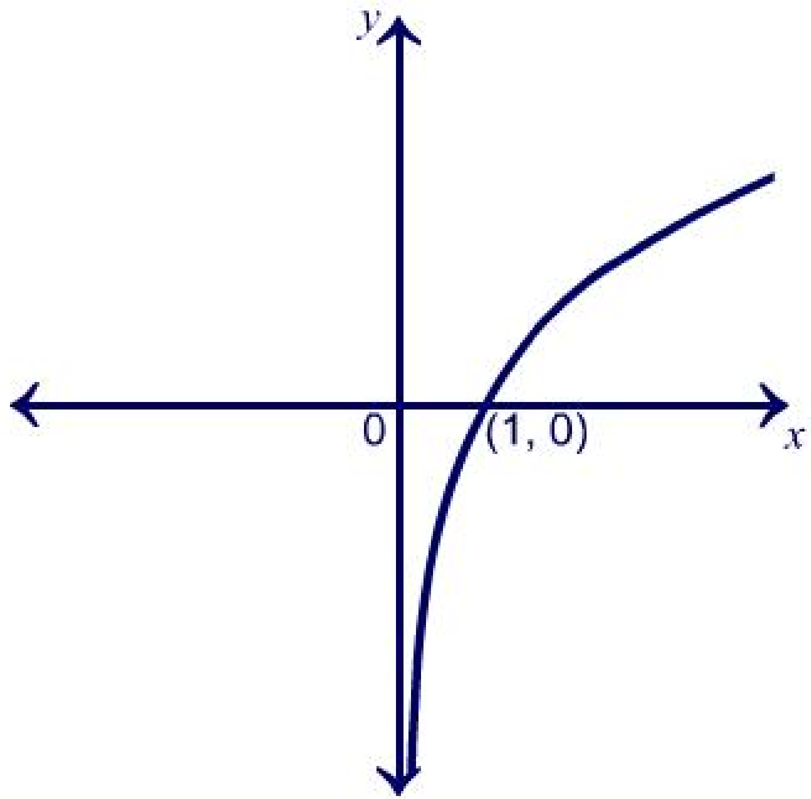
*Press the arrows to transform the function. Press **show** or **hide** to show or hide the transformed functions.*

$f(x) = \ln x$

$y = f(x) + 0$ $y = f(x + 0)$

show show

Reflecting the natural logarithmic function



*Press **show** or **hide** to show or hide the transformed functions.*

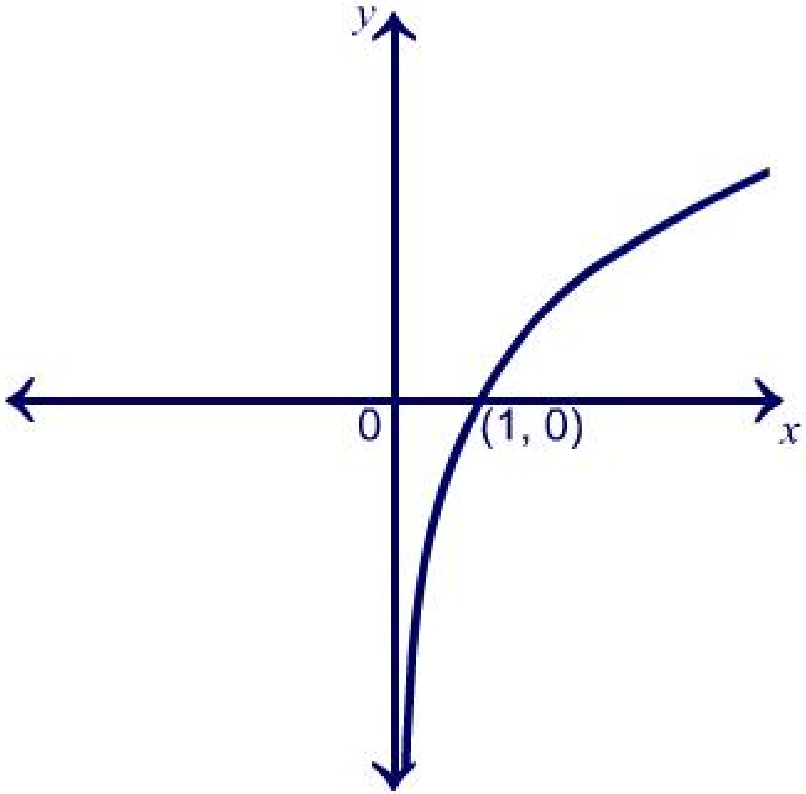
— $f(x) = \ln x$

*Press **show** or **hide** to show or hide the transformed functions.*

— $y = f(-x)$ show

— $y = -f(x)$ show

Stretching the natural logarithmic function



Press the arrows to transform the function. Press ***show*** *or* ***hide*** *to show or hide the transformed functions.*

$f(x) = \ln x$

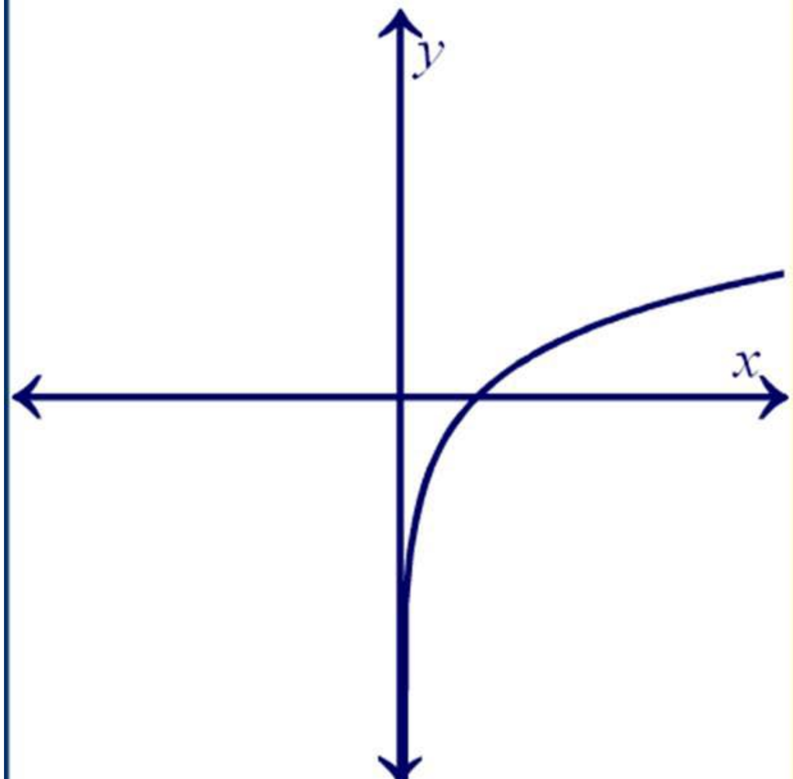
$y = 1f(x)$

$y = f(1x)$

show

show

Combining transformations



— $f(x) = \ln x$

stretch | translate | reflect

Using logarithms

The natural logarithm (ln), can be used to solve exponential equations base e.

natural logarithm: for $x = e^y$, $y = \ln x$

To calculate y, press the “LN” button then the value of x and then press “ENTER”.

Can you find the value of x (to the nearest hundredth) for the function $f(x) = e^x$ when $f(x) = 50$ using logarithms?

	$f(x) = e^x$
substitute $f(x) = 50$:	$50 = e^x$
take the natural logarithm:	$\ln 50 = x$
evaluate:	**3.91 =** x (to the nearest hundredth)

Practice problem

Natural exponential and logarithmic functions

A function g is defined by:
$g(x) = 3e^{x+1} - 4$

a) Describe the sequence of geometrical transformations by which the graph of $g(x)$ can be obtained from that of $f(x) = e^x$.

b) Write an expression for $g^{-1}(x)$ and state its domain and range.

Press the "next" arrow to see the solutions.