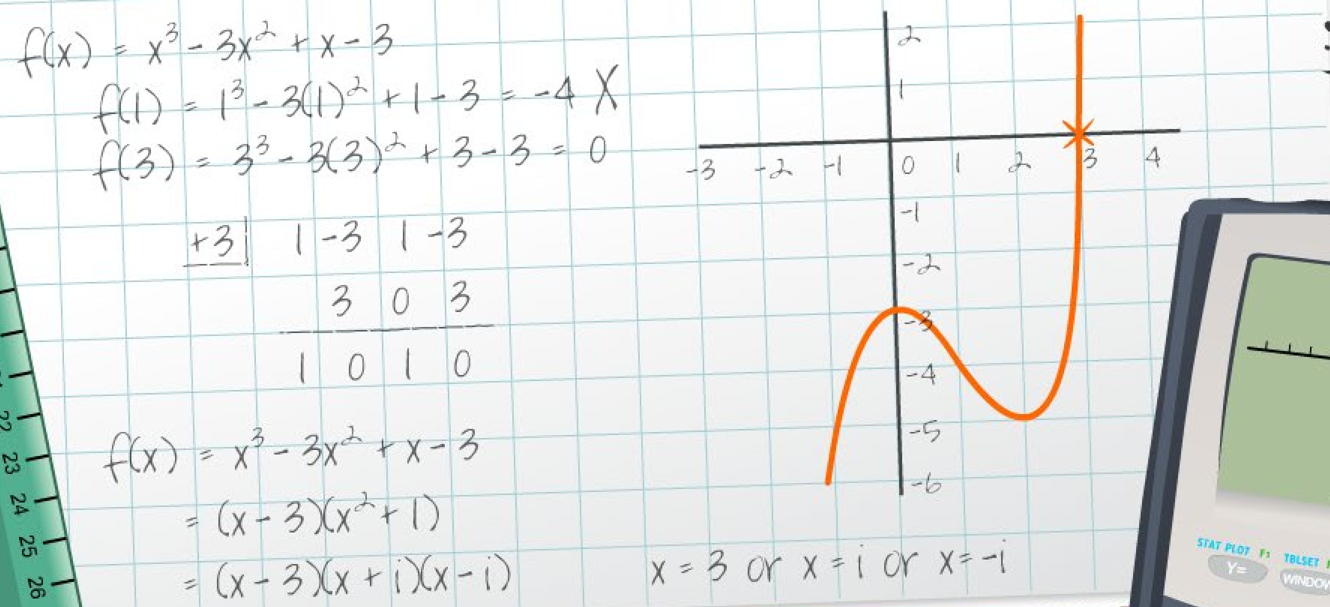
Logarithmic Functions



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

MODELING

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

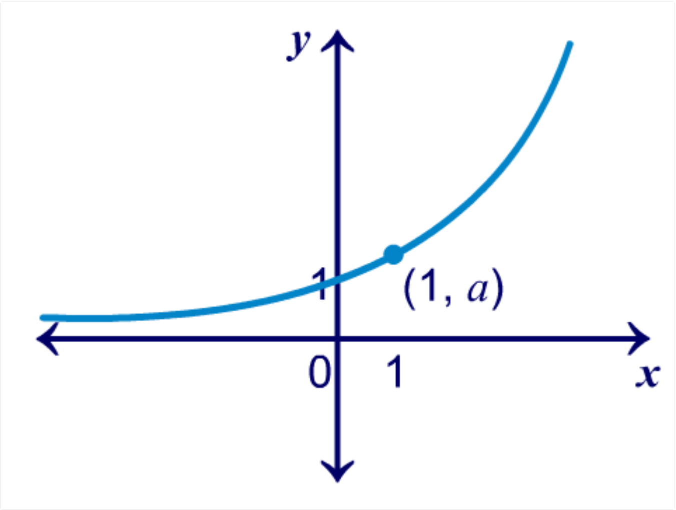
This icon indicates teacher's notes in the Notes field.

Review of exponential functions

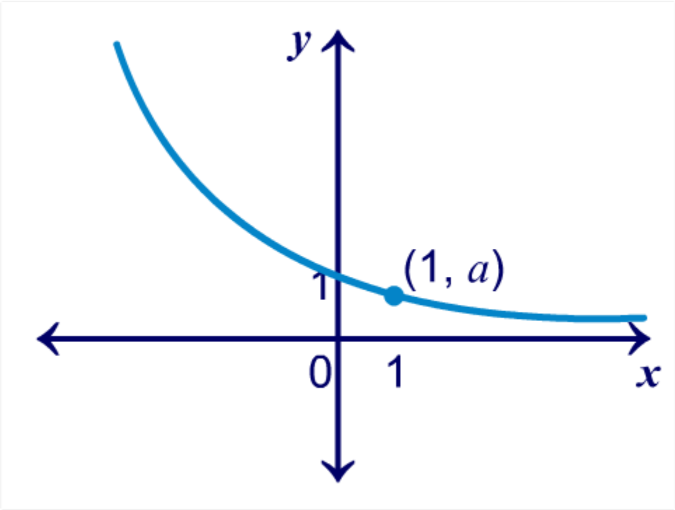
exponential function with base a:

$y = a^x$ where $a > 0$ and $a \neq 1$

When $a > 1$, the graph of $y = a^x$ has the following shape:



When $0 < a < 1$, the graph of $y = a^x$ has the following shape:



In both cases, the graph passes through (0, 1) and (1, a).

Exponential functions

For the function $y = 3^x$, find the value of y when $x = 4$.

$y = 3^x$

substitute $x = 4$: $y = 3^4$

evaluate: $\mathbf{y = 81}$

Now can you find the value of x for the function $y = 3^x$ when $y = 100$, without using a graph?

$y = 3^x$

substitute $y = 100$: $100 = 3^x$

solve: $x = ?$

When the value of x is not obvious, as in this case, the **inverse** of the exponential is required.

What is a logarithmic function?

The inverse of addition is subtraction.
The inverse of multiplication is division.

The inverse of an exponential is called a **logarithm** (log).

logarithmic function:
if $x = a^y$, then $y = \log_a x$

for a positive value of x
and for $a > 0$, $a \neq 1$

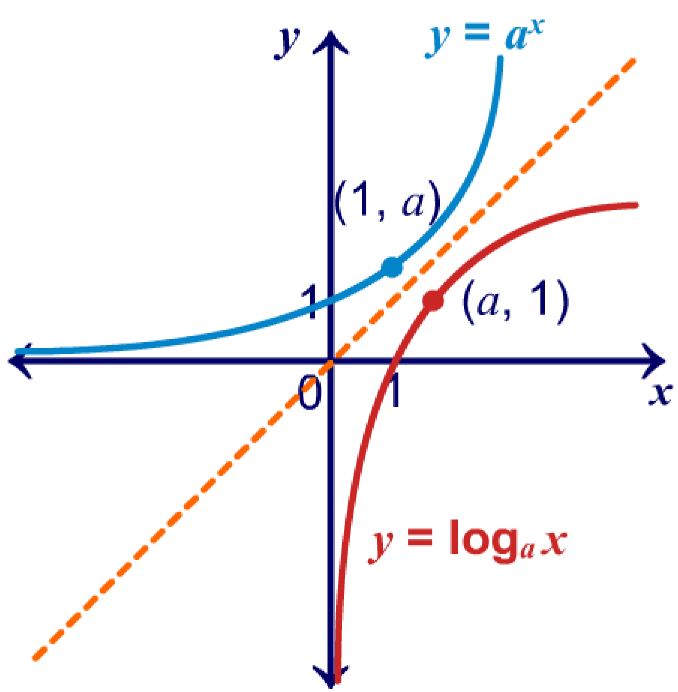
"$\log_a x$" is read as "log base a of x."

The logarithm asks "to what power, y, is a raised to get x?"

Can you explain why $a \neq 1$ must be included in the definition of a logarithmic function?

By thinking about the graphs of some inverse functions, e.g. $f(x) = x + 1$ and $f^{-1}(x) = x - 1$, $f(x) = 2x$ and $f^{-1}(x) = ½x$, sketch the graph of a logarithmic function.

The graph of a function's inverse is a reflection of the function in the line $y = x$.



The graph of the logarithmic function is therefore a reflection of an exponential function in the line $y = x$.

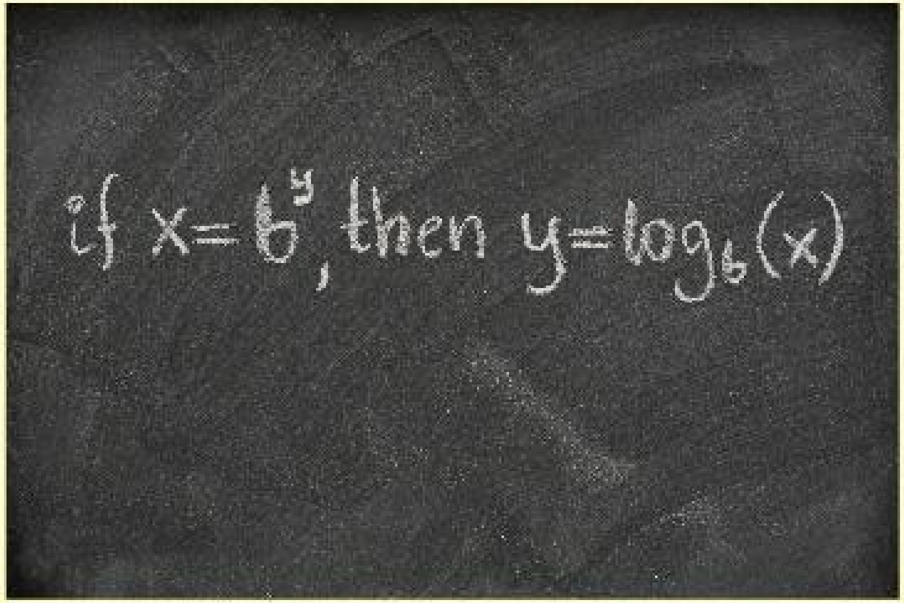
Discuss the key features of the graph of $y = \log_a x$.

Graph observations

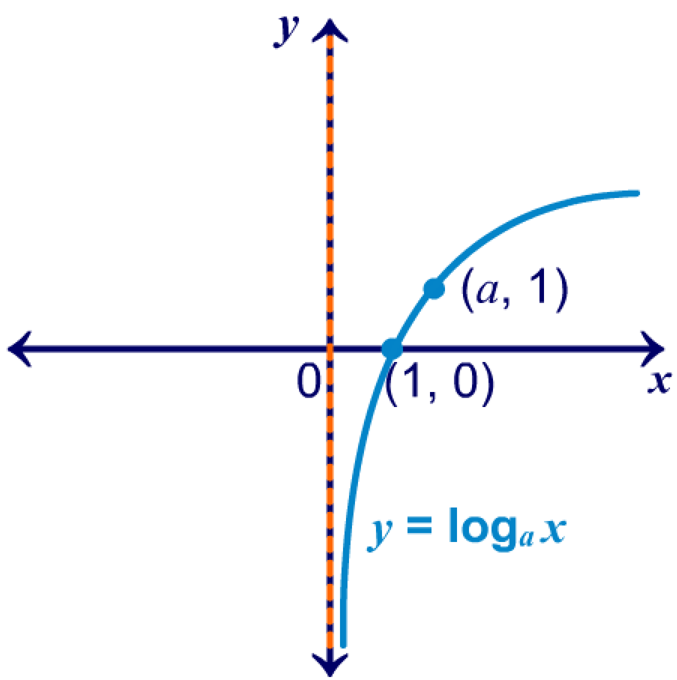
How closely did you observe the graph of $y = \log_a x$?

Press **start** to begin a multiple choice quiz about the graph's key features.

start



parent logarithmic function: $y = \log_a x$



vertical asymptotes:	$x = 0$
horizontal asymptotes:	none
domain:	$(0, \infty)$
range:	$(-\infty, \infty)$
roots:	$(1, 0)$
other key points:	$(a, 1)$

Explain why the curve always passes through the point (1, 0) for all values of a.

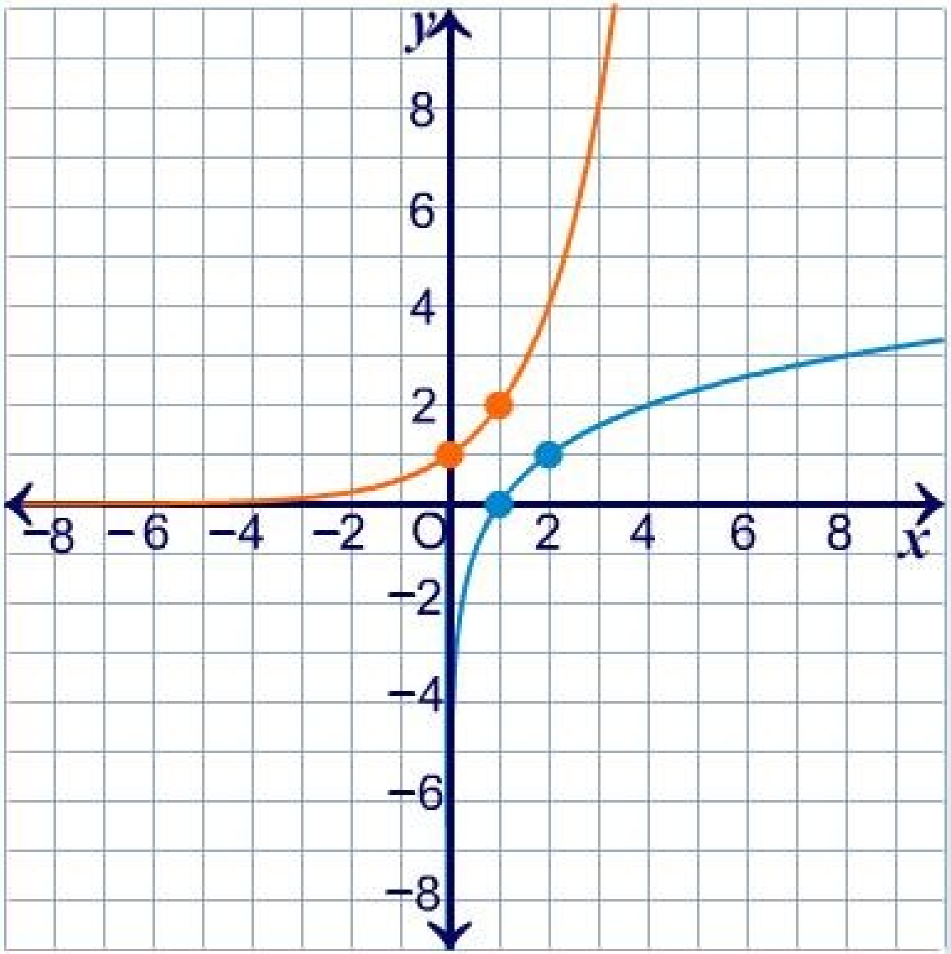
Logarithmic bases

base 2 | base 10 | base *e*

The **base** of a logarithm can be any positive real number, except 1.

The most commonly used bases are **2**, **10**, and e, because they make calculations in certain applications a lot easier.

Press on each tab to learn more about each of these common bases.



Comparing exponential and logarithmic graphs

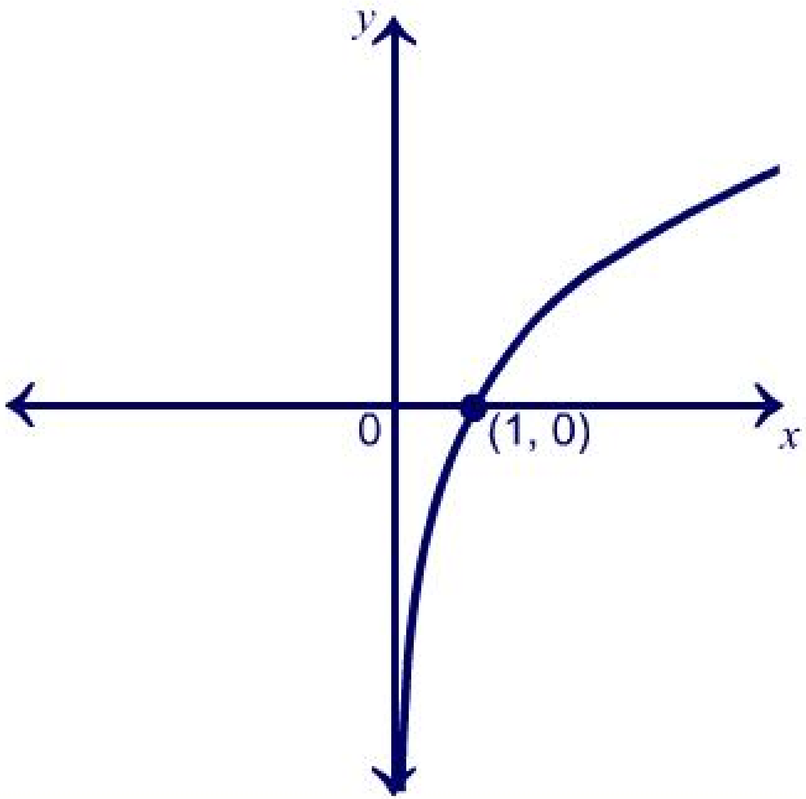
Change the base of the logarithmic function to see how the corresponding logarithmic and exponential graphs change.

$y = 2^x$

$y = \log_2 x$

Translating logarithmic functions

Translating logarithmic functions



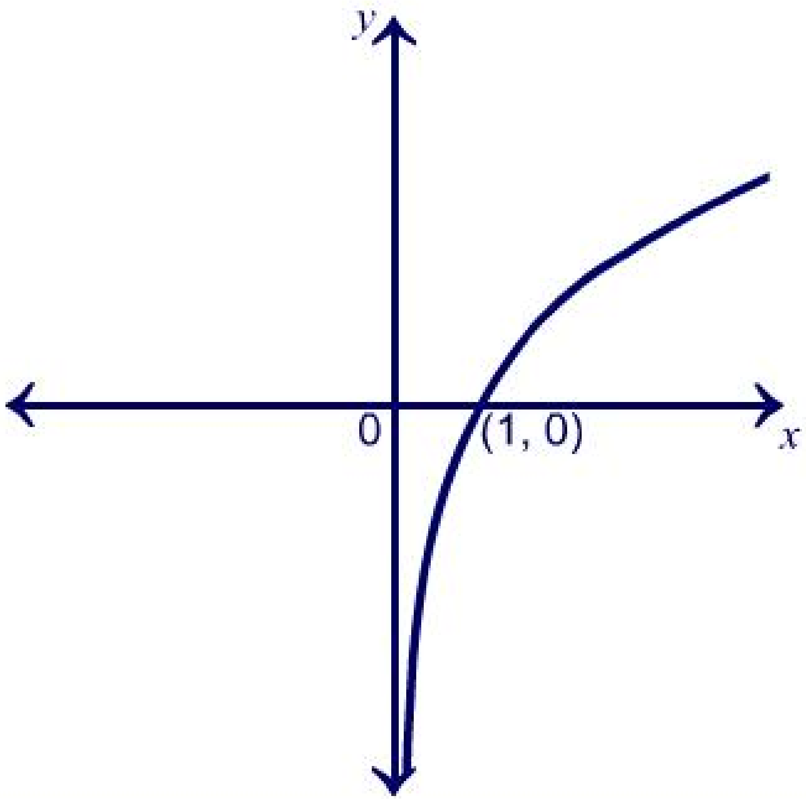
Press the arrows to transform the function. Press ***show*** *or* ***hide*** *to show or hide the transformed functions.*

— $f(x) = \log_2 x$

— $y = f(x) + 0$ — $y = f(x + 0)$

show show

Reflecting logarithmic functions



Press the arrows to change the base of the logarithmic function.

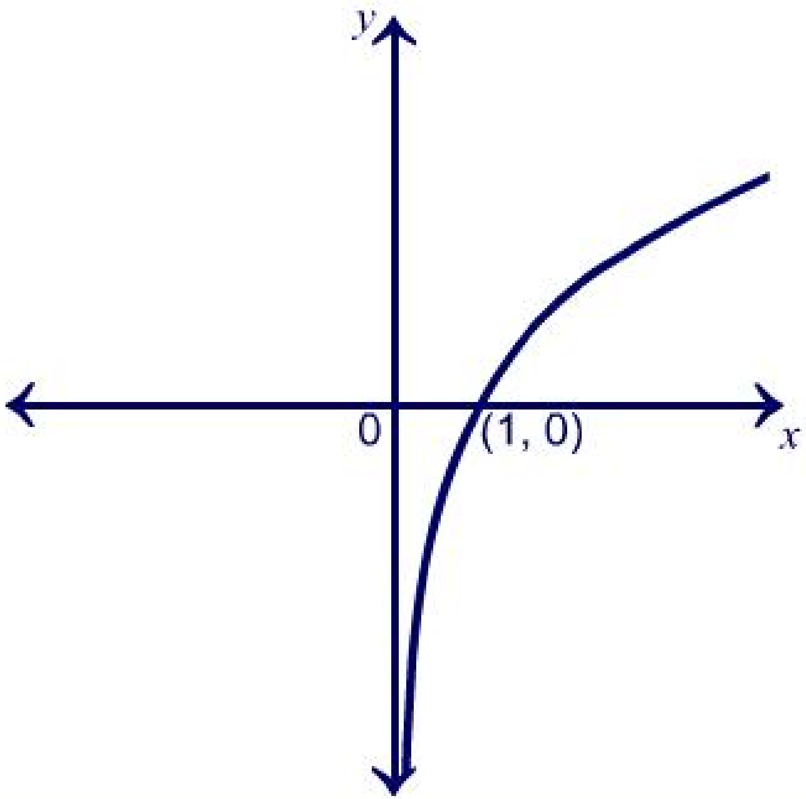
— $f(x) = \log_2 x$

*Press **show** or **hide** to show or hide the transformed functions.*

— $y = f(-x)$ show

— $y = -f(x)$ show

Stretching logarithmic functions



*Press the arrows to transform the function. Press **show** or **hide** to show or hide the transformed functions.*

— $f(x) = \log_2 x$

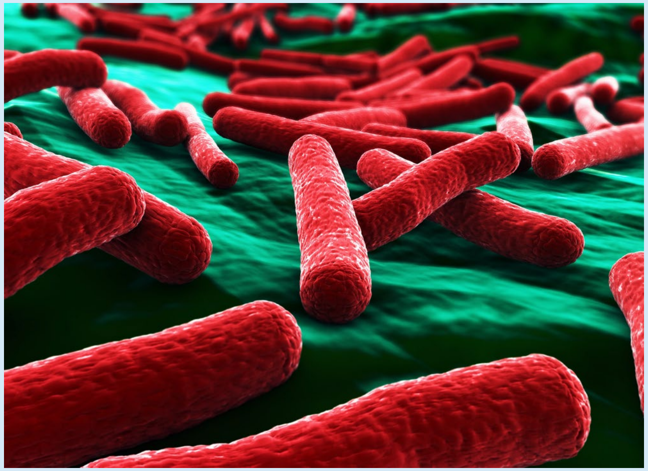
— $y = 1f(x)$ — $y = f(1x)$

show show

Suppose that there is a single bacterium cell that breeds by multiplying by 10 every minute.

Write a logarithmic function describing the number of minutes, t, it takes for the bacteria population to reach B.

How long (to the nearest minute) until there are 8000 bacteria?



write as exponential function: $B = 10^t$

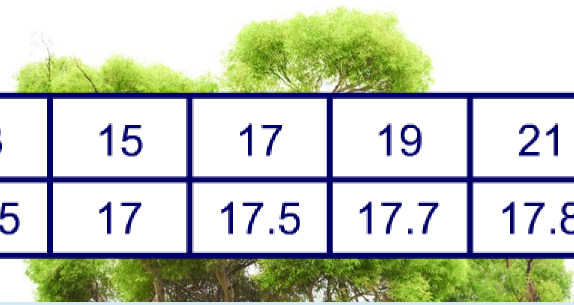
if $x = a^y$, then $y = \log_a x$: $\boldsymbol{t = \log B}$ ("log" implies base 10)

substitute $B = 8000$: $t = \log 8000 = 3.90$ (nearest hundredth)

It will therefore take **4 minutes**.

The growth of a tree

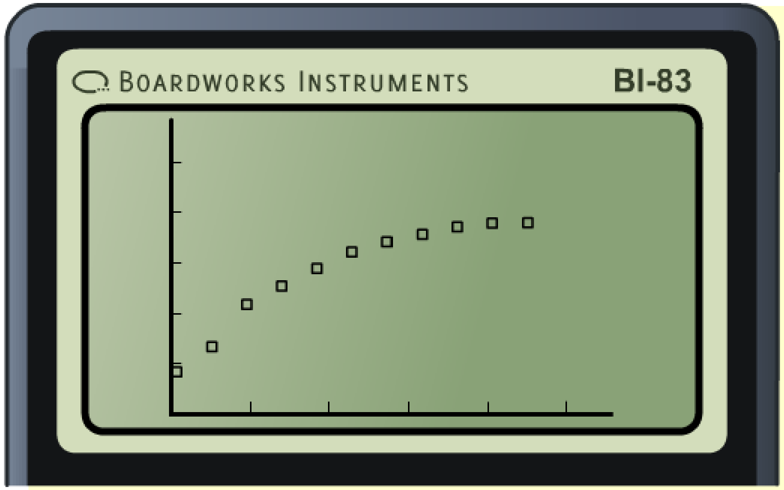
To celebrate Kim's first birthday, her father planted a tree in the front yard of their home. Every two years he went out and recorded the height of the tree. The table below shows the tree's height up to Kim's 21st birthday.



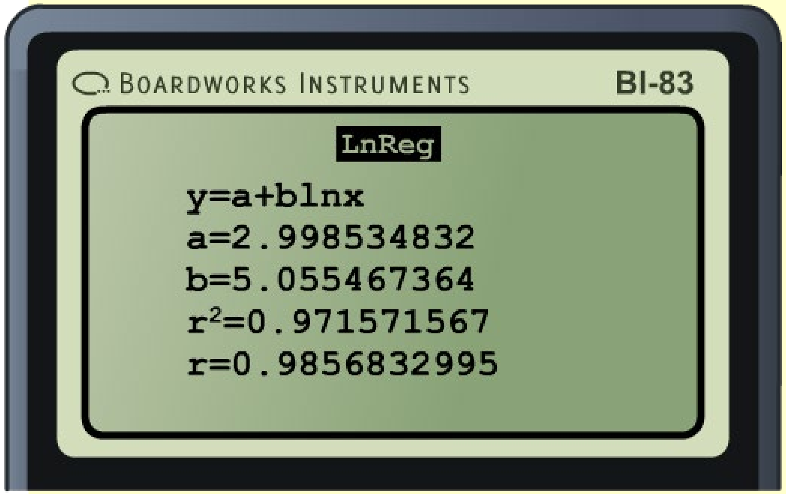
Kim's age	1	3	5	7	9	11	13	15	17	19	21
height (ft)	4	6.5	11	13	14.5	15.5	16.5	17	17.5	17.7	17.8

a) Using a graphing calculator, make a scatter plot of the data in the table.
b) Fit a logarithmic function to the data.
Discuss the fit using the correlation coefficient.
c) Use your model to predict the height of the tree when Kim is 30 years old.

The growth of a tree



a) This is how the data looks when plotted in a scatter plot.



b) Use the logarithmic regression ("LnReg") feature of your graphing calculator to fit a function to the data:

The regression equation is:
$\boldsymbol{y = 2.999 + 5.055 \ln x}$

The correlation coefficient is
$r = 0.986$ so the data is a good fit.

c) By evaluating the function at $x = 30$, we can predict that the tree will be approximately **20.2 ft**.