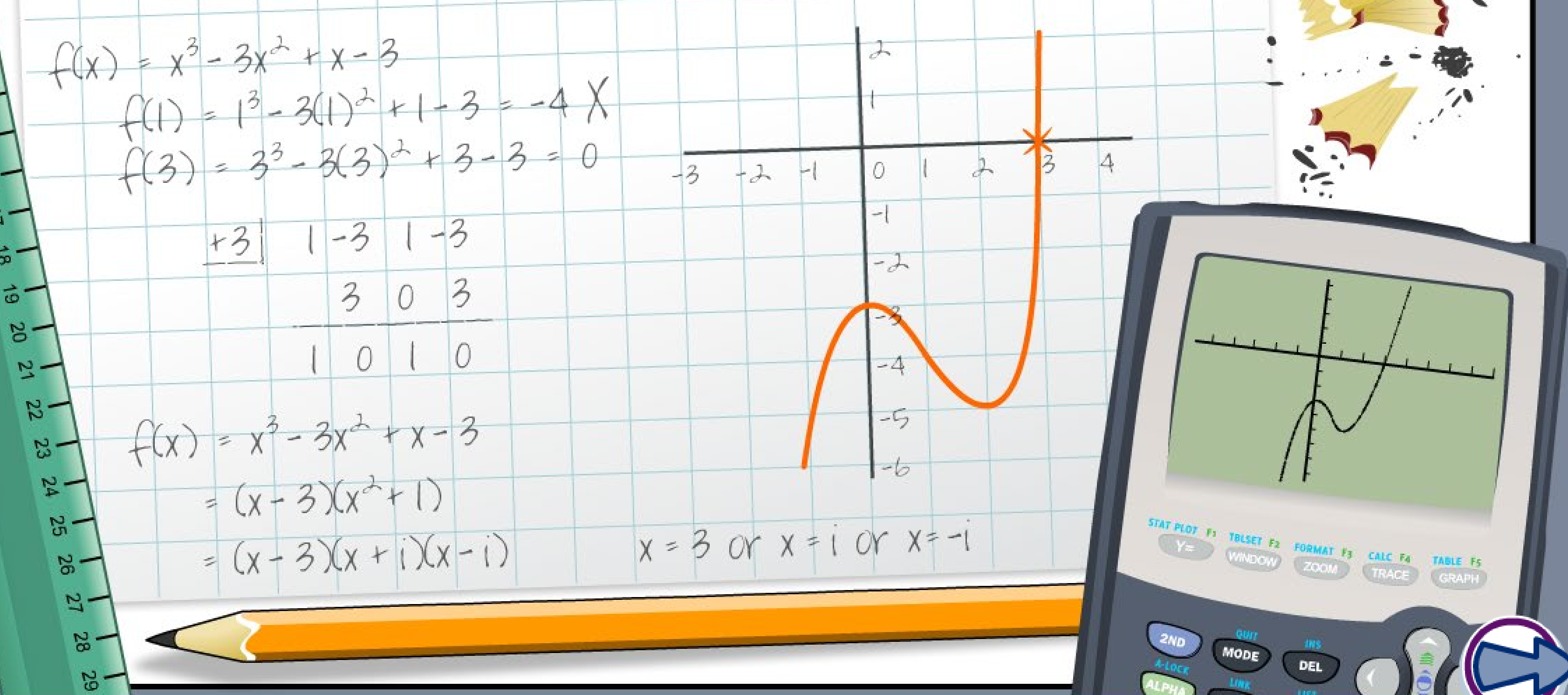
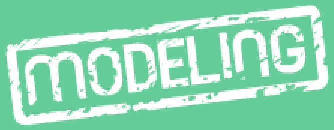
Linear Inequalities



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

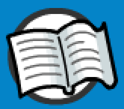
This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

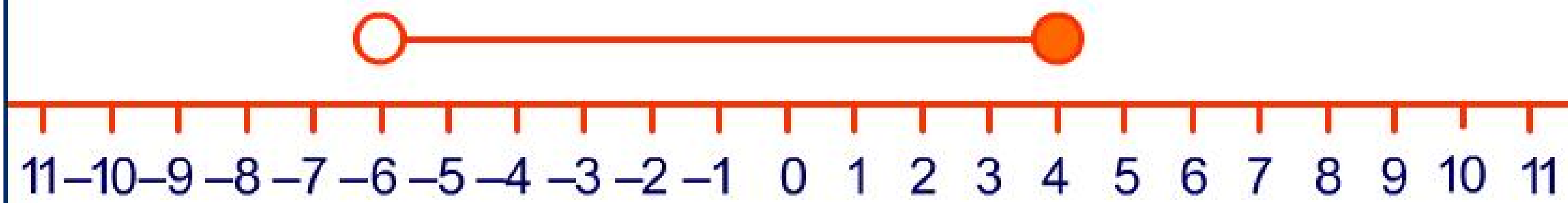
1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Write out the inequality shown on the number line.



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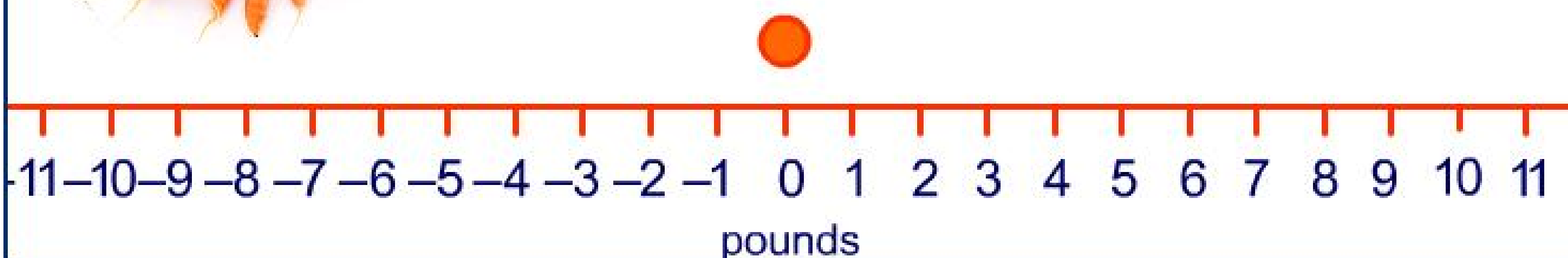
Inequalities in context

MODELING

Represent the following inequality on the number line:

m, the mass of a quantity of carrots weighing between 2 and 7 pounds



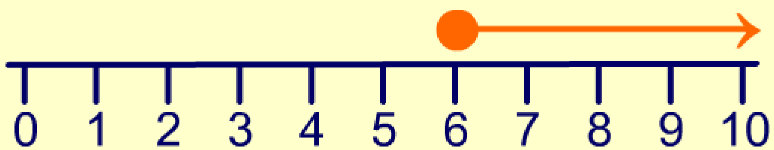
Solving linear inequalities

Look at the following inequality: $x + 2 \geq 8$.

What values of x make this inequality true?

This means "x plus 2 is greater than or equal to 8."

Any value of x greater than or equal to 6 satisfies the inequality.



Finding the values of x that make the inequality true is called **solving the inequality**.

The solution of $x + 2 \geq 8$ is written $\boldsymbol{x \geq 6}$.

Solving linear inequalities

An inequality can be solved using inverse operations, in the same way as an equation containing an equals sign.

Solve the inequality $2x - 5 < 7 - x$.

add 5 to both sides:	$2x < 12 - x$
add x to both sides:	$3x < 12$
divide both sides by 3:	$\mathbf{x < 4}$

How could we check this solution?

test $x = 3$ in the inequality:	$2(3) - 5 < 7 - 3$ $1 < 4$	✓
test $x = 5$ in the inequality:	$2(5) - 5 < 7 - 5$ $5 < 2$	✘

Multiplying or dividing by negatives

What happens if both sides of an inequality are multiplied by a negative number?

Let's look at an example: $-2 < 7$.

multiply both sides by –1: $-2 \times -1 < 7 \times -1$

$2 < -7$ ✘ **This is not true.**

Be careful! When multiplying or dividing an inequality by a negative number, the inequality sign must be **reversed**:

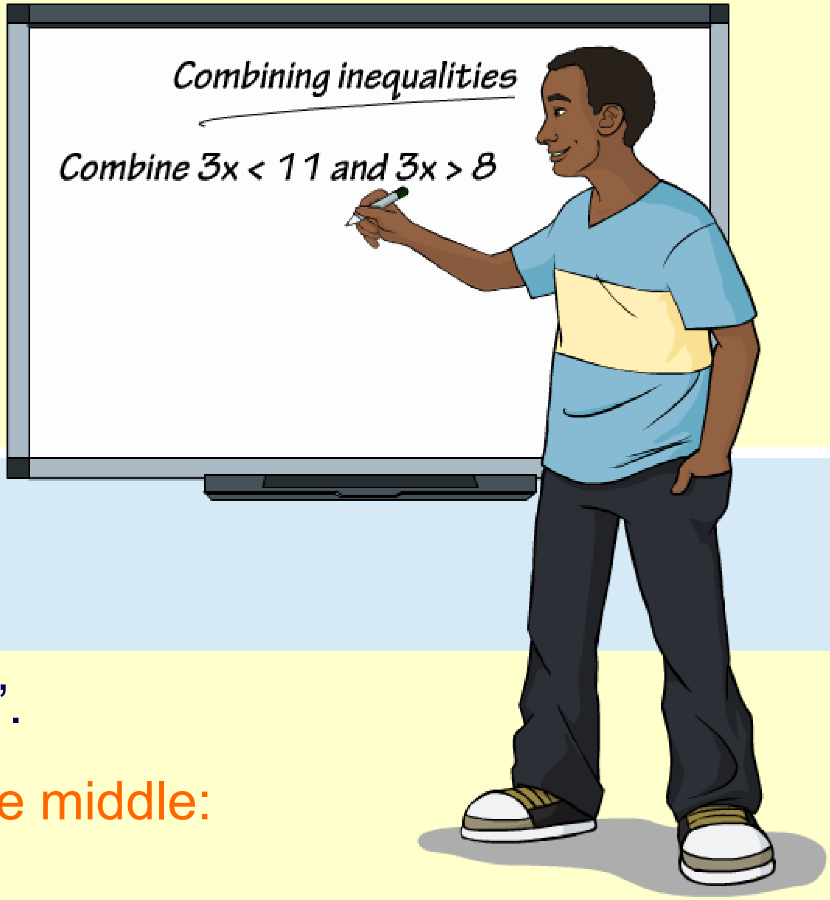
$2 > -7$ ✓

Can you write $3x < 11$ and $3x > 8$ as one single inequality?

Notice that both inequalities contain the term "$3x$".

write a single inequality with "$3x$" in the middle:

$$8 < 3x < 11$$



Combine $21 \geq x - 1$ and $x - 1 \geq 7$ into a single inequality.

Both inequalities contain the term "$x - 1$".

write a single inequality with "$x - 1$" in the middle:

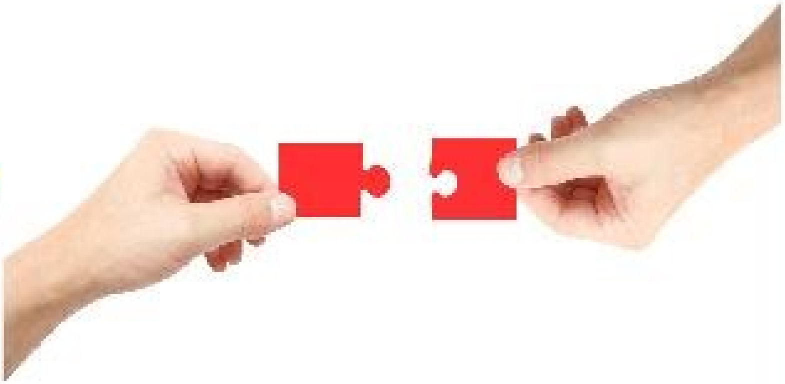
$$7 \leq x - 1 \leq 21$$

Combining inequalities and solving

Two inequalities can be combined into a single inequality.

Press on an element in the flow chart to see an example.

- two inequalities can be combined
  - unknown in just one part, e.g. $3 < 2x + 1 \leq 5$
  - unknown in more than one part, e.g. $x + 2 \leq 4x - 1 < 7$
    - overlapping solutions
    - inconsistent inequalities



Solving linear inequalities

Solve the following inequalities:

1) $5x \geq 8x + 6$?

2) $-3 < 2x + 7 < 1$?

3) $10 \leq 3x + 1 \geq 2x - 1$?

4) $4x - 1 > 5x - 3 > 7$?

5) $11 - 3x < 7 - 2x > -3$?

Hotel elevator

A hotel elevator can carry a maximum load of 1300 lbs.

A group of guests arrive from the airport and need to use the elevator to go up to their rooms. The guests weigh 170 lbs each, and they each have one 65 lb suitcase.

a) Write an inequality describing the maximum number of guests (*G*) and suitcases (*S*) that the elevator can carry.

b) If six guests get into the elevator, how many suitcases can go in with them?

Hotel elevator

a) The inequality is: $\mathbf{170G + 65S \leq 1300}$

b) substitute $G = 6$: $170(6) + 65S \leq 1300$

simplify: $1020 + 65S \leq 1300$

subtract 1020: $65S \leq 280$

divide by 65: $S \leq 4.3\ldots$

S must be an integer: **$S = 4$ suitcases**

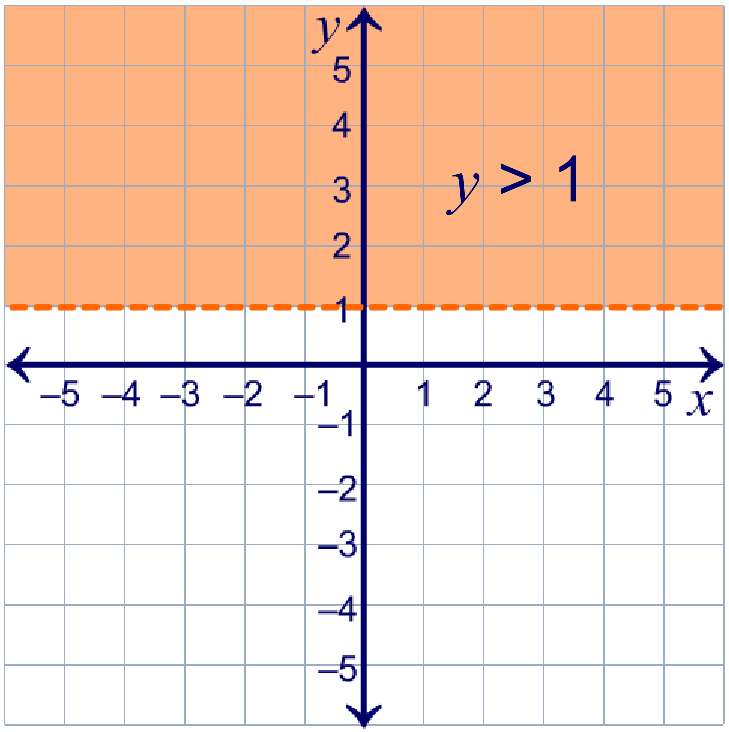
The hotel owner installs a fire extinguisher in the elevator, weighing 29 lbs. Write a new inequality describing the maximum number of guests (G) and suitcases (S) that the elevator can carry (assuming the same weights as before).

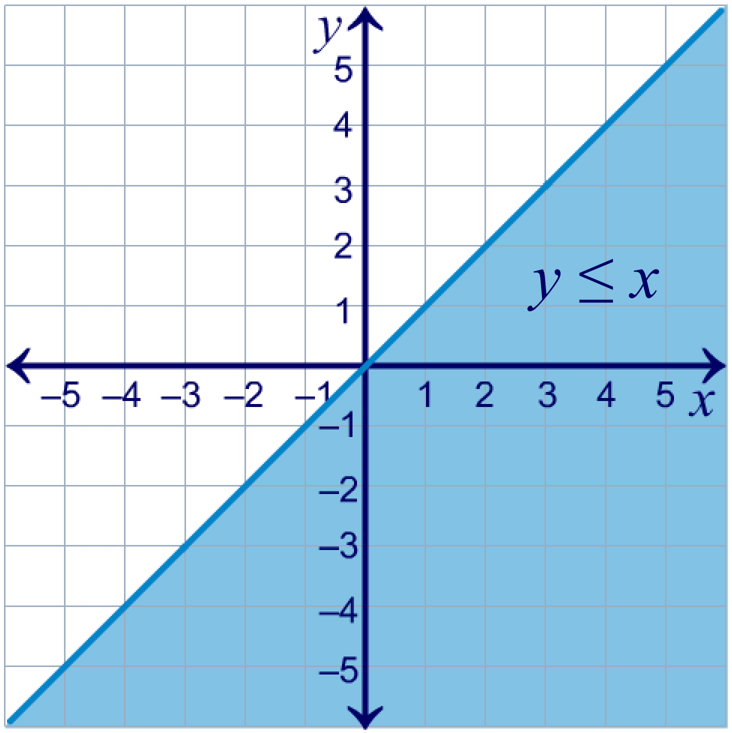
The new inequality is: $170G + 65S + 29 \leq 1300$

subtract 29: $\mathbf{170G + 65S \leq 1271}$

Inequalities can be represented by **regions** on a graph.

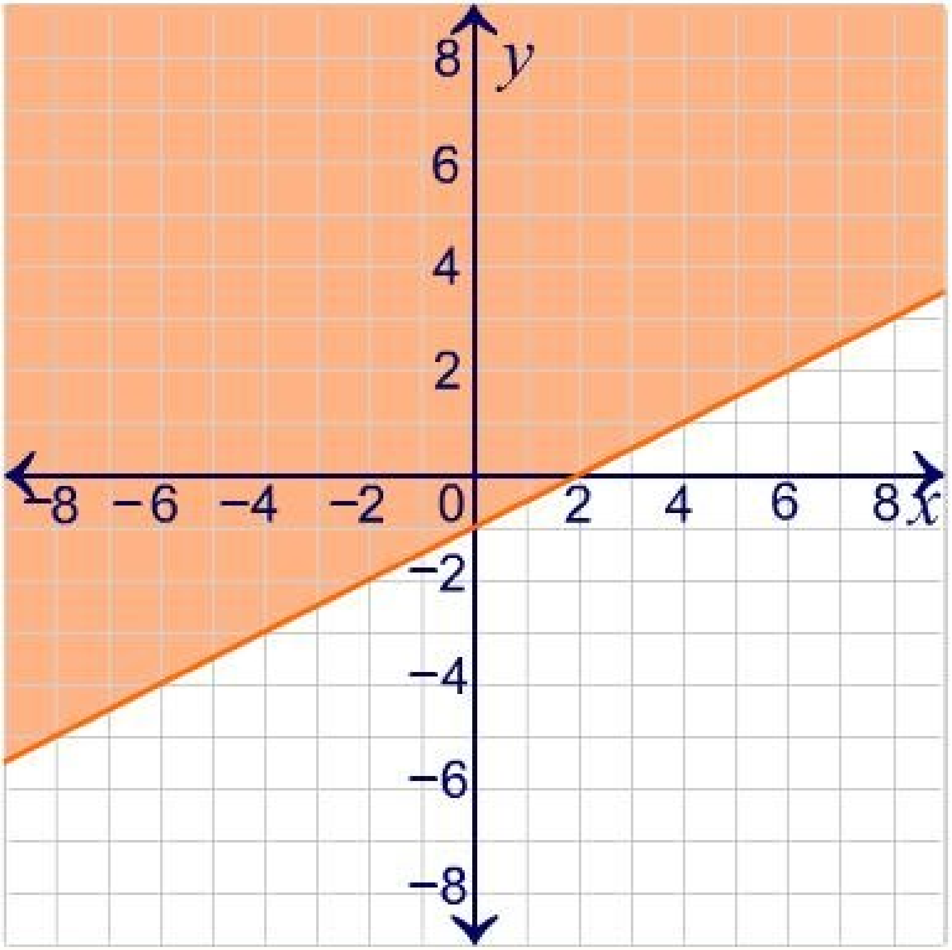
The solution area is shaded in.





What inequality is shown in each graph?

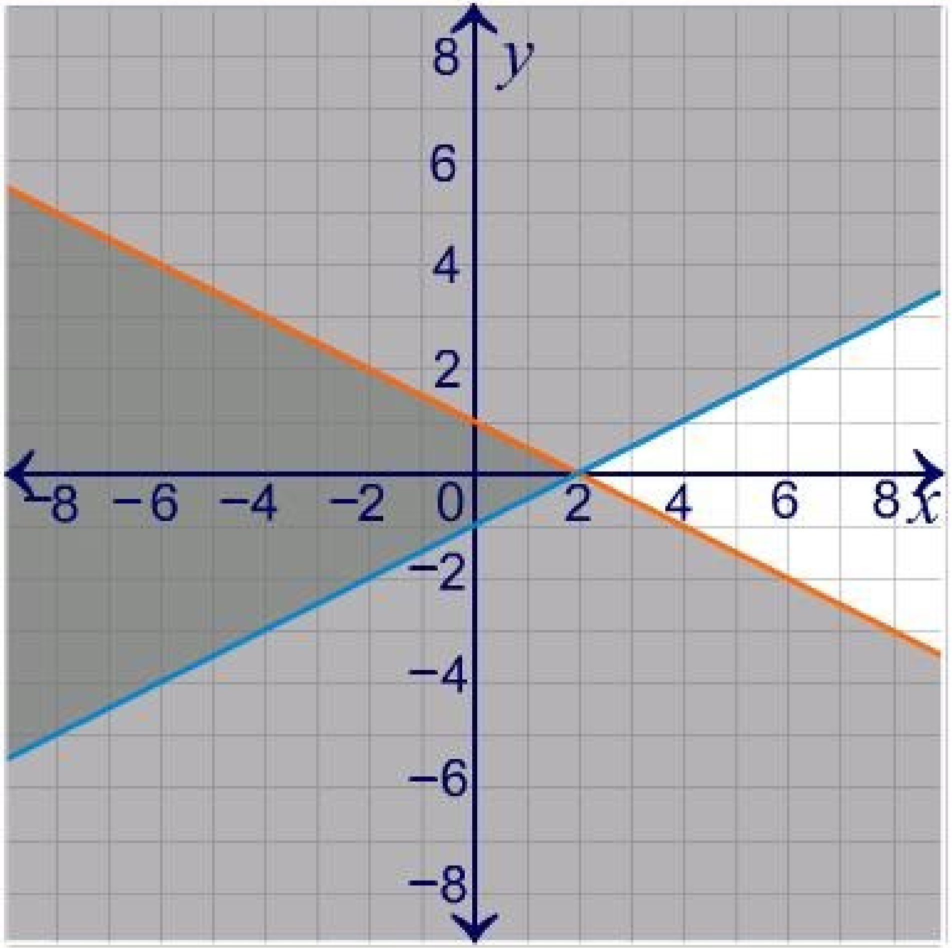
Inequalities in two variables



Change the inequality to see how the shaded region (the solution set) changes.

$1x - 2y \leq 2$

Combining two inequalities



Change the inequalities to see the corresponding solution regions.

$1x + 2y \leq 2$

$1x - 2y \leq 2$

School trip plan

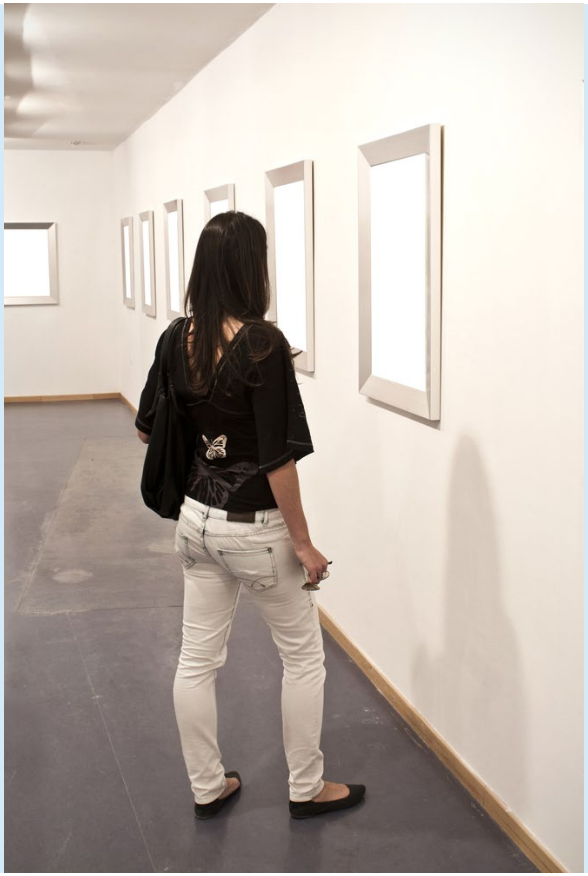
MODELING

More than one inequality can be displayed on the same graph.

The overlap of the inequalities gives the solution region.

A high school is planning a trip to an art exhibition, for which they have 21 tickets available. Health and safety rules state that there must be at least one teacher for every three students on the trip.

Write two inequalities to describe this situation. Graph the inequalities and find the maximum number of students who can go on the trip.



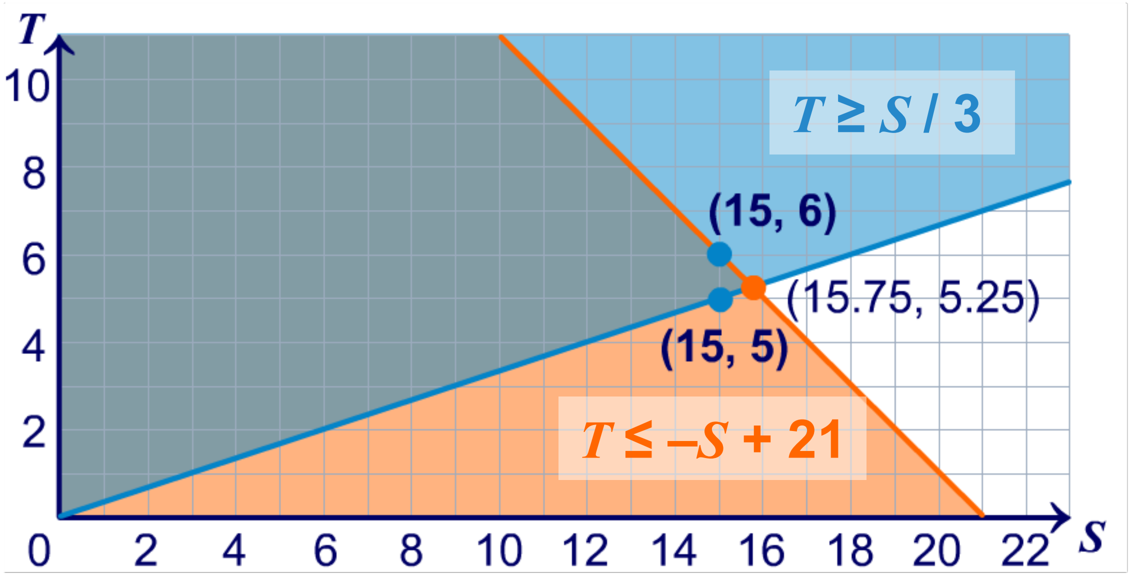
Let's call S = number of students and T = number of teachers.

- There must be at least one teacher for every three students.

$3T \geq S$, which simplifies to: $\boldsymbol{T \geq S / 3}$

- A maximum of 21 people in total can go on the trip.

$S + T \leq 21$, which simplifies to: $\boldsymbol{T \leq -S + 21}$



The intersection does not give whole values.

Round down; a maximum of 15 students can go (with either 5 or 6 teachers).

Absolute value inequalities

The absolute value of a number is its distance from zero to that number on a number line. For example $|-3| = 3$.

When an inequality contains an absolute value, it is really two separate inequalities.

*Press one of the examples to find out more, or press **summary** to see the general rules for inequalities with absolute values.*

$|x| < 5$ $|x - 7| \leq 2$ $|x| \geq 4$ $|x + 3| > 1$ **summary**

Graphing inequalities

Question: 1/5

Graph the solution region of the inequality $|x + 4| > 1$.

Press the "=" button to show the calculations step-by-step.

Linear programming

Linear programming is a technique used to find the minimum or maximum value of some quantity, e.g. profit.

- The maximum or minimum quantity is modeled with an **objective function**.
- Limits on the variables in the objective function are the **constraints**, written as linear inequalities.
- Graph these inequalities and determine the “corner points” of the overlapping region (called the “feasible region”).
- Identify the corner point at which the maximum or minimum value of the objective function occurs.

Candle sale

A school wants to sell candles to raise money for new playground equipment. A candle company offers them jar candles, some 5 inches tall and some 3 inches tall. The school's current budget allows them to spend a maximum of $600 and they expect to sell a maximum of 500 candles.

Here is a price list from the candle company:

Candle price list

5-inch jars:
12 candles per case.
You pay $24 per case.
Sell at $4.50 per candle.

3-inch jars:
20 candles per case.
You pay $15 per case.
Sell at $2.00 per candle.

How many of each size candle should the school order to maximize profit? What is the maximum profit?

Candle sale

Organize the information in a table.

	5-inch jars	3-inch jars	total
number of cases	x	y	
number of candles	$12x$	$20y$	500
cost	$24x$	$15y$	600
profit made	$30x$	$25y$	$30x + 25x$

Each row of terms provides a constraint for the problem.

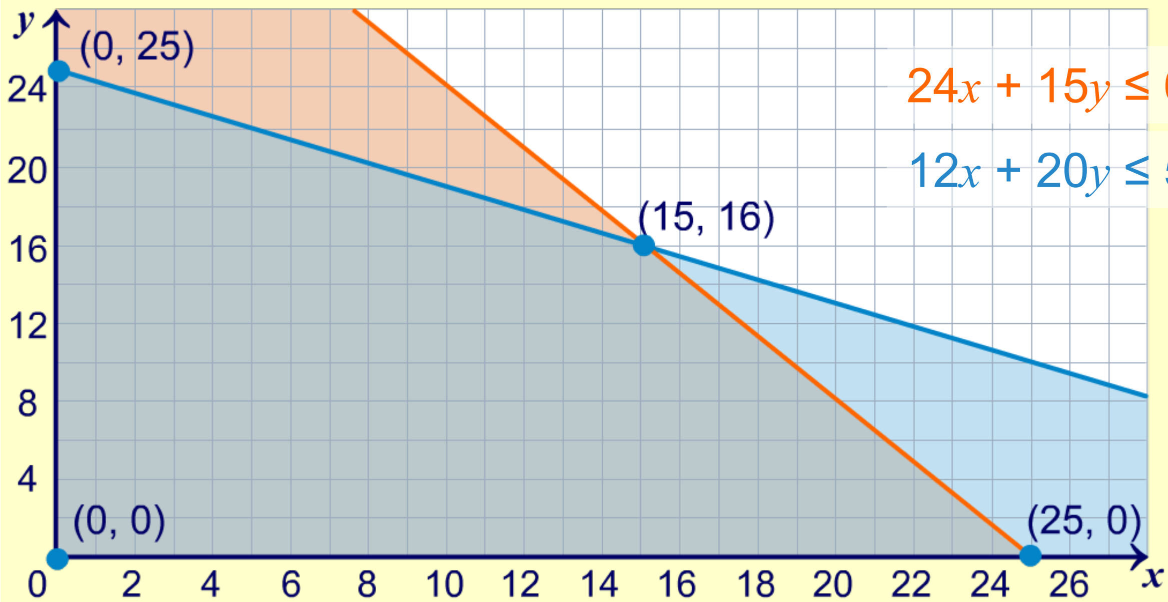
write the objective function (total profit made): $P(x, y) = 30x + 25y$

write the constraints: $x \geq 0$ and $y \geq 0$ (number of candles ordered must be a positive number)

$12x + 20y \leq 500$ (maximum candles sold)

$24x + 15y \leq 600$ (maximum budget)

Candle sale



$24x + 15y \le 600$ (max budget)

$12x + 20y \le 500$ (max candles sold)

Evaluating the objective function at each corner point will identify the maximum profit.

Objective function for maximum profit: $P(x, y) = 30x + 25y$

$P(0, 0)$ = \$0 $P(15, 16)$ = \$850 This is the maximum profit.

$P(0,25)$ = \$625 $P(25, 0)$ = \$750

The school should buy **15 cases of 5-inch candles** and **16 cases of the 3-inch candles** to make a maximum profit of **\$850**.