

Group and Project Work

$$f(x) = x^3 - 3x^2 + x - 3$$

$$f(1) = 1^3 - 3(1)^2 + 1 - 3 = -4 \quad \times$$

$$f(3) = 3^3 - 3(3)^2 + 3 - 3 = 0$$

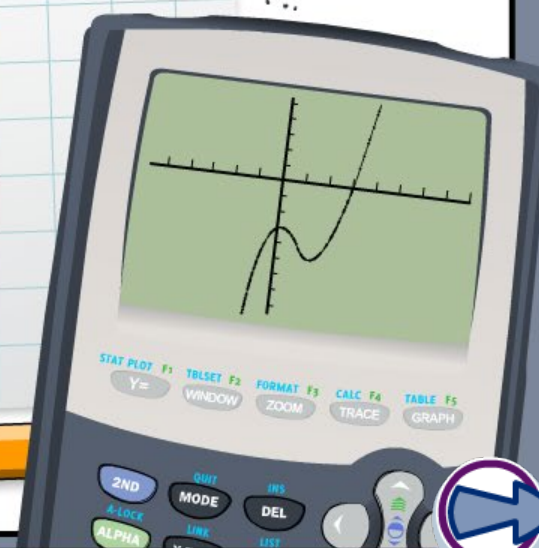
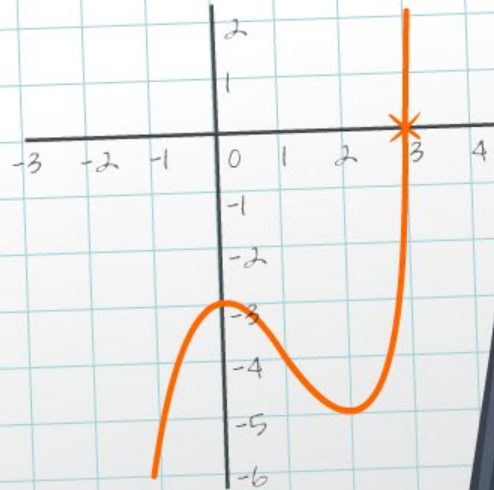
$$\begin{array}{r|rrrr} +3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$f(x) = x^3 - 3x^2 + x - 3$$

$$= (x - 3)(x^2 + 1)$$

$$= (x - 3)(x + i)(x - i)$$

$$x = 3 \text{ or } x = i \text{ or } x = -i$$



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.



This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

- 1) **Make sense of problems and persevere in solving them.**
- 2) **Reason abstractly and quantitatively.**
- 3) **Construct viable arguments and critique the reasoning of others.**
- 4) **Model with mathematics.**
- 5) **Use appropriate tools strategically.**
- 6) **Attend to precision.**
- 7) **Look for and make use of structure.**
- 8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.




This icon indicates teacher's notes in the Notes field.






Packaging company problems

Finding extrema (maximum or minimum points) is known as **optimization**. Press the info button for help solving these problems 

1) Find the dimensions of the cheapest possible box with a volume of 960 cm^3 . The box must be rectangular, with a square base. The material for the top and bottom of the box will cost $\$0.02$ per cm^2 , and the material for the sides will cost $\$0.05$ per cm^2 . 

2) Find the radius, r , of the cylindrical can that will require the least amount of material and be able to hold 1 liter of oil. State the height of this can. 





Estelle's doctor tells her that she needs to increase her daily intake of magnesium by 20 mg and calcium by 300 mg, by taking vitamin supplement tablets. She finds two brands of vitamins; brand A contains 6 mg of magnesium and 50 mg of calcium, and brand B contains 4 mg of magnesium and 100 mg of calcium. Each brand A tablet costs 8¢, and each brand B tablet costs 10¢.

How many tablets of each brand should Estelle take each day in order to obtain the desired amounts of magnesium and calcium as cheaply as possible?



Press **start** to see the solution.

start



- 1) In groups, find another use of the linear programming method. Look for a situation in which the objective function needs to be maximized, rather than minimized.
- 2) Each group should write a linear programming problem and its solution. They should then pass it to another group who must solve it and present its solution to the class. The group who made the problem should assess the answer.





The Golden Ratio

There is a special ratio called the **golden ratio**, denoted by the greek letter τ ("tau"). Two quantities are said to be in the golden ratio if the ratio of the smallest to the largest quantity equals the ratio of the largest quantity to the sum of the quantities:



$$\frac{a}{b} = \frac{a + b}{a} = \tau$$

Press **start** to see your task.

start



Reminder:
A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and “ i ” is the imaginary unit.



- 1) Investigate how complex numbers are represented on a coordinate grid.
- 2) Draw such a grid, labeling the axes accordingly, then plot the following numbers:
a) $-4 + 3i$ b) $2 - i$ c) i
- 3) Determine how to find the length of the line joining any point on the grid to the origin, and hence find the distance from the origin to the numbers in question 2.

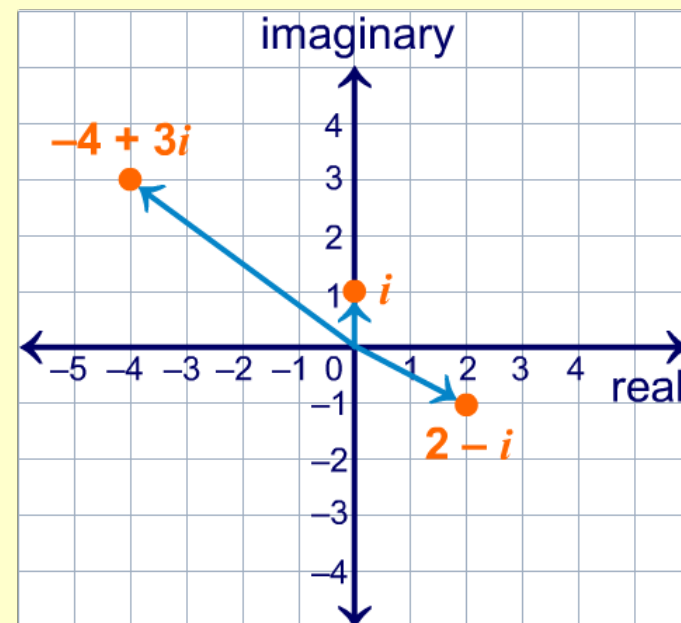


The complex plane (2)

1) When plotting points on the complex plane, the horizontal axis is the real axis and the vertical axis is the imaginary axis.

2) Plotting $-4 + 3i$ is like plotting the ordered pair $(-4, 3)$. Similarly, $2 - i$ is like $(2, -1)$ and i is like $(0, 1)$.

3) The length of the line from the origin to a number $a + bi$ plotted on the grid can be found using the Pythagorean theorem.



$$a) |-4 + 3i| = \sqrt{(-4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = \mathbf{5 \text{ units}}$$

$$b) |2 - i| = \sqrt{(2)^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5} \mathbf{\text{ units}}$$

$$c) |i| = \sqrt{(0)^2 + (1)^2} = \sqrt{1} = \mathbf{1 \text{ unit}}$$

The distance on the complex plane from the origin to $z = a + bi$, where a and b are real, is the **absolute value** or **magnitude** of z , i.e. $|z|$.



Choose one of the arches shown below or find another example of a parabolic archway on the internet, in a textbook or in a magazine. Superimpose a coordinate plane on it and algebraically determine the equation of the parabola that best fits it.



Top: Key Bridge, Washington, D.C.
Left: Gateway Arch, St. Louis.





Designing a tunnel

We want to build a tunnel over an existing two-lane road and I want you to design it. The road is 30 feet wide, and there is also a walkway that is 3 feet wide on each side of the road. A restriction exists on this road saying that no trucks over the height of 11 feet, or wider than 13 feet, can use it. Show proof that your design will accommodate these trucks. You'll need to find the equation of a parabolic tunnel that meets these requirements, then create a scale drawing on graph paper that I can present to the construction team.

Press this button to see a possible solution:



Research: logarithm tables



Before the widespread use of calculators, students had to use tables of values in the back of their textbooks to find the logarithm of a number to help them with calculations.

Prepare a report and demonstration for the class on the use of logarithm tables and be prepared to show how to evaluate the log of a number that is not an integral power of 10, such as 3.35.

