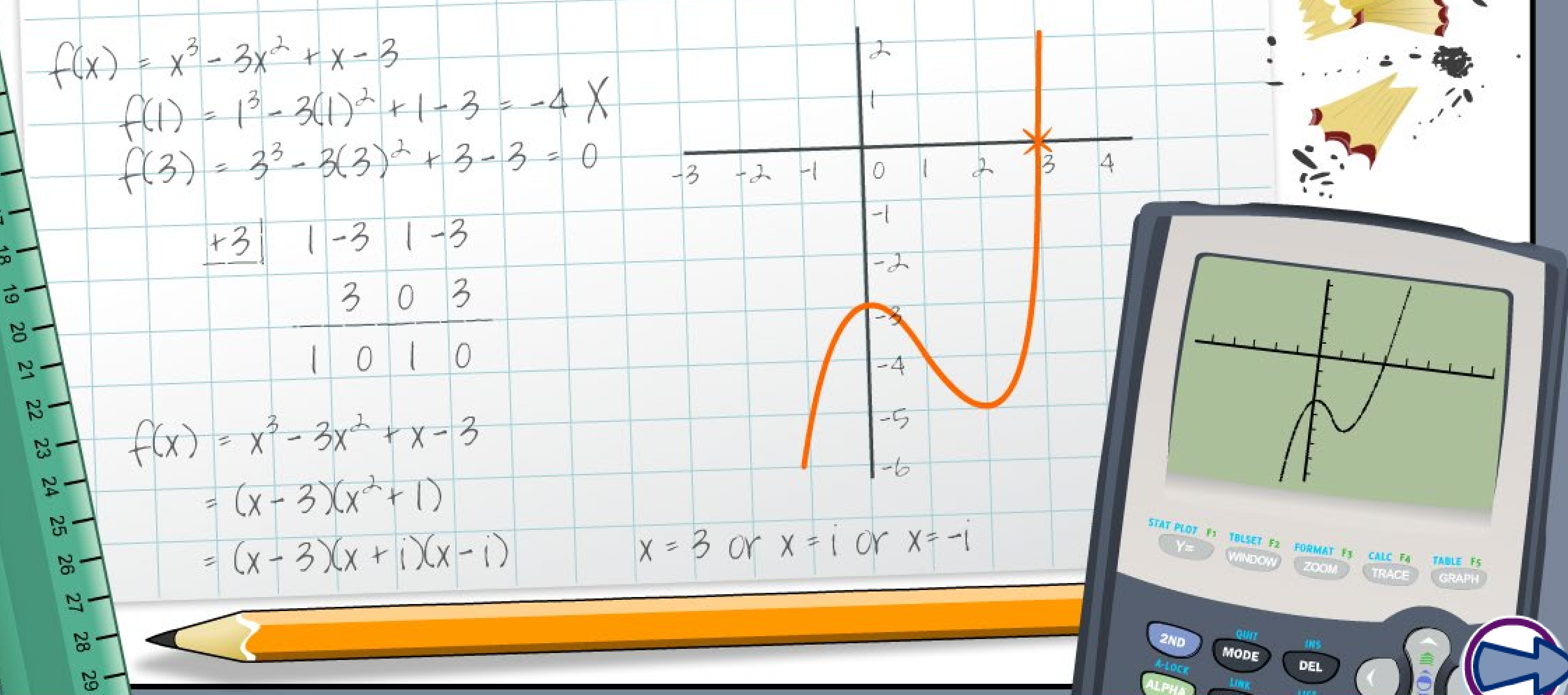
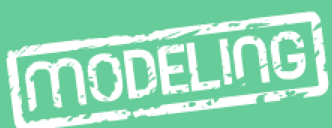
Geometric Sequences and Series



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

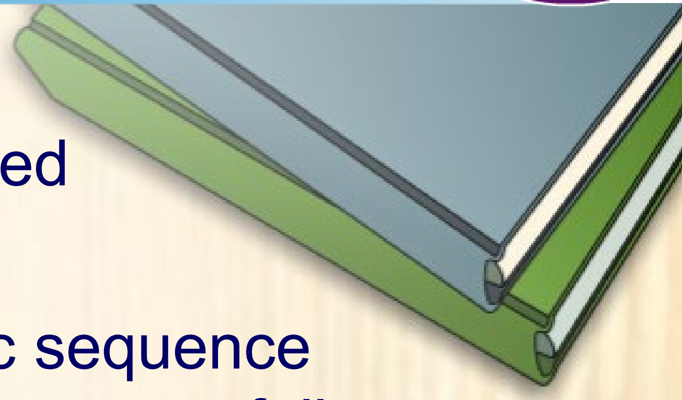
7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

The n^{th} term of a geometric sequence



Sequences that progress by multiplying each term by a common ratio, r, are called **geometric sequences**.

In general, if the first term of a geometric sequence is a_1 and the common ratio is r, the terms are as follows:

- a_1
- $a_2 = a_1 \times r$,
- $a_3 = a_1 \times r^2$,

  …
- $a_n = a_1 \times r^{n-1}$

This continues up to the n^{th} term.

n^{th} term of a geometric sequence with first term a_1 and common ratio r is:

$$a_n = a_1 r^{n-1}$$

Review of geometric sequences

Find the next two terms in this geometric sequence:

8192 4096 2048 [ ] [ ]

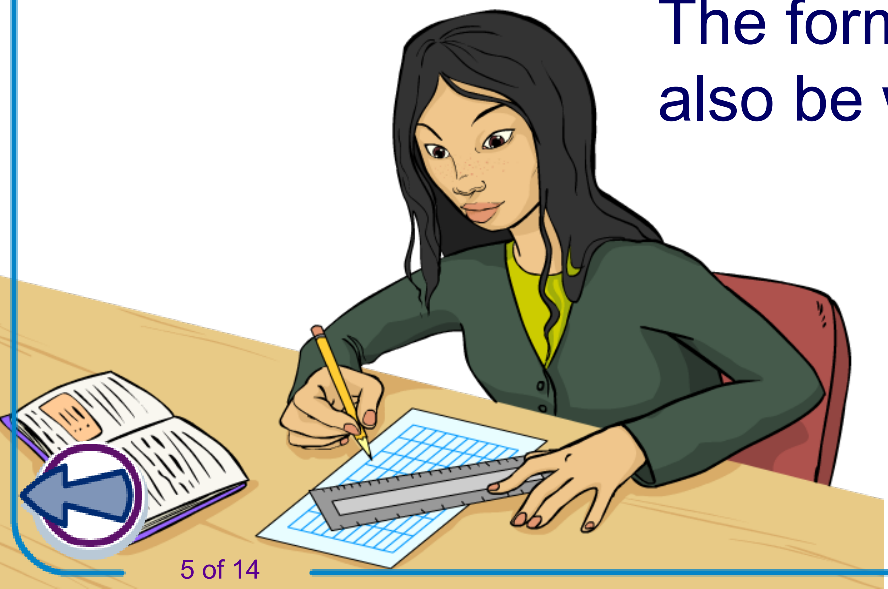
Press the yellow boxes to reveal the next two terms.

A **geometric series** is a sum in which all the terms form a geometric sequence.

The sum of n terms of a geometric series can be found using the formula:

sum of n terms of a geometric series:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$



The formula for a geometric series can also be written using summation notation.

summation notation for an geometric series:

$$S_n = \sum_{k=1}^{n} a_1 r^{k-1}$$

The sum of a finite number of terms of a geometric sequence is called a **geometric series**. How can we write a formula for a geometric series?

Press **play** to see how.

Using Σ notation

Find $\sum_{k=1}^{7} 5(2)^{k-1}$.

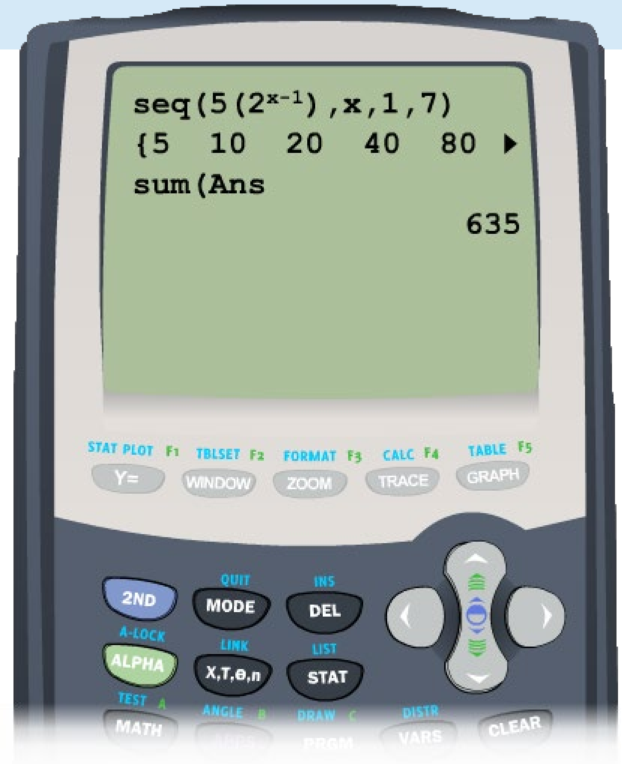
$$\sum_{k=1}^{7} 5(2)^{k-1} = 5 + 10 + 20 + \ldots + 5(2)^6$$

write the formula for a geometric series: $S_n = \dfrac{a_1(1 - r^n)}{1 - r}$

substitute $a_1 = 5$, $r = 2$, and $n = 7$: $S_n = \dfrac{5(1 - 2^7)}{1 - 2} = \mathbf{635}$

Using a graphing calculator

Find $\sum_{k=1}^{7} 5(2)^{k-1}$ using a graphing calculator.



- Go to “LIST” (“2ND”, “STAT”).
- Select “seq(” on the “OPS” menu.
- Enter the sequence: 5(2^(x–1)). The variable is “x” and “end” is 7 since n = 7. Ignore “step” and press “Paste”.
- Press “ENTER” to see a list of the terms.
- To sum the terms, press “LIST” then select “sum(” on the “MATH” menu. Press “ANS” to insert the sequence then “ENTER” to see the sum. The sum is **635**.

Don't break the chain!

Todd wanted to see how far-reaching a 'chain email' would be. He emailed it to 3 of his friends and asked them to email 3 different friends the following day with the same request.

1. **If no one breaks the chain or receives the email twice, how many people will the email reach by the end of the first week (day 7)?**

2. **If 40 people received the email (including Todd), how many days have passed since Todd started the chain?**

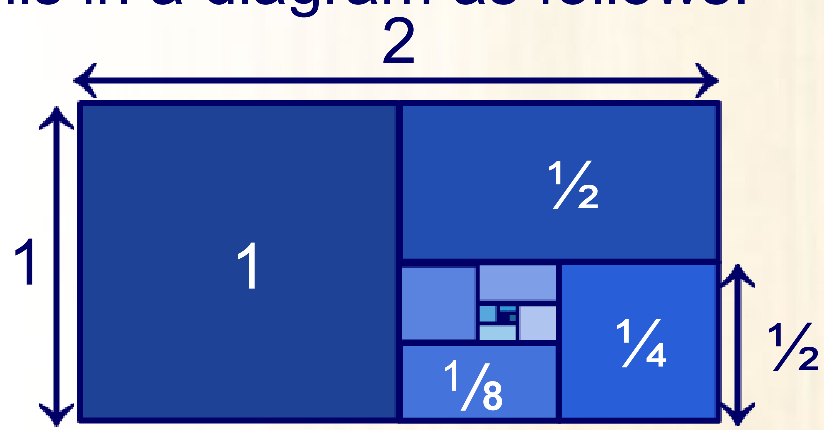
When the common ratio of a geometric series is between –1 and 1 exclusive, the sum of the series will approach a particular value as more terms are added.

For example, the sum of the geometric series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$$

approaches 2 as the number of terms increases.

We can show this in a diagram as follows:



Formula for the sum to infinity

The formula $S_n = \dfrac{a_1(1 - r^n)}{1 - r}$ sums the first n terms of a geometric series.

How can we adapt this to sum an infinite number of terms of a series with a common ratio $|r| < 1$? Consider what happens as n approaches infinity.

When a number $|r| < 1$ is raised to a higher and higher power, the value gets smaller and smaller, e.g. $0.5^2 = 0.25$, $0.5^3 = 0.125$, $0.5^4 = 0.0625$, etc.

So, as $n \to \infty$, $r^n \to 0$.

Therefore, $1 - r^n \to 1$.

sum to infinity of a geometric series: $S_\infty = \dfrac{a_1}{1 - r}$

Practice questions: sum of a geometric series

1. What is the sum of an infinite geometric series with $a_1 = 34$ and $r = 0.5$?
2. How many terms are in the sequence with sum 248 and first terms: 8, 16, 32, ...?
3. Find $\sum_{k=1}^{8} 0.3(3)^{k-1}$.
4. The common ratio in a geometric series is ¼. The sum to infinity is 108. What is the 2nd term?
5. Is it possible for the sum to infinity of a geometric series to be zero if $a_1 \neq 0$? Explain why/why not.

Donations for breast cancer

A local women's club wants to raise money for breast cancer research. They decide to put a donation of $1000 in an account.

Each year they will donate 10% more than the previous year. A list of their donations for the next 6 years is given in the table.

Show that the donations form a geometric sequence.

find the common ratio of the terms:

1100/1000 = 1.1; 1210/1100 = 1.1;
1331/1210 = 1.1; 1464.10/1331 = 1.1;

The donations progress by a common ratio of **1.1**.
Therefore, they form a geometric sequence.

year	donation
1	$1000
2	$1100
3	$1210
4	$1331
5	$1464.10
6	$1610.51

What is the total amount of money donated by the end of year 10?

find a, r, and n:

$a = 1000$ starting donation

$r = 1.1$ common ratio

$n = 10$ number of donations, 10 years

state the formula: $$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

substitute in values: $$S_{10} = \frac{1000(1 - 1.1^{10})}{1 - 1.1}$$

simplify: $$S_{10} = \frac{-1593.742}{-0.1} = \$15{,}937.42$$

$15,937.42 donated over 10 years

year	donation
1	$1000
2	$1100
3	$1210
4	$1331
5	$1464.10
6	$1610.51