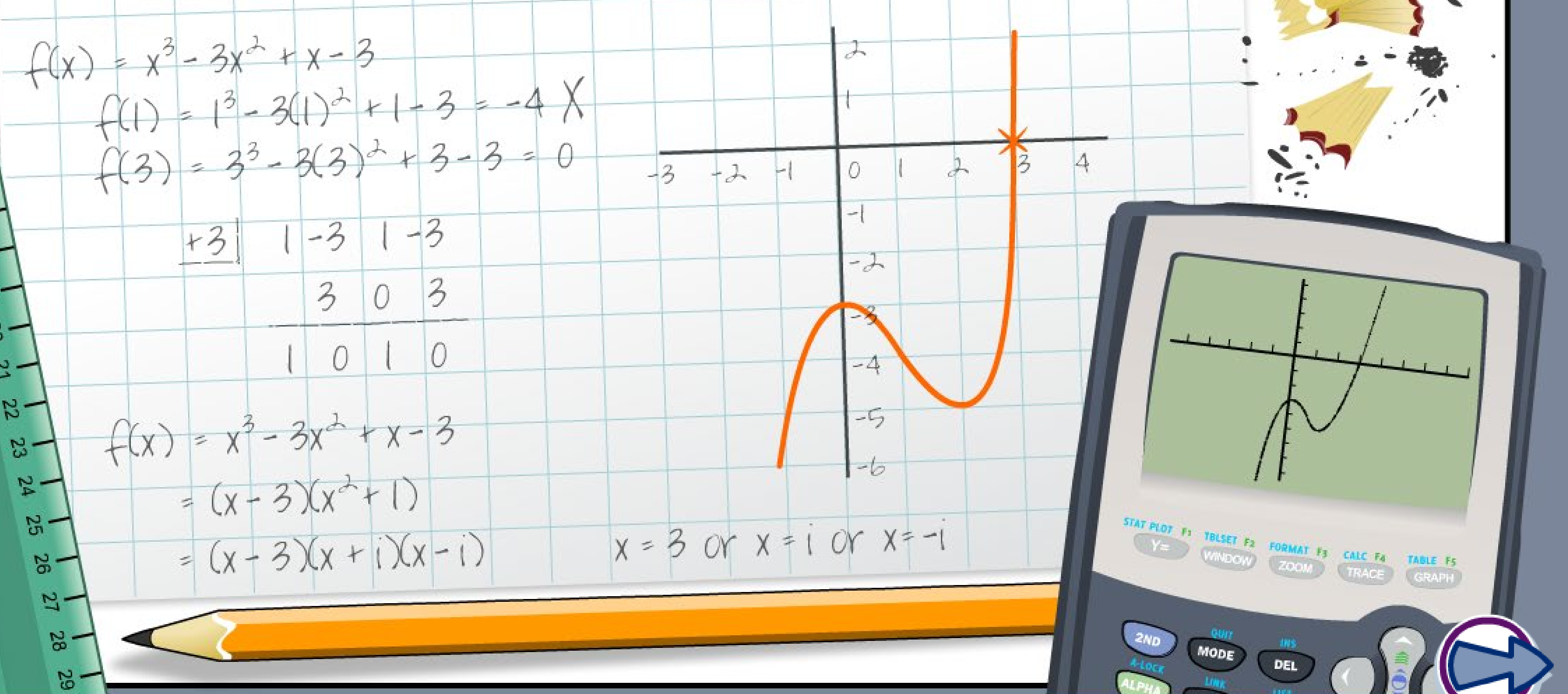
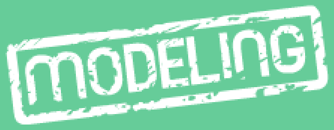
Factoring Polynomials



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

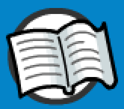
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Methods of factoring

A polynomial can be factored if it can be written as a product of two or more lower order polynomials.

Some of the methods used to factor quadratic expressions can also be used to factor other polynomial expressions.

Press a method to see an example.

greatest common factor

by grouping

difference of squares

Factor $f(x) = x^3 + 1$ given that $(x + 1)$ is a factor.

given:	$x^3 + 1 = (x + 1)(ax^2 + bx + c)$
by inspection:	$a = 1$ and $c = 1$
substitute:	$x^3 + 1 = (x + 1)(x^2 + bx + 1)$
expand:	$x^3 + 1 = x^3 + bx^2 + x + x^2 + bx + 1$
group like terms:	$x^3 + 1 = x^3 + (b + 1)x^2 + (b + 1)x + 1$
equate coefficients:	$b + 1 = 0$
	$b = -1$
substitute:	$\mathbf{x^3 + 1 = (x + 1)(x^2 - x + 1)}$

Can this polynomial be factored further?

Sum or difference of cubes

An expression of the form $a^3 \pm b^3$ is a **sum** or **difference of cubes**. It can be factored using a rule:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor $f(x) = x^3 - 8$.

write as difference of cubes: $f(x) = x^3 - 2^3$

factor: $f(x) = \mathbf{(x - 2)(x^2 + 2x + 4)}$

check: $f(x) = x^3 + 2x^2 + 4x - 2x^2 - 4x - 8$

$f(x) = x^3 - 8$ ✓

Match the equivalent expressions

$x^3 + 1$	$(x^2 + 3)(x^2 – 3)$
$3x^5 + 6x^2$	$(x – 1)(x^2 + 4)$
$x^4 – 9$	$(2x – 3)(4x^2 + 6x + 9)$
$8x^3 – 27$	$(x + 1)(x^2 – x + 1)$
$x^3 – x^2 + 4x – 4$	$3x^2(x^3 + 2)$

Suppose that $(x - a)$ is a factor of a polynomial $f(x)$.

What is $f(a)$?

write as factor: $f(x) = (x - a)g(x)$

where $g(x)$ is a polynomial of a lower order than $f(x)$.

substitute a: $f(a) = (a - a)g(a)$

simplify: $f(a) = 0\,g(a)$

$f(a) = \mathbf{0}$

The factor theorem: $(x - a)$ is a factor of a polynomial $f(x)$ if and only if $f(a) = 0$.

We can check whether a linear polynomial $(x - a)$ is a factor of a higher order polynomial $f(x)$ by evaluating $f(a)$.

Factoring with the factor theorem

Use the *factor theorem* to show that (x + 2) is a factor of $f(x) = 3x^2 + 5x - 2$. Factor $f(x)$.

factor theorem: (x + 2) is a factor of $f(x) = 3x^2 + 5x - 2$ if $f(-2) = 0$

evaluate: $f(-2) = 3(-2)^2 + 5(-2) - 2$

$f(-2) = 12 - 10 - 2$

$f(-2) = 0$ ✓

therefore: $3x^2 + 5x - 2 = (x + 2)(bx + c)$

expand: $3x^2 + 5x - 2 = bx^2 + (2b + c)x + 2c$

compare coefficients: $b = 3$ and $c = -1$

substitute: $\mathbf{3x^2 + 5x - 2 = (x + 2)(3x - 1)}$

Factor theorem practice

The factor theorem can be used to factor polynomials by systematically looking for values of x that will make the polynomial equal to 0.

Press **start** to see an example.

start

Factor the polynomials

Factor the polynomial expressions below

1. $x^4 + 8x^3 + 23x^2 + 28x + 12$	?
2. $x^3 + 7x^2 + x + 7$	?
3. $3x^3 - 7x^2 + 5x - 1$	?
4. $x^4 - 36x^2 - 32x + 192$	?
5. $2x^4 - 3x^3 - 4x^2 + 3x + 2$	?
6. $x^4 - 1$	?

Press the gray boxes to reveal the factored expressions.