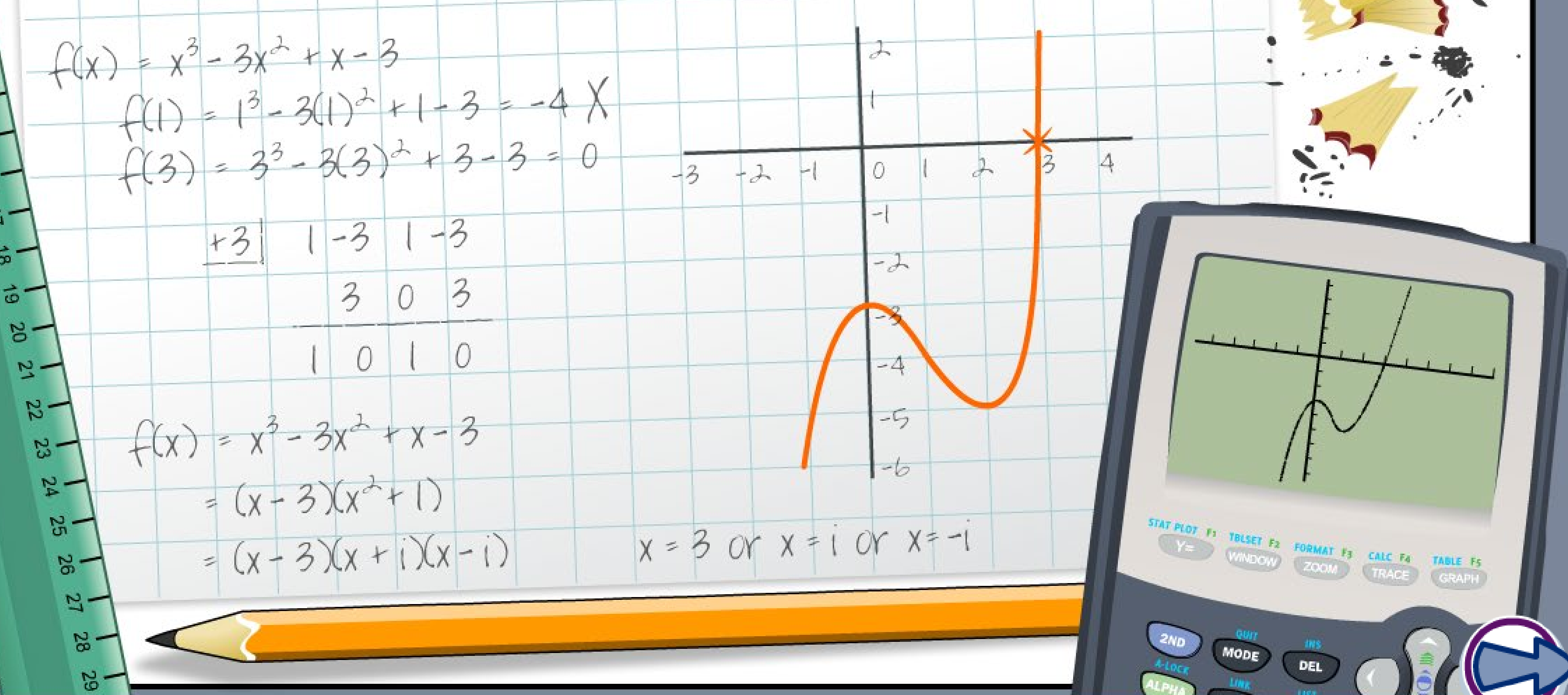
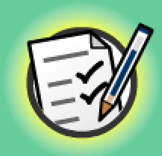
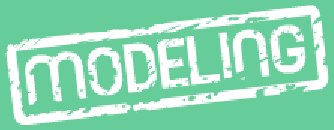
Exponential Functions



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

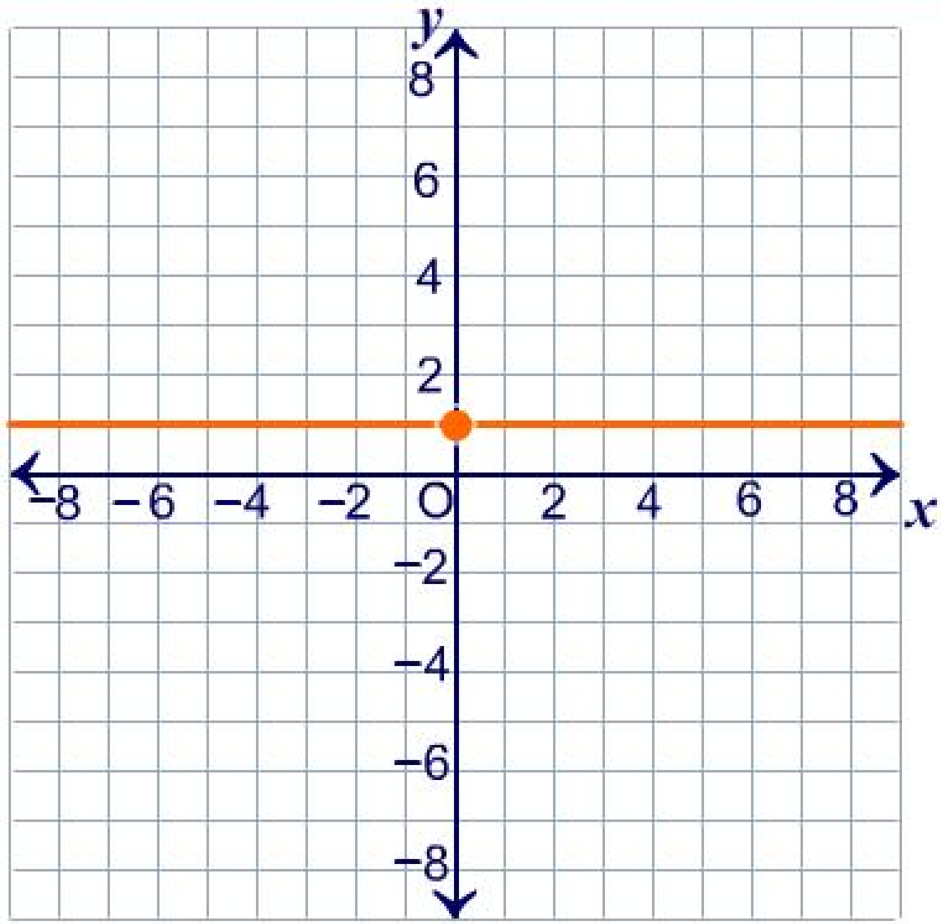
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Two genies

A very lucky man discovers two genies. He tells them he wishes to be rich and both genies make him an offer. He can only accept one of the offers...

Explain to the man which is the best offer. Press your chosen genie.



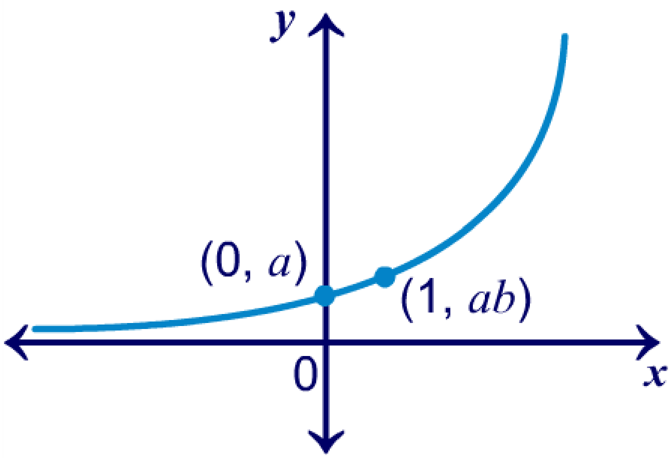
x	y
-3	1
-2	1
-1	1
0	1
1	1
2	1

$y = 1^x$

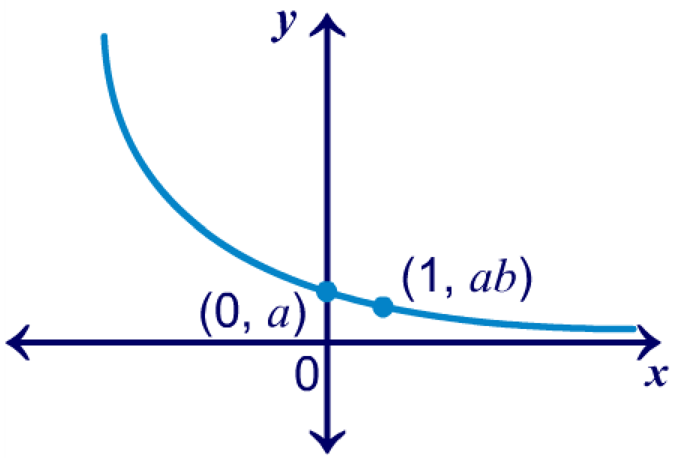
parent exponential function base b: $f(x) = ab^x$

where x is a real number, $b > 0$, $b \neq 1$ and $a \neq 0$

When $b > 1$, the graph $y = ab^x$ has this shape:



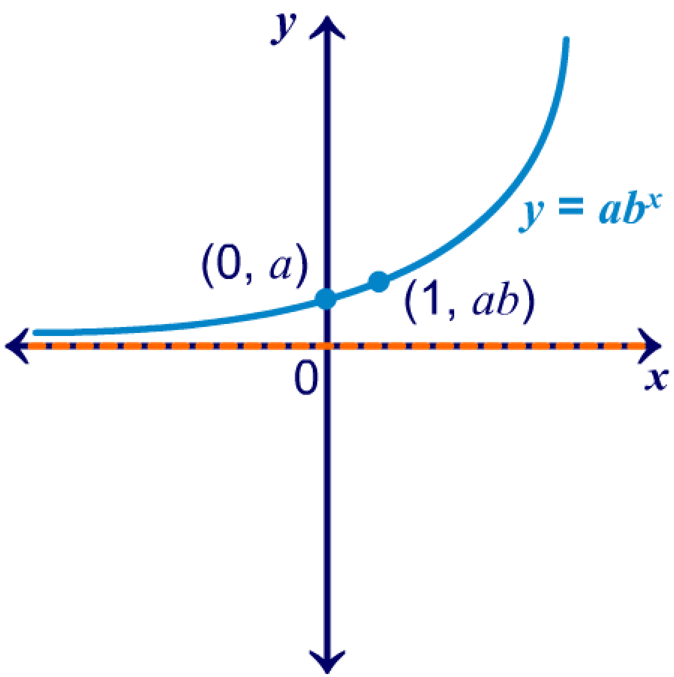
When $0 < b < 1$, the graph $y = ab^x$ has this shape:



In both cases, the graph passes through $(0, a)$ and $(1, ab)$.

Can you explain why this happens?

parent exponential function: $f(x) = ab^x$



vertical asymptote:	none
horizontal asymptote:	$y = 0$
domain:	$(-\infty, \infty)$
range:	$(0, \infty)$
roots:	none
y-intercept:	$(0, a)$
other key points:	$(1, ab)$

Discrete and continuous exponential growth

What do you think it means if a quantity undergoes

a) discrete exponential growth
b) continuous exponential growth?

Can you give an example of each type of growth?

Press on a type of exponential growth to learn more.

discrete exponential growth

continuous exponential growth

The world's population

In 1985, a town's population was approximately 26,000. It has been growing continuously at an annual rate of 1.8%.

A) Write a function to model the growth of the town's population from 1985.

B) Estimate the population of the town in 2030.

Continuous growth is modeled by $A = A_0e^{kt}$, where k is the growth rate and t is the time. The function modeling the population will therefore be in this form.

The world's population

In 1985, a town's population was approximately 26,000. It has been growing continuously at an annual rate of 1.8%.

A) Continuous exponential growth is modeled by $A = A_0e^{kt}$.

The town's population can be modeled by the function:

$\boldsymbol{P(t) = 26000e^{0.018t}}$, where $P(t)$ is the population and t is the number of years after 1985.

B) The year 2030 is 45 years after 1985, i.e. $t = 45$.

$P(t) = 26000e^{(0.018 \times 45)}$

$P(t) = 58{,}445.6$ = **58,446 people (to the nearest person)**

Time spent online

An annual survey investigating internet usage was carried out on a random selection of Americans over a period of 5 years.

year	hours per week spent online
2006	6
2007	7.2
2008	8.6
2009	10.4
2010	12.4

A) Write a function to model the number of hours (h) per week spent online t years after 2006.

B) Predict the number of hours spent online in 2023. Discuss whether this prediction is realistic.

A) The values in the table increase year-by-year by a factor of about 1.2, starting at 6 in 2006 (when h = 0).

The data can therefore be modeled by the function: $\boldsymbol{h = 6(1.2^t)}$

Time spent online

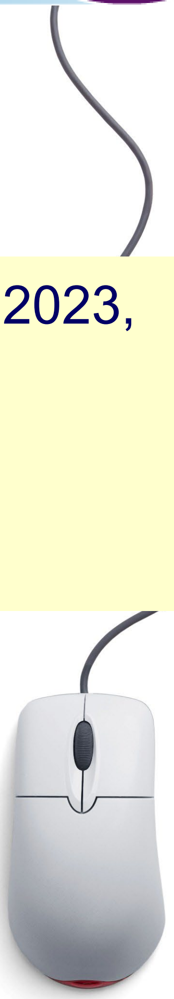
year	hours per week spent online
2006	6
2007	7.2
2008	8.6
2009	10.4
2010	12.4
...	...
2023	?

The data can be modeled by the function $\boldsymbol{h = 6(1.2^t)}$.

B) To predict the hours spent online in 2023, substitute in $t = 17$:

$$h = 6(1.2^{17})$$

$$\boldsymbol{h = 133.1 \text{ hours}}$$



This value is unrealistic – in one week there are 168 hours, and the model is predicting that nearly all of this time will be spent online.

Be careful when using models to predict values; it cannot always be assumed that a trend will continue.

Radioactive substances have a **half-life**. This is the time that it takes for one half of the atoms of that substance to decay.

Write a function describing the remaining amount A of a radioactive substance, in terms of the original amount A_0 and the number of half-lives that have passed, t.

The quantity of the original substance (A_0) will be multiplied by 0.5 (i.e. halved) t times to give A: $A = A_0(0.5^t)$

How many atoms remain of a substance after 6 half-lives, if there were originally 192 atoms?

substitute $A_0 = 192$ and $t = 6$: $A = 192(0.5^6)$

A **= 3 atoms**

Radioactive half-life

Radioactive substances have a **half-life** – the time that it takes for one half of the atoms of that substance to decay.

A scientist records the mass of a sample of the isotope Carbon-10 as 12.8 g. 2 minutes and 20 seconds later, the mass of the same sample has only 0.1 g of Carbon-10.

A) How many half-lives have passed during this time?

B) What is the half-life of the isotope Carbon-10?

C) Graph the mass of Carbon-10 in the sample over 20 half-lives from the first mass recording of 12.8 g.

D) How long until there is no Carbon-10 left at all?