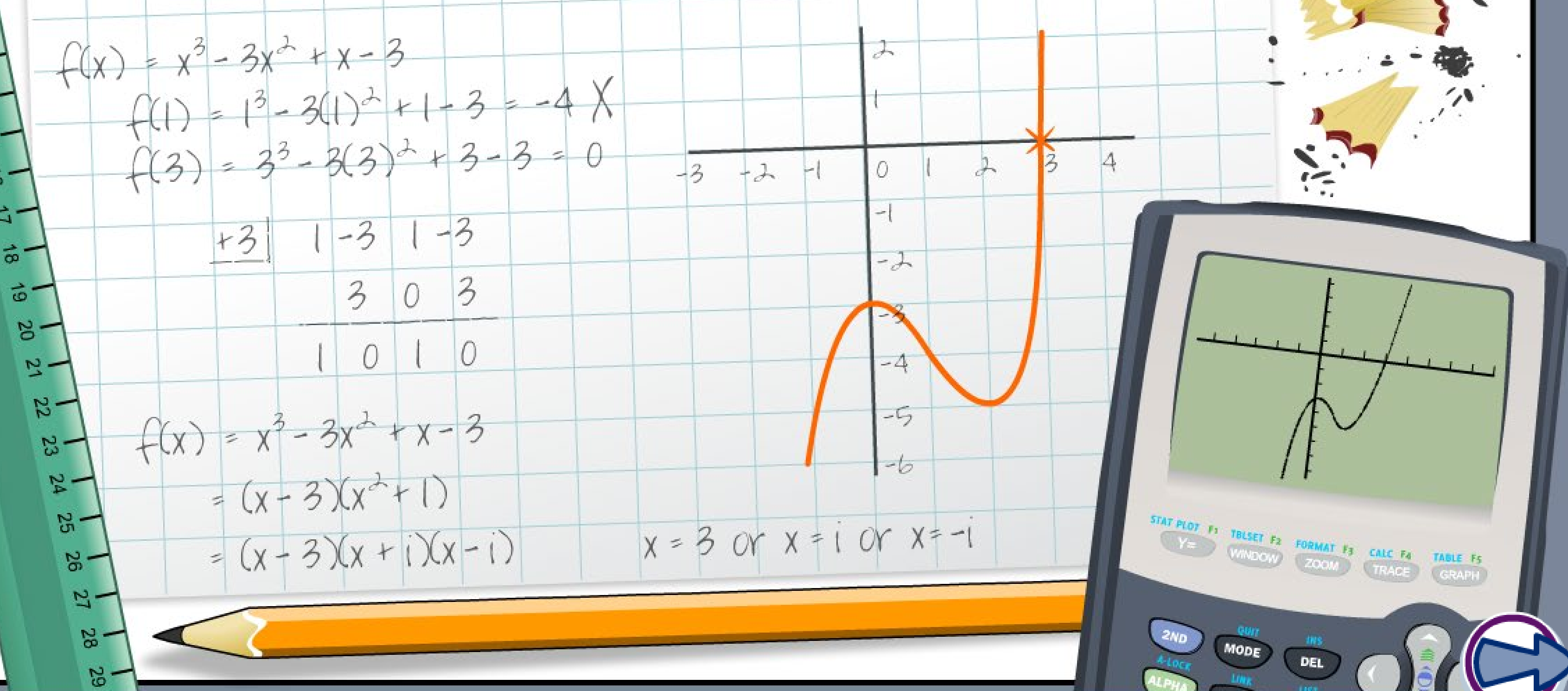
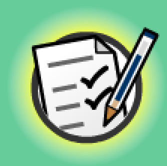
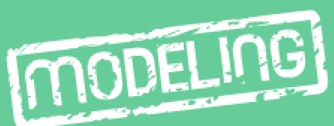
Comparing Functions



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

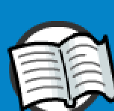
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Describing functions

Press a function to view its graph and then analyze the graph using the list of characteristics.

domain:	?
range:	?
continuity:	?
increasing or decreasing:	?
symmetry:	?
bounded:	?
extrema:	?
asymptotes:	?
end behavior:	?

$y = x$ | $y = x^2$ | $y = x^3$

$y = e^x$ | $y = \frac{1}{x}$ | $y = \sqrt{x}$

$y = \ln(x)$ | $y = |x|$ | $y = \text{int}(x)$

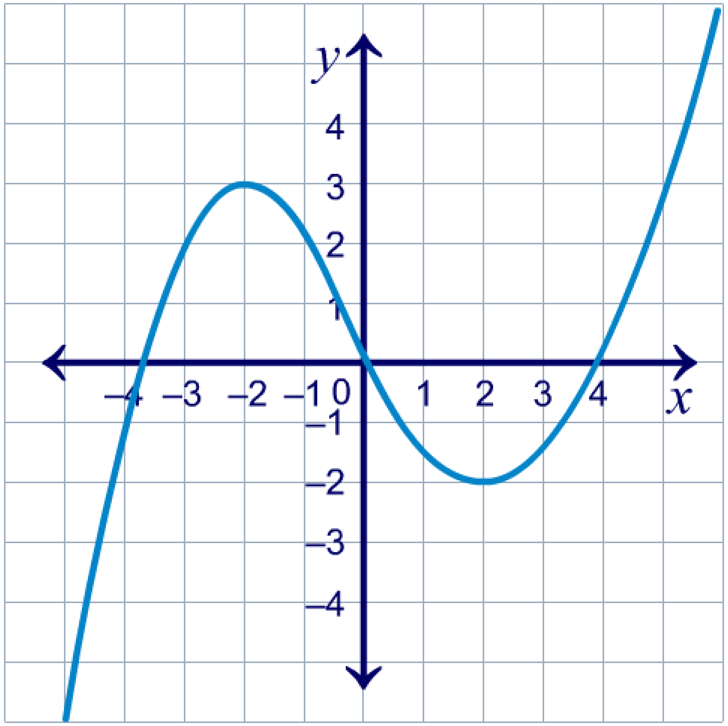
How well do you know the basic functions?

Press **start** to begin a multiple choice quiz about the key features of different functions.

start

A function can have an **absolute** maximum or minimum, and it can have **local** maxima or minima.

A local maximum or minimum is a point with a higher or lower y-value than the points to each side of it, but it might not be the highest or lowest overall.



Are there any local maxima or minima on this graph? Give coordinates.

local maximum: (–2, 3)

local minimum: (2, –2)

Extrema multiple choice quiz

Can a function have **no** maximum or minimum values?

Can a function have an **absolute** and a **local** maximum?

Test how well you understand local and absolute extrema. Press **start** to see graphs and figure out whether they have any absolute or local maximum or minimum values.

start

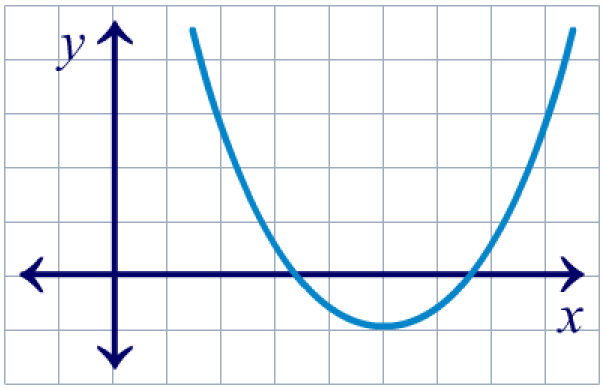
Quadratic functions are polynomial functions of degree 2. Their graphs are parabolas and can be represented algebraically in either **standard form** or in **vertex form**.

The different forms reveal different features of the function:

	standard form	vertex form
	$f(x) = ax^2 + bx + c$	$f(x) = a(x - h)^2 + k$
y-intercept:	$y = c$	$y = ah^2 + k$
vertex:	$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	(h, k)
axis of symmetry:	$x = -\frac{b}{2a}$	$x = h$
x-intercepts:	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Lead coefficient

For a quadratic $f(x) = ax^2 + bx + c$ or $f(x) = a(x - h)^2 + k$, the sign of the lead coefficient, a, gives clues about the behavior of the graph.

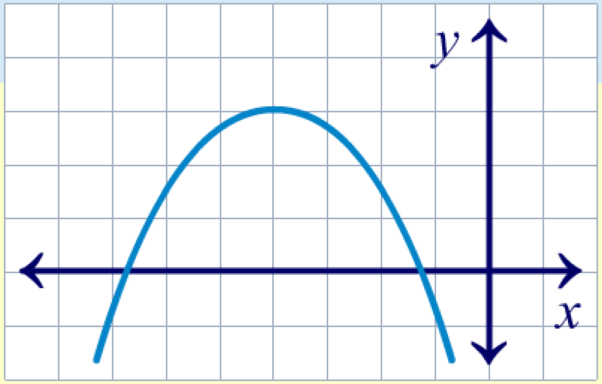


What happens if $a > 0$?

- the parabola opens upwards
- the vertex contains the minimum
- $f(x) \rightarrow \infty$ as $x \rightarrow \infty$ and as $x \rightarrow -\infty$.

What happens if $a < 0$?

- the parabola opens downwards
- the vertex contains the maximum
- $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$ and as $x \rightarrow -\infty$.



Comparing quadratics

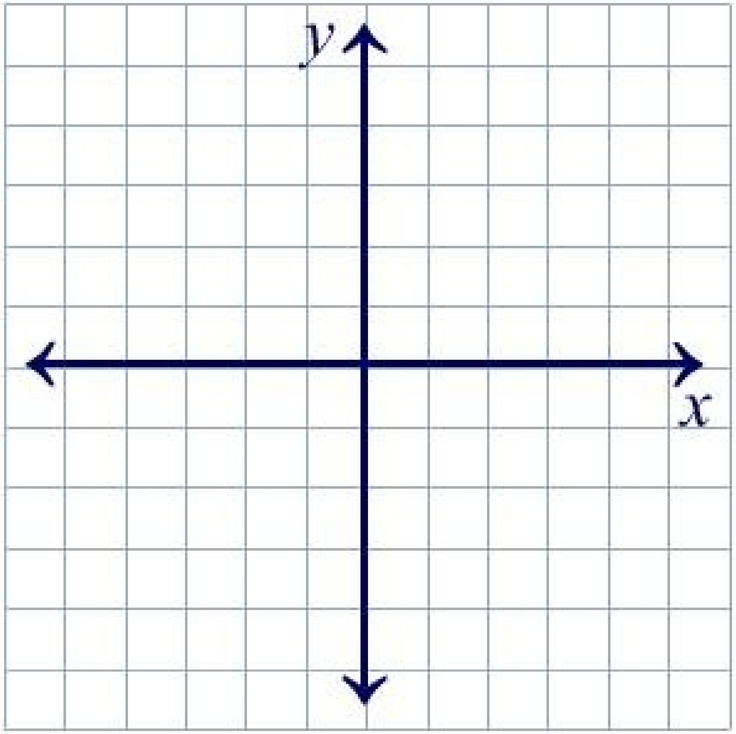
In a science competition, two rockets are fired upwards from a platform and rise into the air before falling to the ground below the platform. The height of each rocket in feet, t seconds after launching, is modeled by a quadratic function:

Rocket A: $f(t) = -16t^2 + 48t + 16$

Rocket B: $g(t) = -16(t - 2)^2 + 80$

1. **Which rocket achieved the highest maximum height? What was it and at what time?**
2. **What domain and range applies to each function?**
3. **Find the height of the platform and compare the initial velocities of the rockets.**

polynomial function: $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$



The degree n and the sign of the leading coefficient a_n determine the end behavior of the graph.

Press the buttons to see how.

$a > 0$, n odd	$a > 0$, n even
$a < 0$, n odd	$a < 0$, n even

If $(x - a)^n$ is a factor of a polynomial then a is a root of the polynomial and a has **multiplicity n**.

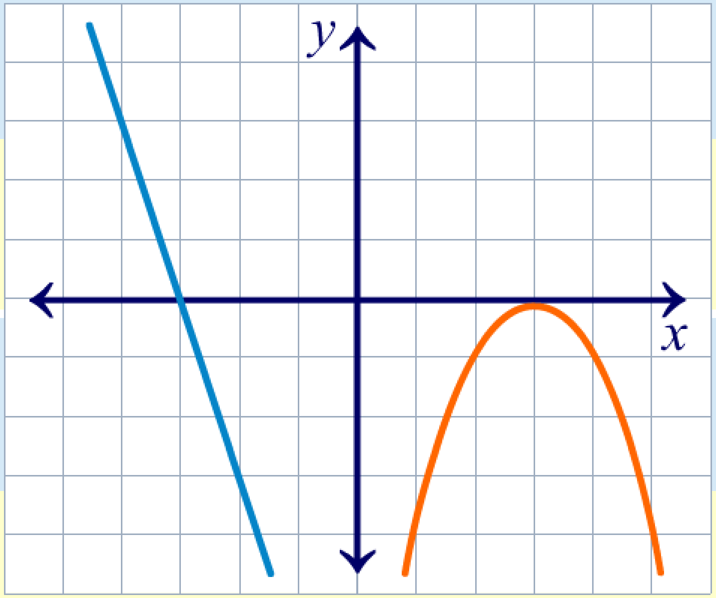
The multiplicity determines the behavior of the graph about the point $(a, 0)$.

How does the graph behave at a root with odd multiplicity?

It intersects the x-axis.

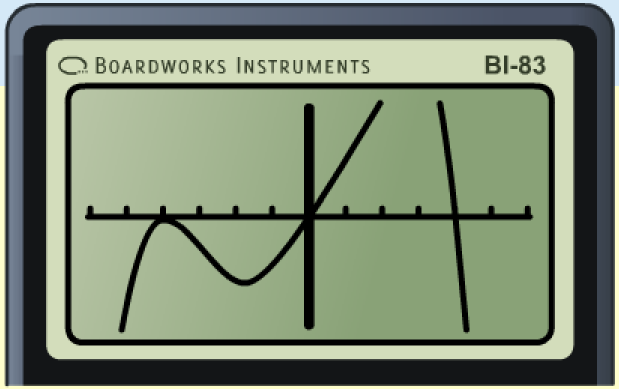
How does the graph behave at a root with even multiplicity?

It is tangent to the x-axis.



Predicting behavior (1)

Predict the end behavior and the behavior near the x-axis of the graph of $f(x) = -x(x + 4)^2(x - 4)$. Confirm using a graphing calculator.



degree of $f(x)$: 4

lead coefficient: $a = -1$

end behavior given even degree, $a < 0$: as $x \to \infty$ or $-\infty$, $f(x) \to -\infty$

multiplicity of zeros: $x = -4$, multiplicity 2

graph is tangent to the x-axis at $x = -4$.

$x = 0$ and $x = 4$ have multiplicity one

graph intersects the x-axis at $x = 0$ and $x = 4$.

Predicting behavior (2)

Factor $f(x) = x^3 - x^2 - 6x$ and then predict the end behavior and the behavior near the x-axis.
Confirm using a graphing calculator.

factored form: $f(x) = x(x + 2)(x - 3)$

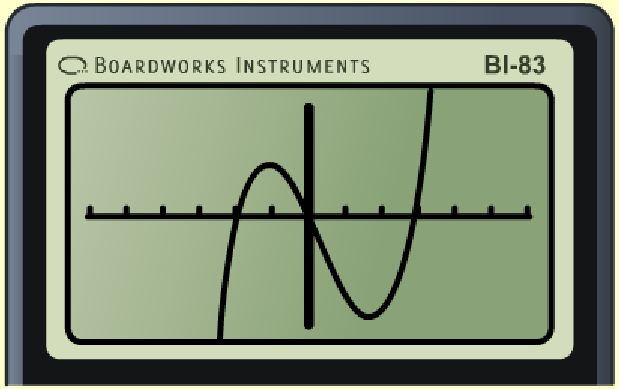
degree of $f(x)$: 3

lead coefficient: $a = 1$

end behavior given odd degree, $a > 0$: as $x \to \infty$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to -\infty$

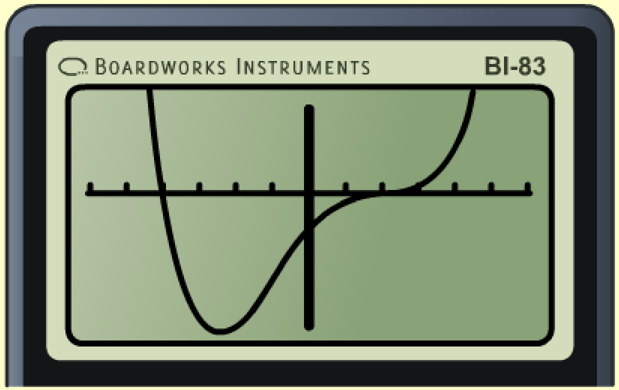
multiplicity of zeros: all zeros have multiplicity one

graph crosses the x-axis at $x = -2$, $x = 0$, and $x = 3$.

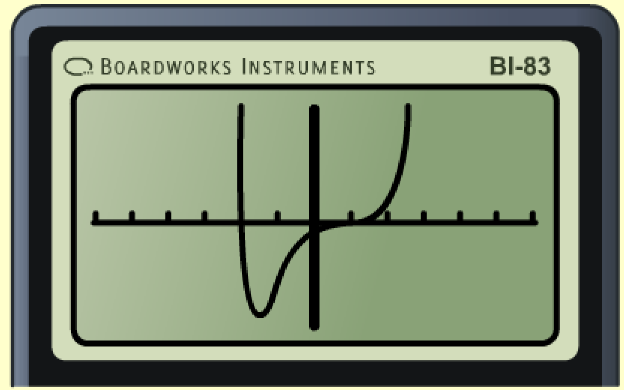


Odd multiplicity greater than 1

Graph $f(x) = (x + 4)(x - 3)^3$ and $g(x) = (x + 2)(x - 1)^5$. Describe the behavior of each graph near the x-axis.



$f(x) = (x + 4)(x - 3)^3$



$g(x) = (x + 2)(x - 1)^5$

Both graphs cross the x-axis at each of their zeros.

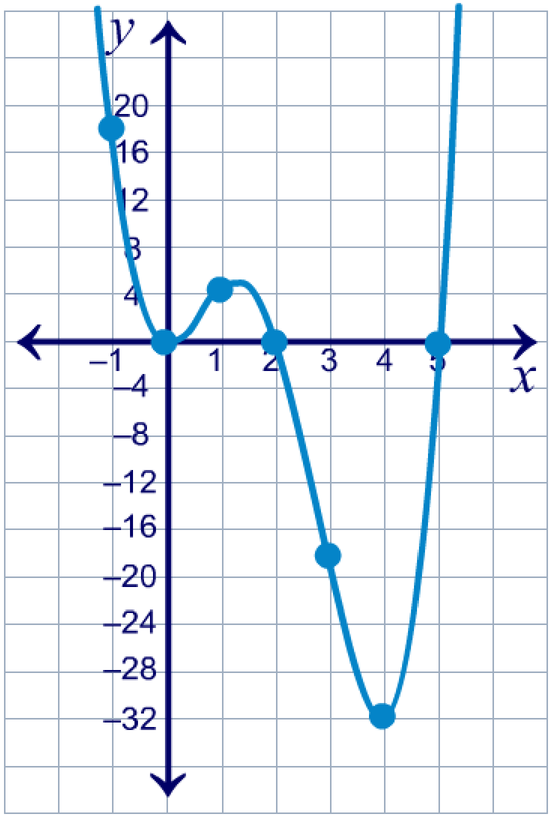
All zeros have odd multiplicity so this was expected.

The graphs flatten out near the x-axis around the zeros with odd multiplicity greater than 1.

There is still only one point on the x-axis at each zero.

Draw a scatter plot from the table of values below.

x	y
–1	18
0	0
1	4
2	0
3	–18
4	–32
5	0



The data is a polynomial function. What is the minimum degree polynomial that could fit this data exactly?

There are three x-intercepts so it is at least degree 3.

The shape shows it must be a **quartic**.

Find a quartic regression to model the data.

$x^4 - 7x^3 + 10x^2$