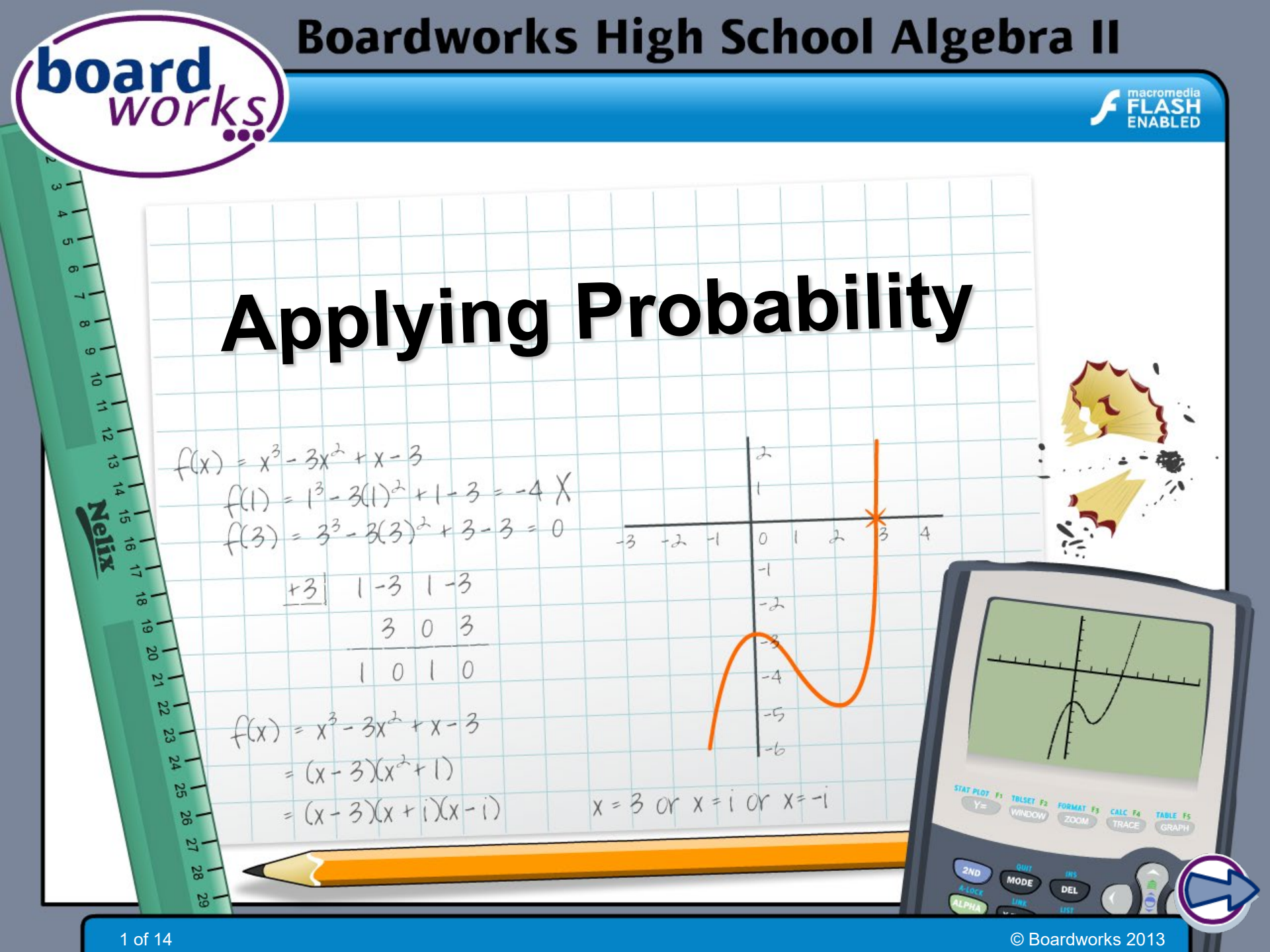
# Applying Probability



# Information

## Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.


Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

**1) Make sense of problems and persevere in solving them.**

**2) Reason abstractly and quantitatively.**

**3) Construct viable arguments and critique the reasoning of others.**

**4) Model with mathematics.**

**5) Use appropriate tools strategically.**

**6) Attend to precision.**

**7) Look for and make use of structure.**

**8) Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

# Which company?

**Should Jasmine buy company A or company B?**

Her research into the companies yielded the results shown.

| buy company A | | | |
|---|---|---|---|
| **event** | lose \$25,000 | break even | make \$65,000 |
| **probability** | 25% | 55% | 20% |

| buy company B | | | |
|---|---|---|---|
| **event** | lose \$35,000 | break even | make \$115,000 |
| **probability** | 30% | 60% | 10% |

Jasmine's decision will be based on expected value.

$E(X_A) = (0.25)(-\$25{,}000) + (0.55)(0) + (0.20)(\$65{,}000)$

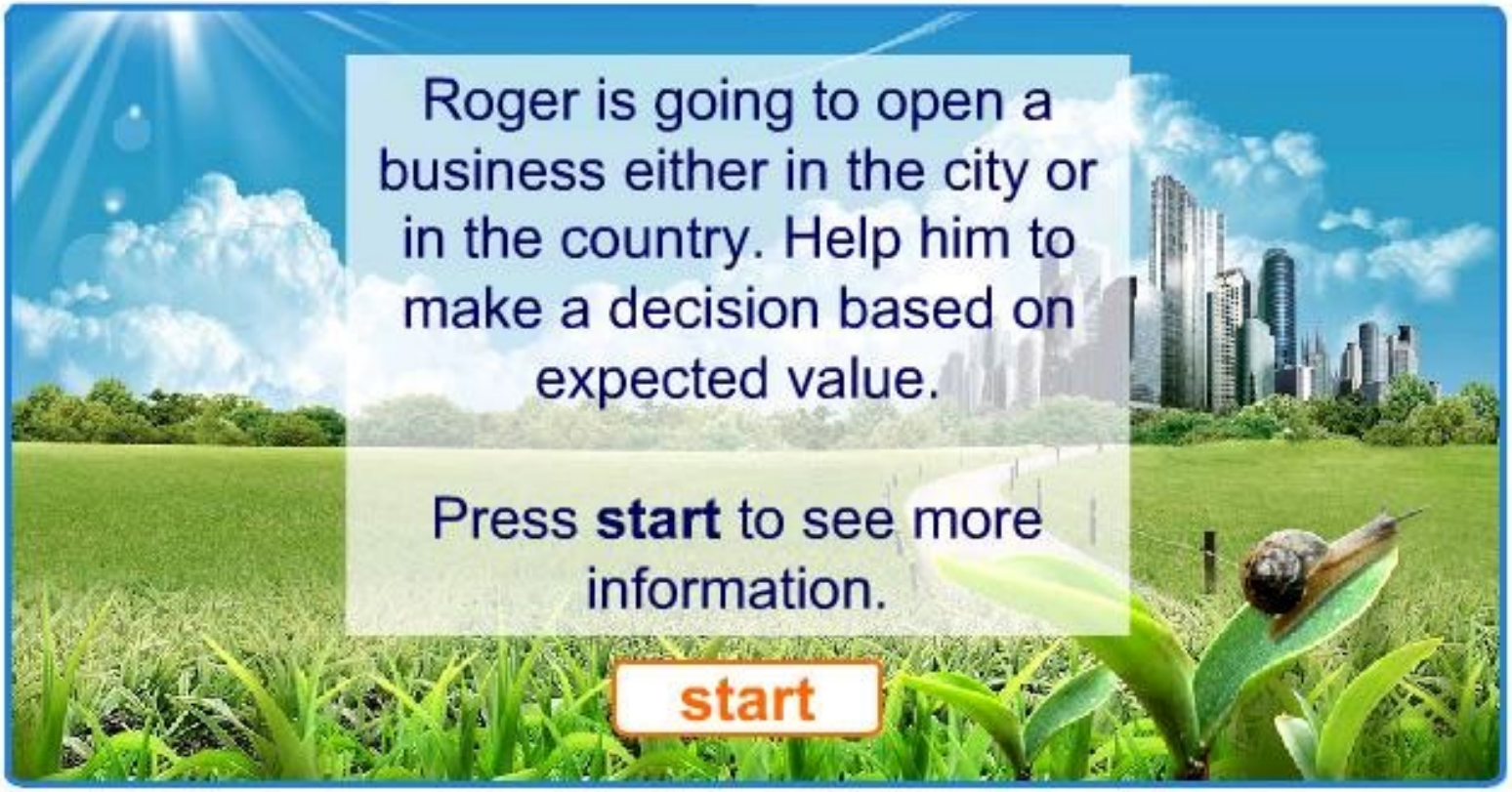
$= -\$6{,}250 + \$13{,}000 = \$6{,}750$

$E(X_B) = (0.30)(-\$35{,}000) + (0.60)(0) + (0.10)(\$115{,}000)$

$= -\$10{,}500 + \$11{,}500 = \$1{,}000$

Based on expected value, Jasmine should buy **company A**.

# Choosing a location



## Two-stage treatment

A doctor has developed a two-stage treatment to reduce arthritic pain. The first treatment has an 85% success rate. If this is a success, the second treatment has a 95% success rate. If the first treatment is not a success, the second treatment has a 35% success rate.

**1) What is the probability that:**

**a) both treatments are unsuccessful**

**b) both treatments are successful**

**c) the first treatment is unsuccessful and the second treatment is successful?**

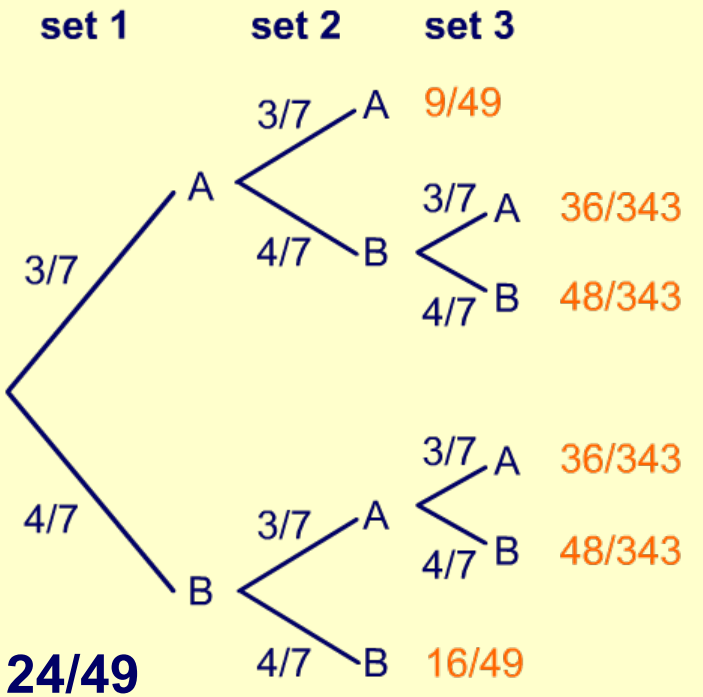
**2) Discuss whether or not to get the treatments.**

# Tennis

Adam and Boris are playing tennis. The first player to win two sets wins the match. The probability that Adam wins a set is 3/7 and the probability that Boris wins a set is 4/7.

**What is the probability that:**
**a) Adam wins b) Boris wins**
**c) Adam wins in exactly 2 sets,**
**d) either man wins after 3 sets?**

Draw a tree diagram:




a) $P$(A wins) = 9/49 + 36/343 + 36/343
= **135/343**

b) $P$(B wins) = 48/343 + 16/49 + 48/343
= **208/343**

c) $P$(2 sets, A wins) = **9/49**

d) $P$(3 sets played) = 2(36/343) + 2(48/343) = **24/49**

# Guessing answers

A multiple choice test contains 10 questions with 4 choices for each answer.

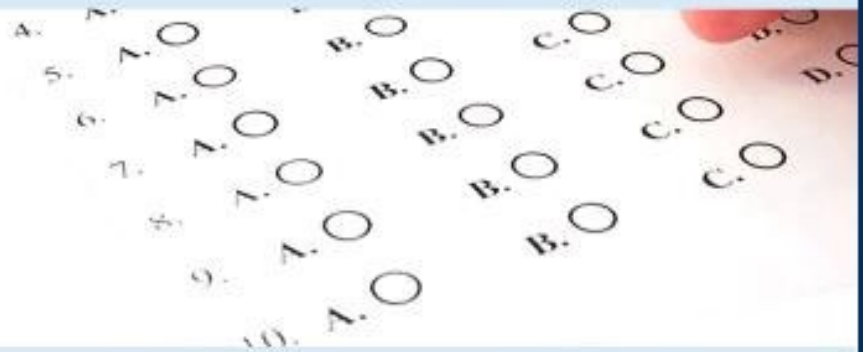
**1) If you guess all of the answers, what is the probability that you get 6 correct answers?**

?

formula method:

calculator method:

simulation method:



**2) If you guess all of the answers, what is the expected value for the correct answers?**

?

## Guessing test answers

A multiple-choice test contains 25 questions with 4 choices for each answer. You guess all the answers.
*Drag the correct values to the boxes.*

| | |
|---|---|
| probability of 5 correct answers | |
| probability of 10 correct answers | |
| expected value for correct answers | |
| expected value for the wrong answers | |

18.75 | 6.25 | 0.04 | 17.75 | 7.25 | 0.11 | 0.16 | 0.07

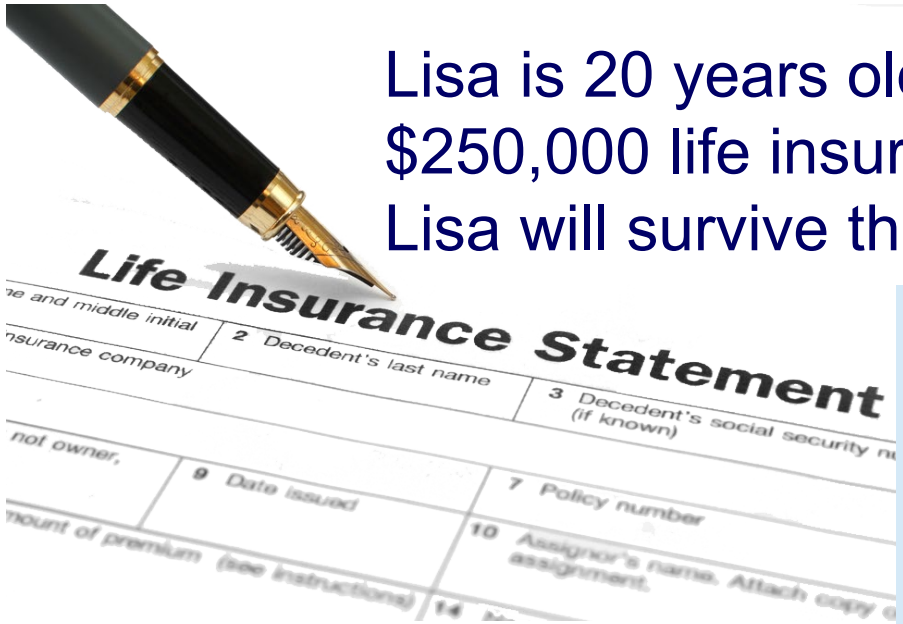
## Fire insurance policy

An annual premium for a fire insurance policy for Hugo's home is $275. The insurance company will pay Hugo $225,000 if there is a fire. The probability of a fire in Hugo's neighborhood is 0.0003.

**What is the expected value of the policy for the insurance company?**

$$E(X) = \text{probability of a fire} \times \text{net money paid for a fire} + \text{probability of no fire} \times \text{annual cost of policy}$$

# Life insurance policy




Lisa is 20 years old and pays $250 for a year-long $250,000 life insurance policy. The probability that Lisa will survive the policy period is 0.9995.

**What is the expected value for the insurance company of:**
**a) Lisa's policy**
**b) 10,000 similar policies?**

a) $E(X)$ = (policy cost × $P$(surviving)) – (payout × $P$(not surviving)):

$E(X)$ = ($250)(0.9995) – ($250,000 – $250)(1 – 0.9995)

= ($250)(0.9995) – ($249,750)(0.0005)

= ($249.88) – ($124.88) = **$125.00**

b) expected value of 10,000 policies:

10,000 $E(X)$ = (10,000)( $125.00) = **$1,250,000**

# Quality control

**A quality control manager is testing a plastic bottle manufacturing machine that has a 0.01 probability of producing a defective bottle. What is the probability that less than 3 of 20 of the bottles will be defective?**

Let the random variable $x$ = number of defective bottles.

Add the probabilities of 0, 1, and 2 defective bottles.

$P(x = 0) = {}_{20}C_0\,(0.01)^0(0.99)^{20} = 0.818$

$P(x = 1) = {}_{20}C_1\,(0.01)^1(0.99)^{19} = 0.165$

$P(x = 2) = {}_{20}C_2\,(0.01)^2(0.99)^{18} = 0.016$

$P(x < 3) = 0.818 + 0.165 + 0.016 = \mathbf{0.999}$

**How does this compare to a machine with 0.1 probability of producing less than 3 defective bottles out of 20?**

# The Monty Hall problem

On a game show, there are three mystery doors, behind each of which there is a prize to be won. Behind one door is a car, behind the others there are goats. The host asks the contestant to choose a door.

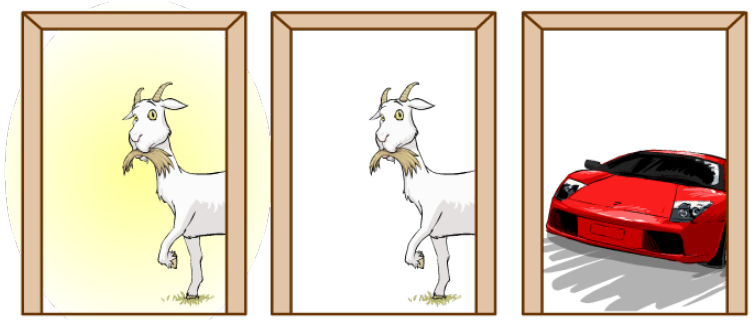
The contestant picks a door, and the host, who knows what is behind the doors, opens one of the others to reveal a goat.

**The host asks the contestant if they would like to change their choice to the other unopened door. Should they?**

# The Monty Hall problem

Figure out the probability of getting a car by looking at all possible outcomes.

Scenario 1

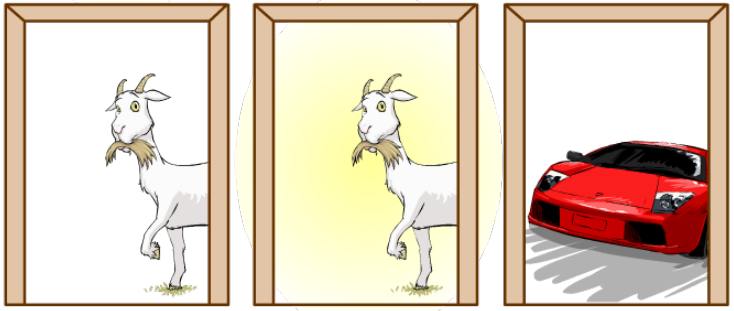


Choose goat.

Stick = goat

Change = car

Scenario 2

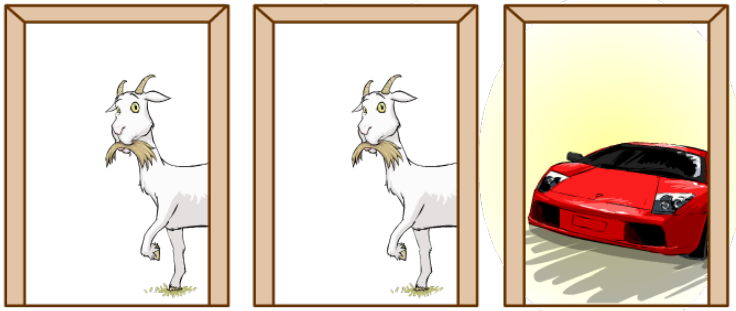


Choose goat.

Stick = goat

Change = car

Scenario 3



Choose car.

Stick = car

Change = goat

# The Monty Hall problem


On a game show, there are three mystery doors, behind each of which there is a prize to be won. Behind one door is a car, behind the others two there are goats. The game show host asks you to choose one of the doors.
You pick a door, and the host, who knows what is behind the doors, opens one of the others to reveal a goat.

He asks if you would like to change your choice to the other unopened door, or stick with your choice.
What should you do? Press **start** to play the game!

start

hide