

Adding and Subtracting Rational Expressions

$$f(x) = x^3 - 3x^2 + x - 3$$

$$f(1) = 1^3 - 3(1)^2 + 1 - 3 = -4 \quad \times$$

$$f(3) = 3^3 - 3(3)^2 + 3 - 3 = 0$$

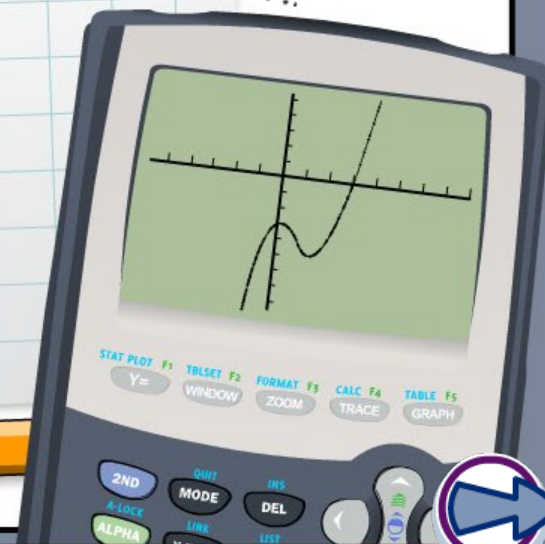
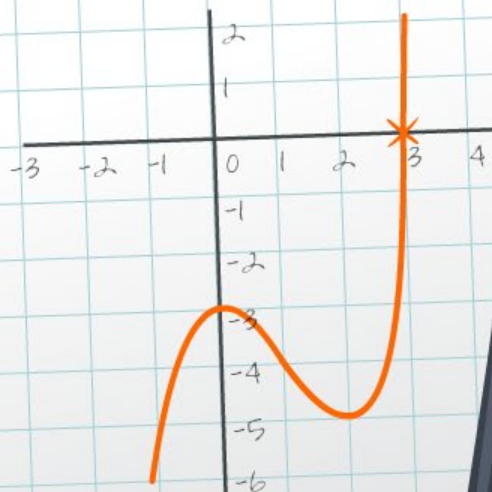
$$\begin{array}{r|rrrr} +3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$f(x) = x^3 - 3x^2 + x - 3$$

$$= (x - 3)(x^2 + 1)$$

$$= (x - 3)(x + i)(x - i)$$

$$x = 3 \text{ or } x = i \text{ or } x = -i$$



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.



This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

- 1) **Make sense of problems and persevere in solving them.**
- 2) **Reason abstractly and quantitatively.**
- 3) **Construct viable arguments and critique the reasoning of others.**
- 4) **Model with mathematics.**
- 5) **Use appropriate tools strategically.**
- 6) **Attend to precision.**
- 7) **Look for and make use of structure.**
- 8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.



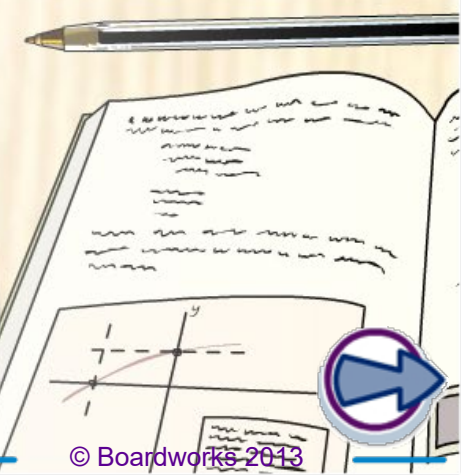
This icon indicates teacher's notes in the Notes field.



Like with numerical fractions, the denominators of rational expressions must be the same before adding.

General approach to adding rational expressions:

1. Check if the expressions have common denominators.
2. If not, factor the denominators and find their lowest common multiple. This is their least common denominator (LCD).
3. Rewrite the given fractions as equivalent fractions using the LCD.
4. Add the numerators.
5. Factor, then reduce, if possible.



Find $\frac{x^2}{x+2} + \frac{-4}{x+2}$ and write it in simplest form.

check to see if the expressions have common denominators:

$$\frac{x^2}{(x+2)} + \frac{-4}{(x+2)} \quad \checkmark$$

add the numerators:

$$\frac{x^2 - 4}{(x+2)}$$

factor, then reduce if possible:

$$\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} = x - 2 \quad x \neq -2$$

The parentheses around $(x + 2)$ in the denominator indicate that it is a group – a factor – and must stay together.

Parts of sums or differences cannot be reduced.



Lowest common multiple

Match the expressions to their lowest common multiple

$2x$ and 3

$x^2 + 6x + 5$ and $x^2 + 10x + 25$

$3x^2$ and x

$x^2 - 5x + 6$ and $x - 3$

x^2 and $2x + 1$

$x + 2$ and $x^2 - 4$

$x - 10$ and $2x - 6$

$2(x - 10)(x - 3)$

$3x^2$

$(x - 3)(x - 2)$

$(x + 2)(x - 2)$

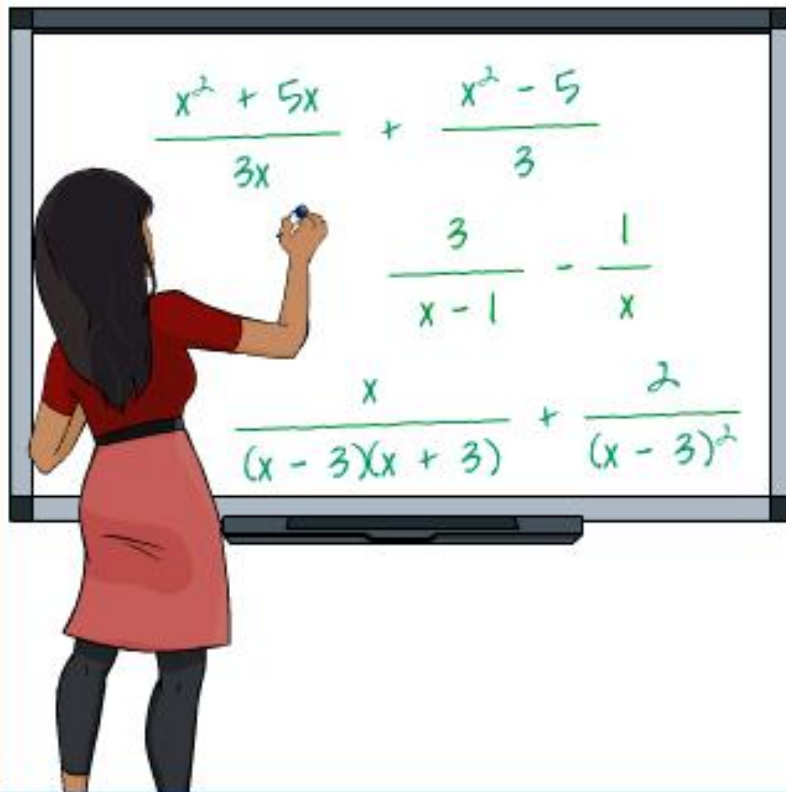
$x^2(2x + 1)$

$(x + 1)(x + 5)^2$

$6x$



Adding and subtracting rational expressions with unlike denominators



Press on a type of denominator to see an example.

monomial denominator

binomial denominator

trinomial denominator



Adding and subtracting rational expressions practice

1) $\frac{1}{y} + \frac{2}{y}$

?

3) $\frac{8}{z+1} + \frac{-3z}{z^2-1}$

?

2) $\frac{1}{2x} - \frac{3}{2x}$

?

4) $\frac{5y+3}{xy} + \frac{3}{y^2}$

?

5) $\frac{a}{a^2+a-2} - \frac{a+2}{a^2+2a-3}$

?

6) $\frac{1}{x^2} \cdot \frac{x}{2x+4} - \frac{x^2}{x^2+4x+4}$

?





Transfer functions and rational functions



what is a transfer function?

Press on a type of control system to see an example.

single component

summation junction

challenge: feedback loop



multiplication

division

addition

subtraction

Remember that polynomials are closed under addition, subtraction and multiplication, but not division.

Are **rational expressions** closed under all operations?

Press the tabs to find out.

