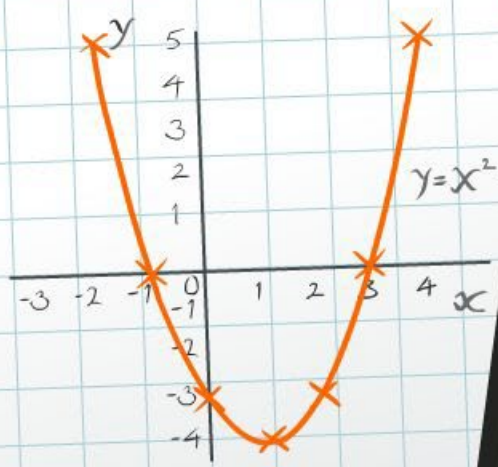
Using formulas

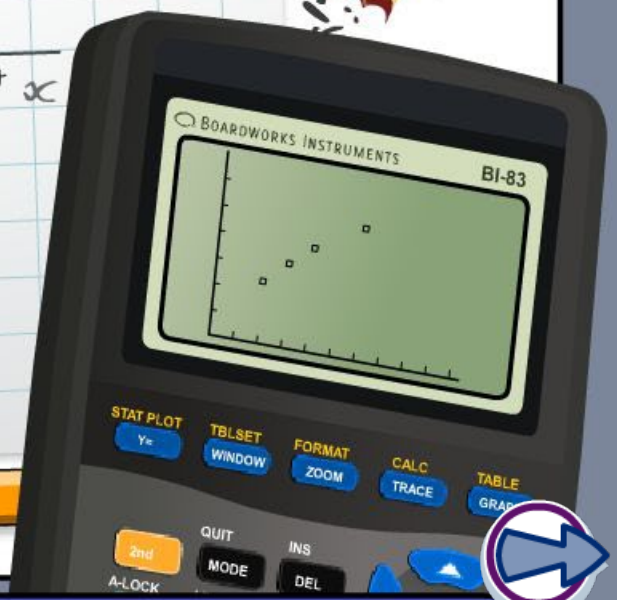
x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$





Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

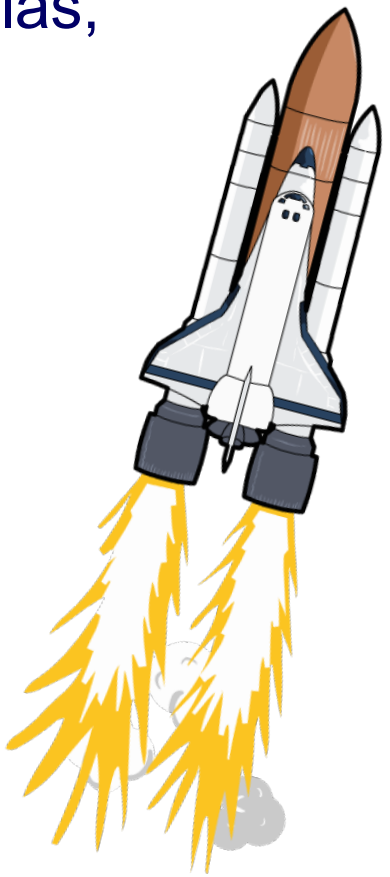
This icon indicates teacher's notes in the Notes field.

Using units

Formulas deal mainly with real-life quantities such as length, mass, temperature or time, and the variables often have **units**.

Units are usually defined when writing formulas, but they are not included in the calculations.

For example: $s = \frac{d}{t}$



This does not mean much unless we say:

- s is the average speed **in m/s**
- d is the distance travelled **in meters**
- t is the time taken **in seconds**.

Speed, distance and time

If a car travels 2 km in 1 min 40 secs, at what speed is it traveling?

- Use the formula: $s = \frac{d}{t}$
- Write the distance and the time using the correct units before substituting them into the formula:

2 kilometers = 2000 meters

1 min and 40 seconds = 100 seconds

- Now substitute these numerical values into the formula: $s = \frac{2000}{100}$
- Write the units in at the final step in the calculation: **= 20 m/s**

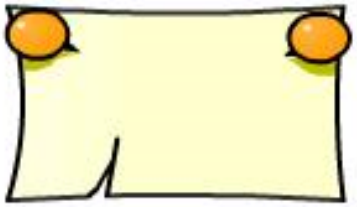
Now find the speed in kilometers per hour.

Substituting with units

$$s = \tfrac{1}{2}(u + v)t$$

Where s is the distance travelled by an object (in m) with initial velocity u (in m/s) and final velocity v (in m/s), t seconds later.

Find s when u is 6m/s, v is 9m/s and t is 15 seconds.



Rearranging formulas

Formulas are usually (but not always) arranged so that a single variable is written on the left-hand side of the equal sign. For example, the formula below has v as the **subject**, and is said to be **solved for v**:

$$v = u + at$$

What happens if we are given values of v, u and a, and asked to find t?

Rearrange the formula to make t the subject, then substitute values in to solve the equation.

$$t = \frac{v - u}{a}$$

Inverse operations

To rearrange a formula, find the **inverse operations** that undo the operations in the formula.

Here is a formula for the area of a rectangle, A, using its length, L, and width, W:

$$A = LW$$

The formula is written with A as the subject. To rearrange it to make L the subject:

rewrite the formula with L on the left:

$$A = LW$$

solving for A

$$LW = A$$

remove W from the left-hand side by dividing both sides by W:

$$L = A \div W$$

solving for L

$$L = \frac{A}{W}$$

Practice rearranging equations

Rearrange the equation to make b the subject.

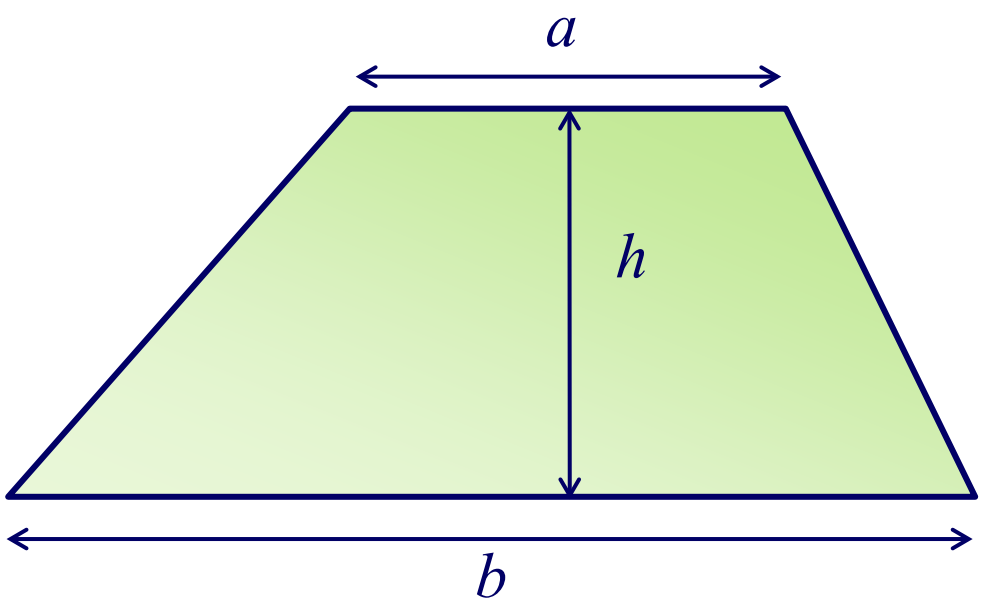
$$d = \frac{b(5 + c)}{a}$$

$b =$ 

Problems leading to equations

The formula used to find the area, A, of a trapezoid with parallel sides a and b and height h is:

$$A = \frac{1}{2}(a + b)h$$



By substituting values into the formula and rearranging the equation, it is possible to calculate an unknown.

What is the height of a trapezoid with an area of 40 cm^2 and parallel sides of length 7 cm and 9 cm?

Problems leading to equations

What is the height of a trapezoid with an area of 40 cm² and parallel sides of length 7 cm and 9 cm?

$$A = \frac{1}{2}(a + b)h$$

$$40 = \frac{1}{2}(7 + 9)h$$

write **40** instead of A, **7** instead of a, and **9** instead of b

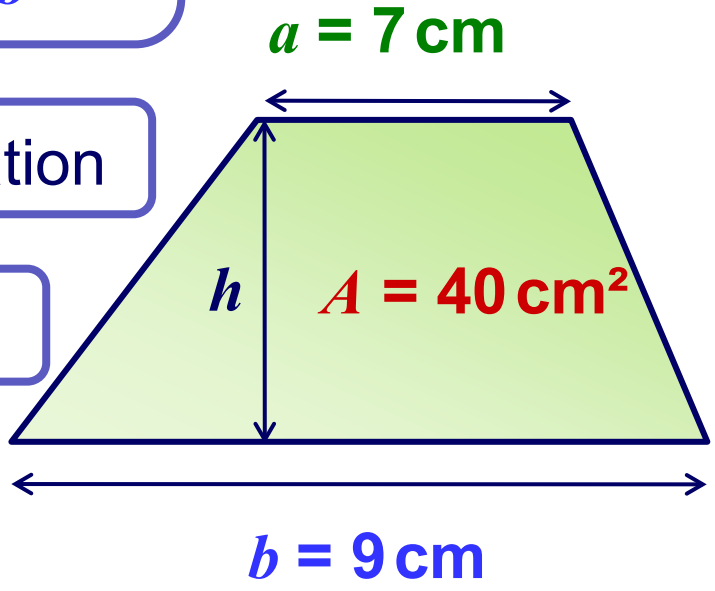
$$40 = \frac{1}{2} \times 16h$$

simplify the equation

$$40 = 8h$$

divide both sides by 8

$$5 = h$$



The height of the trapezoid is **5 cm**.

Find the equivalent formulas

Decide which cards show equations equivalent to this one.

$c = 14 - 2d$

$d = \frac{c - 14}{2}$

$c = 2d - 14$

$d = \frac{14 - c}{2}$

$d = 7 - \frac{c}{2}$

$d = \frac{c - 2}{14}$

$c = 2(7 - d)$

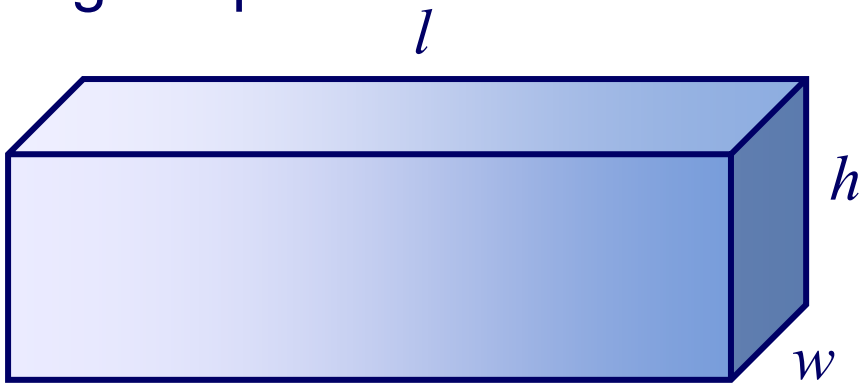
Repeated variables

Sometimes a variable appears more than once in a formula.

For example, three of the variables appear twice in this formula for the surface area of a rectangular prism:

$$S = 2lw + 2lh + 2hw$$

- S is the surface area of a rectangular prism
- l is its length
- w is its width
- h is its height.



Rearrange this formula to make w the subject.

Repeated variables

Rearrange this formula to make w the subject:

$$S = 2lw + 2lh + 2hw$$

Write the formula so that the terms containing w are on the left:

$$2lw + 2lh + 2hw = S$$

Subtract $2lh$ from both sides so that all terms containing w are together:

$$2lw + 2hw = S - 2lh$$

Isolate w by **factoring**:

$$w(2l + 2h) = S - 2lh$$

Divide both sides by $2l + 2h$:

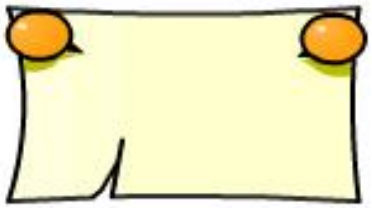
$$w = \frac{S - 2lh}{2l + 2h}$$

Problems leading to equations

$$v^2 = u^2 + 2as$$

Where v is the final velocity (in m/s), u is the initial velocity (in m/s), a is the acceleration (in m/s²) and s is the distance traveled (in m).

Find a where u is 9m/s, v is 25m/s and s is 58m.



Stopping distances

This formula can be used to calculate the stopping distance of a car:

$$\frac{x^2}{20} + x = S$$

- x is the car's speed in mph
- S is the stopping distance in feet.

As an estimate, what will be the stopping distance of a car traveling at 30 mph?

What speed would a car have been doing if it took 315 feet to stop?

Stopping distances

Drag the stopping distance to the car driving at the appropriate speed.

$$\frac{x^2}{20} + x = S$$

$x = 91.8$ feet

Van hire

The cost of hiring a van depends on the number of days it is hired for and the distance driven.

Press "**start**" to see some van-hire problems that can be solved using formulas.

start

Van hire solution

Can you find values for d and m where the cost is the same for both firms? How did you figure this out?

$C = 50d + 0.2m$ $C = 25d + 0.4m$

The cost is the same for both firms, so we can write:

$$50d + 0.2m = 25d + 0.4m$$

$$50d - 25d = 0.4m - 0.2m$$

$$25d = 0.2m$$

$$125d = m$$

The cost is the same if the vans drive **125 miles in a day**.