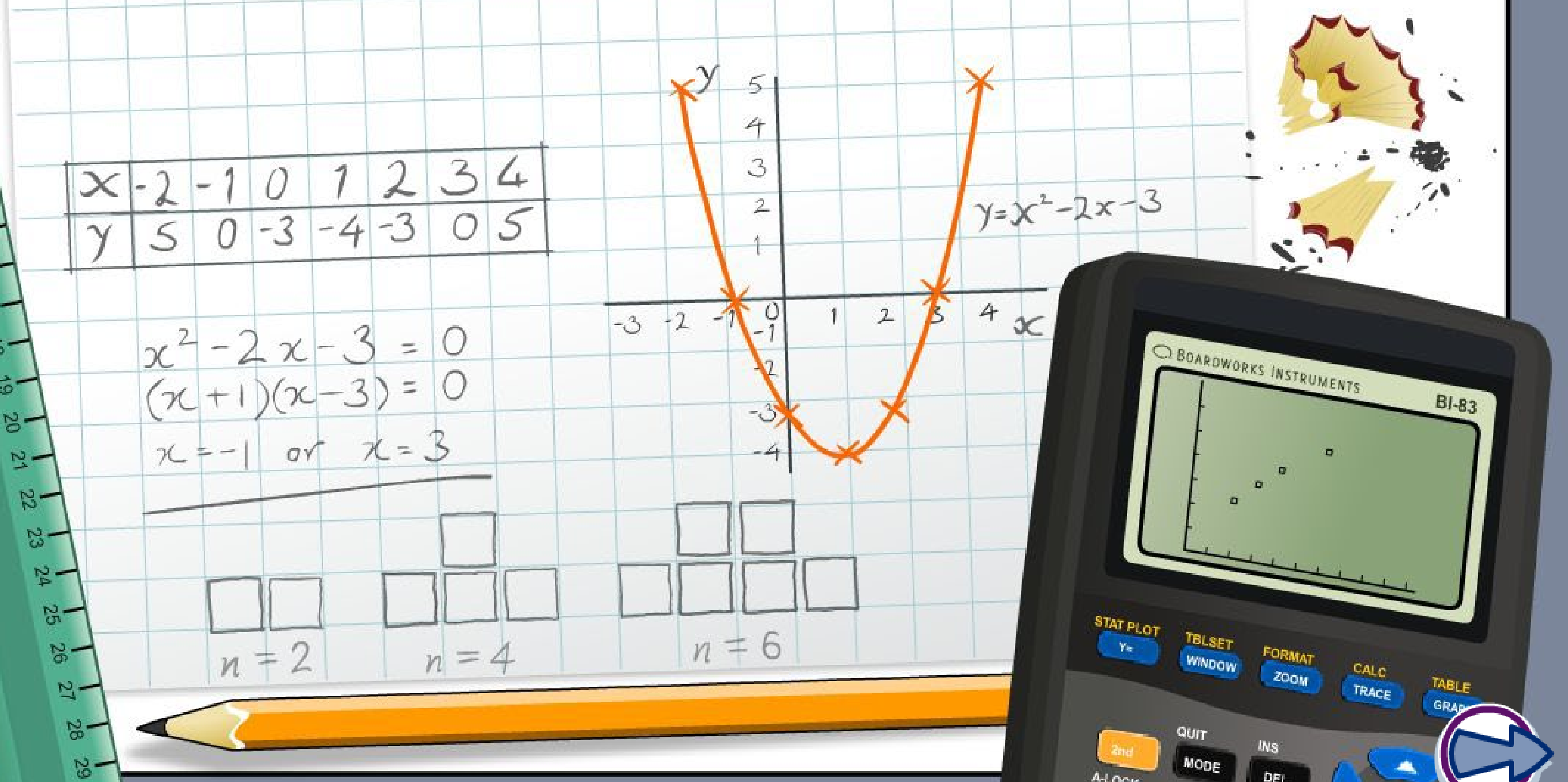
Transforming functions



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

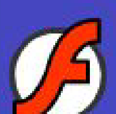
Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

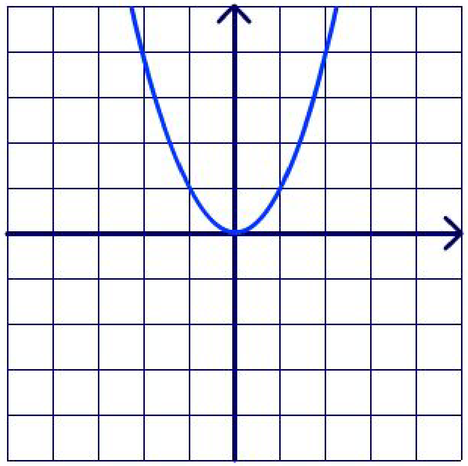
This icon indicates teacher's notes in the Notes field.

Graphs can be transformed by **translating**, **reflecting**, **stretching** or **rotating** them.

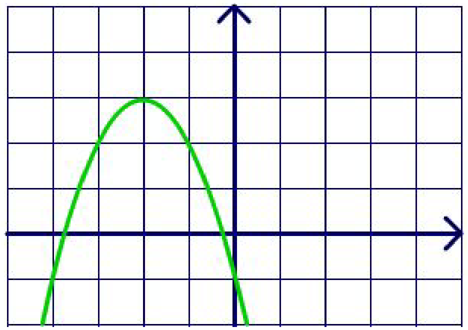
The equation of the transformed graph is related to the equation of the original graph.

When investigating transformations it is useful to distinguish between **functions** and **graphs**.

For example, to investigate transformations of the function $f(x) = x^2$, the equation of the graph of $y = x^2$ can be written as $y = f(x)$.

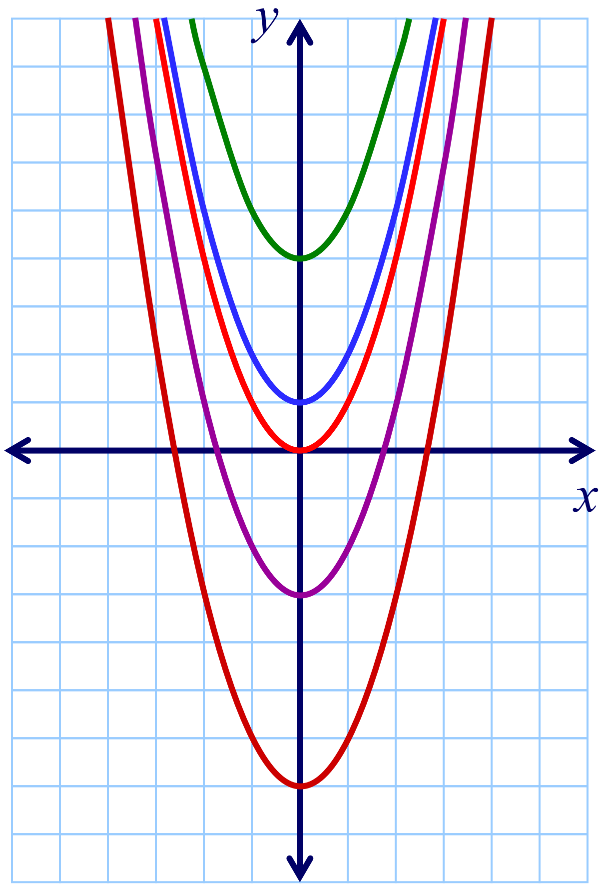


$y = f(x)$



$y = 3 – f(x + 2)$

Here is the graph of $y = x^2$, where $y = f(x)$.



This is the graph of $y = f(x) + 1$

and this is the graph of $y = f(x) + 4$.

What do you notice?

This is the graph of $y = f(x) - 3$

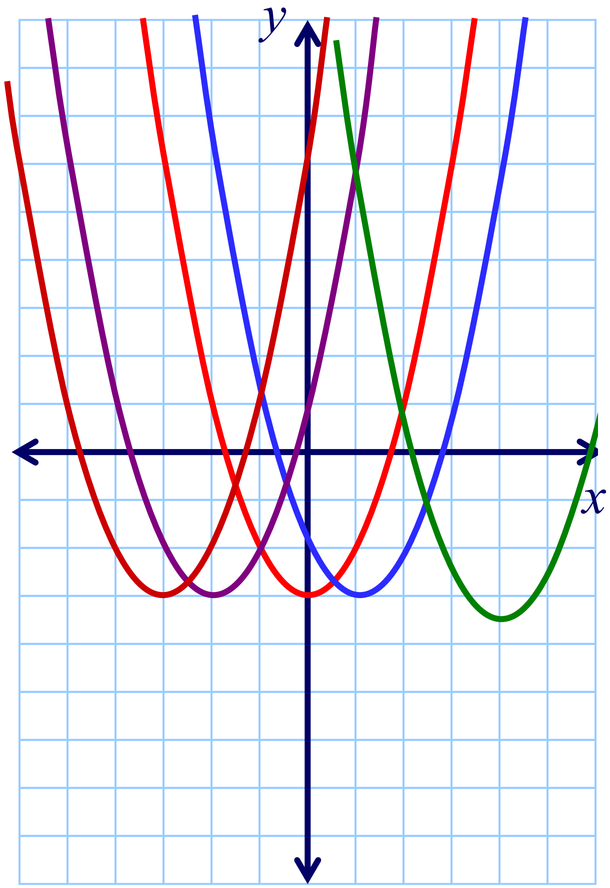
and this is the graph of $y = f(x) - 7$.

What do you notice?

The graph of $y = f(x) + a$ is the graph of $y = f(x)$ translated vertically by a units.

Write a table of values comparing these functions.

Here is the graph of $y = x^2 - 3$, where $y = f(x)$.



This is the graph of $y = f(x - 1)$,

and this is the graph of $y = f(x - 4)$.

What do you notice?

This is the graph of $y = f(x + 2)$,

and this is the graph of $y = f(x + 3)$.

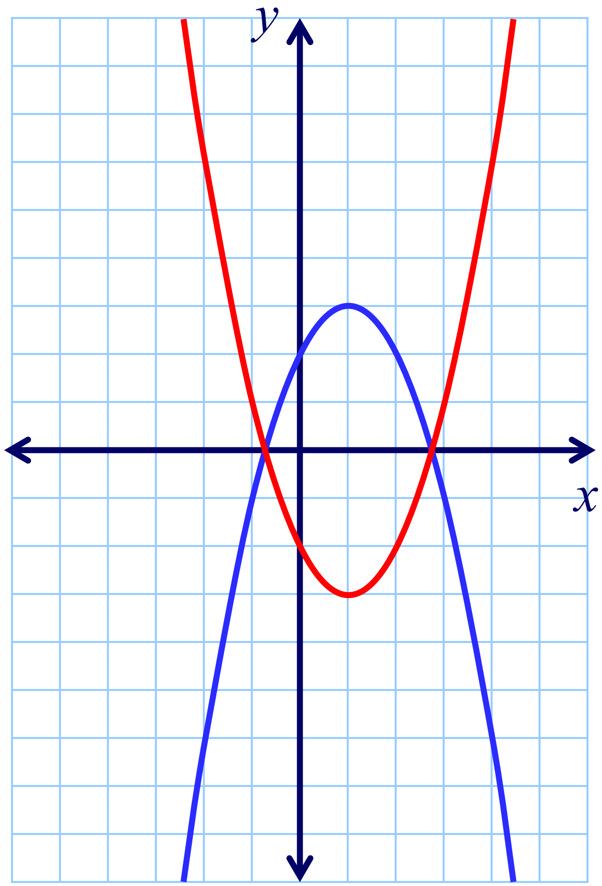
What do you notice?

The graph of $\boldsymbol{y = f(x + a)}$ is the graph of $\boldsymbol{y = f(x)}$ translated horizontally by $-a$ units.

Write a table of values comparing these functions.

Reflections across the x-axis

Here is the graph of $y = x^2 - 2x - 2$, where $y = f(x)$.



This is the graph of $y = -f(x)$.

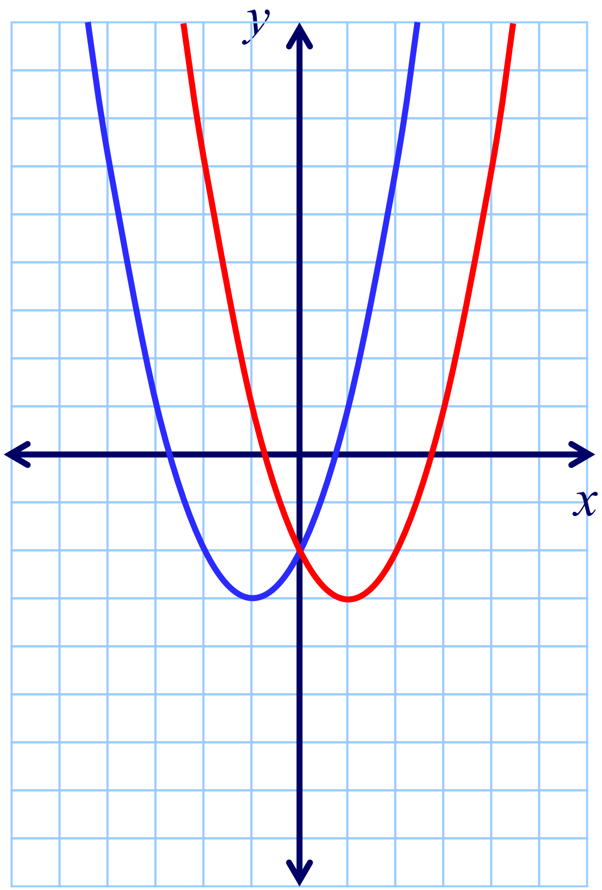
What do you notice?

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis.

Here is the table of values:

x	1	2	3	4	5
$f(x)$	–3	–2	1	6	13
$-f(x)$	3	2	–1	–6	–13

Here is the graph of $y = x^2 – 2x – 2$, where $y = f(x)$.



This is the graph of $y = f(–x)$.

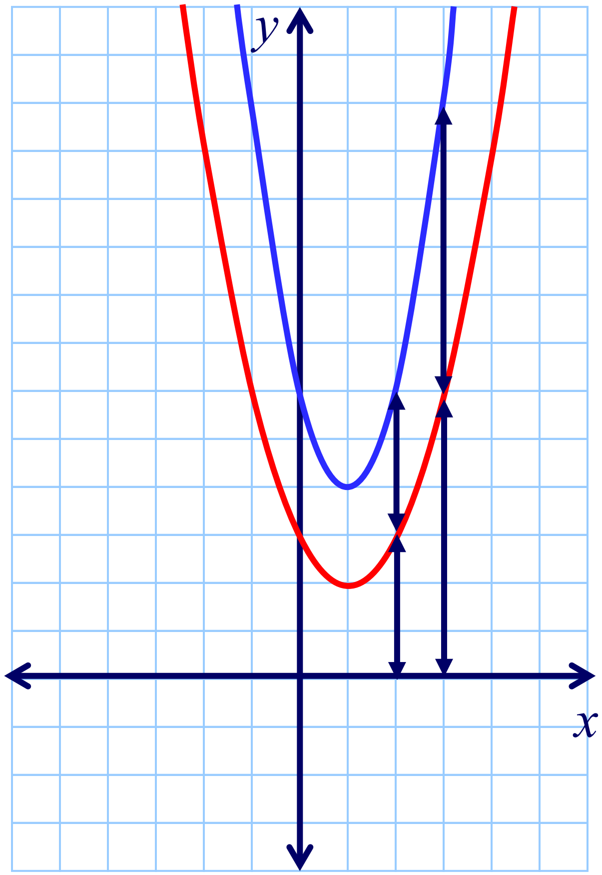
What do you notice?

The graph of $y = f(–x)$ is the graph of $y = f(x)$ reflected across the y-axis.

Here is the table of values:

x	–2	–1	0	1	2
$f(x)$	6	1	–2	–3	–2
$–f(x)$	–2	–3	–2	1	6

Here is the graph of $y = x^2 - 2x + 3$, where $y = f(x)$.



This is the graph of $y = 2f(x)$.

What do you notice?

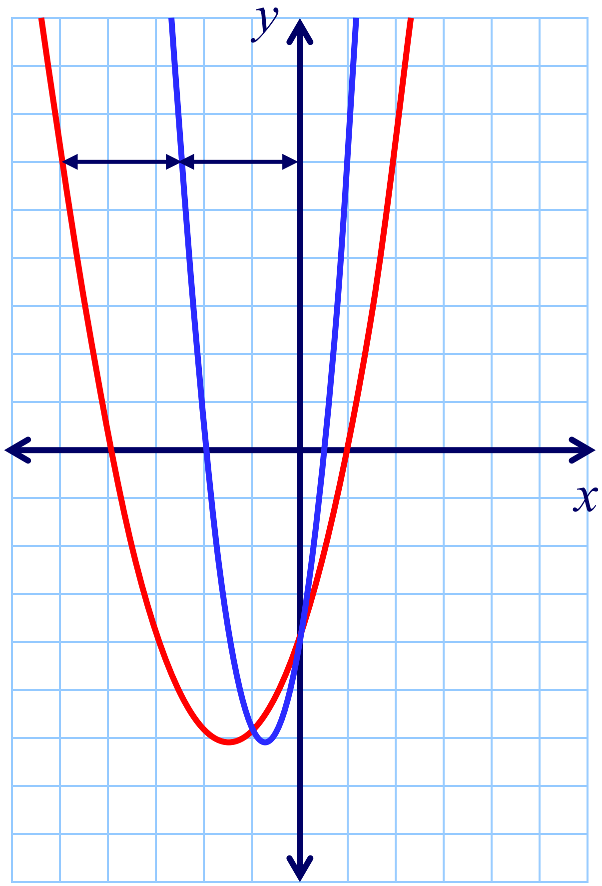
This graph is produced by doubling the y-coordinate of every point on the original graph $y = f(x)$.

This has the effect of **stretching** the graph in the vertical direction.

The graph of $y = af(x)$ is the graph of $y = f(x)$ stretched parallel to the y-axis by scale factor a.

What happens when $a < 1$?

Here is the graph of $y = x^2 + 3x - 4$, where $y = f(x)$.



This is the graph of $y = f(2x)$.

What do you notice?

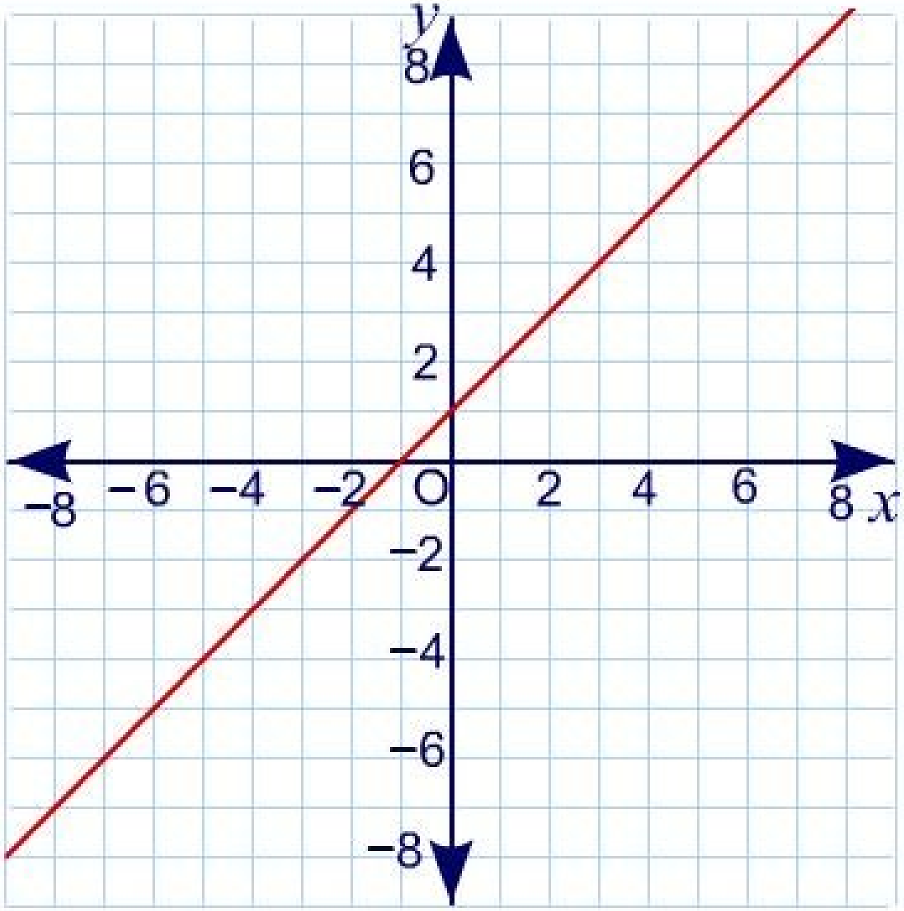
This graph is produced by halving the x-coordinate of every point on the original graph $y = f(x)$.

This has the effect of compressing the graph in the horizontal direction.

The graph of $y = f(ax)$ is the graph of $y = f(x)$ compressed parallel to the x-axis by scale factor $\frac{1}{a}$.

What happens when $a < 1$?

Transforming linear functions



$y =$ x $+ 1$

1 $f(x) + a$

2 $f(x + a)$

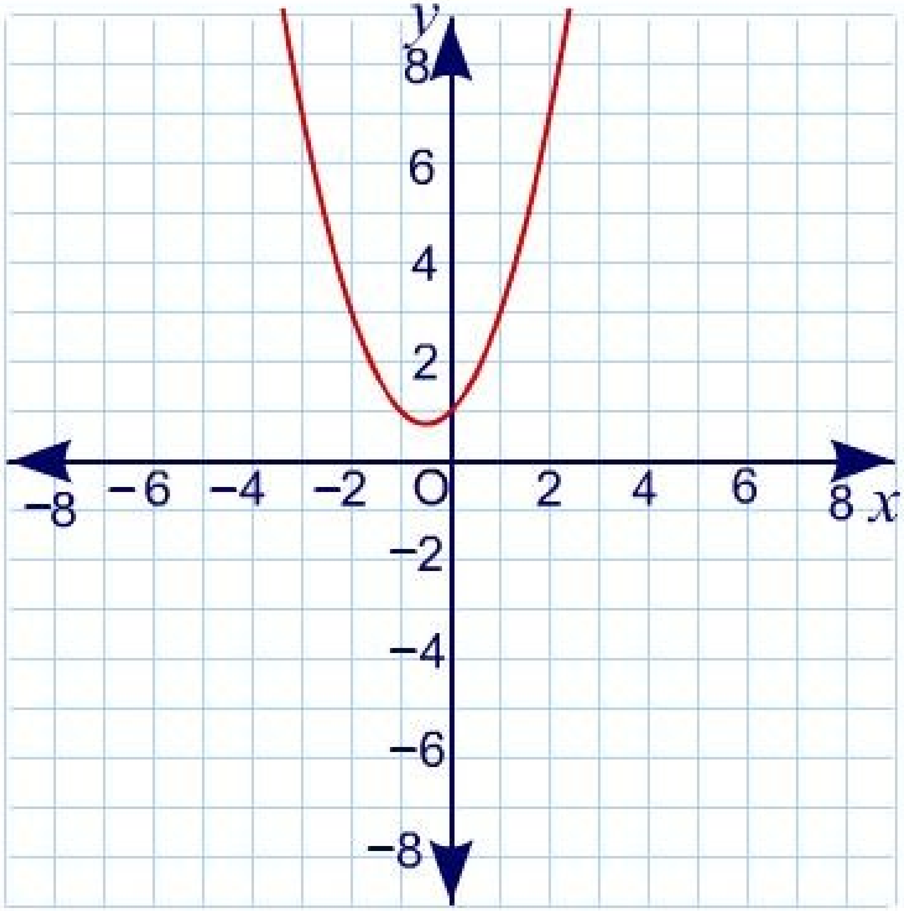
3 $af(x)$

4 $f(ax)$

$a = 0$

$y' = f(x)$

Transforming quadratic functions



$y = + x^2 \; + x \; + 1$

1 $f(x) + a$

2 $f(x + a)$

3 $af(x)$

4 $f(ax)$

$a = 0$

$y' = f(x)$

Match the transformation to the function

a translation by k units

a tr

a re

a re

a

a horizontal shrink by scale factor $^1/_k$

$y = f(-x)$

Match the description of each transformation with the function formed from $y = f(x)$.

Press the info buttons to see an example of each transformation.

Press "**start**" to begin.

start

Combining transformations

We can now look at what happens when we combine any of these transformations.

For example, since all quadratic curves have the same basic shape, any quadratic curve can be obtained by performing a series of transformations on the curve $y = x^2$.

Write down the series of transformations that must be applied to the graph of $y = x^2$ to give the graph $y = 2x^2 + 4x – 1$.

Complete the square to distinguish the transformations:

$$
\begin{aligned}
2x^2 + 4x - 1 &= 2(x^2 + 2x) - 1 \\
&= 2((x + 1)^2 - 1) - 1 \\
&= 2(x + 1)^2 - 3
\end{aligned}
$$

Combining transformations solution

These are the transformations that must be applied to $y = x^2$ to give the graph $y = 2x^2 + 4x - 1$:

$y = x^2$

↓

$y = (x + 1)^2$

↓

$y = 2(x + 1)^2$

↓

$y = 2(x + 1)^2 - 3$

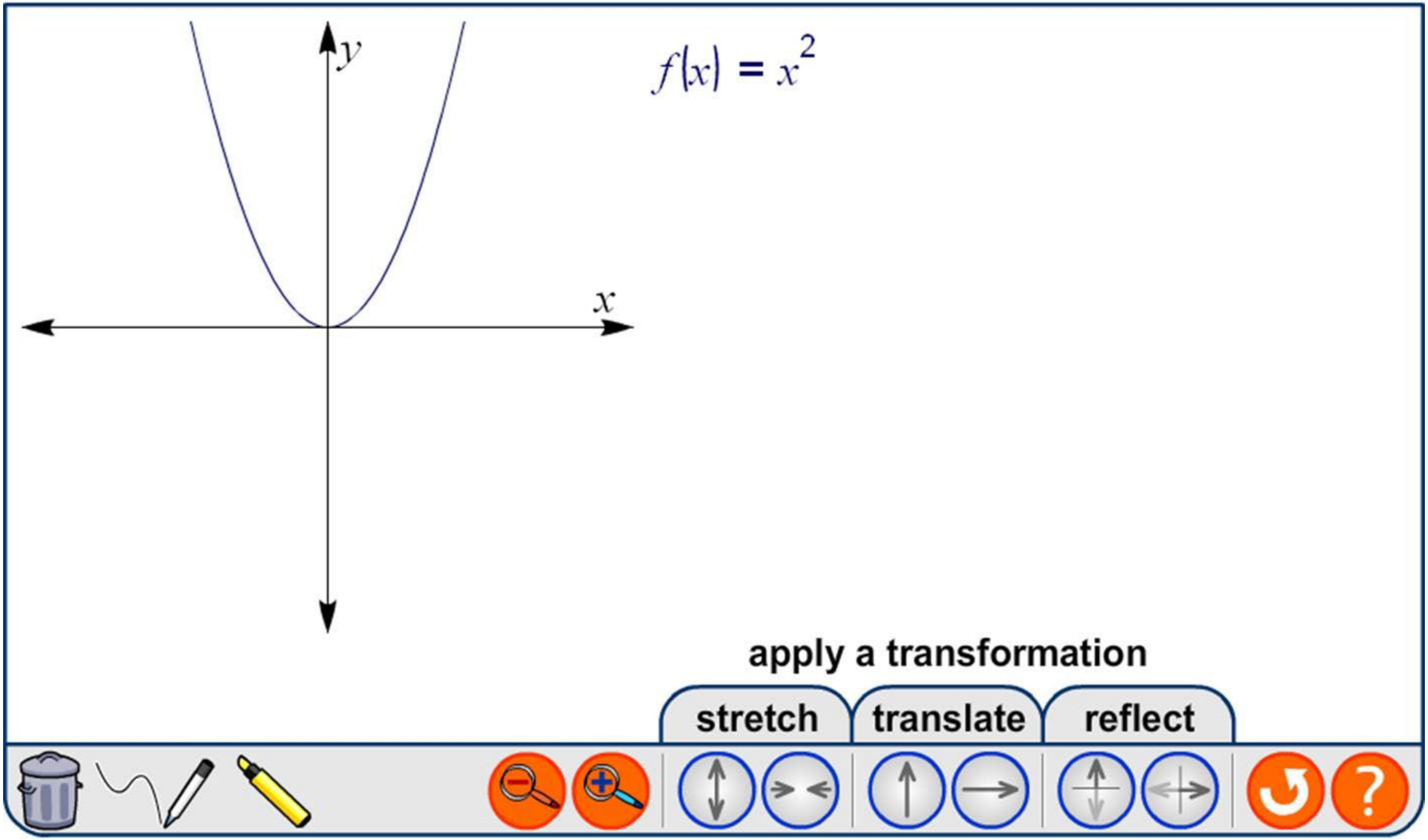
1. Translate –1 units horizontally.

2. Stretch by a scale factor of 2 vertically.

3. Translate –3 units vertically.

These transformations must be performed in the correct order.

Combining transformations



Absolute value functions