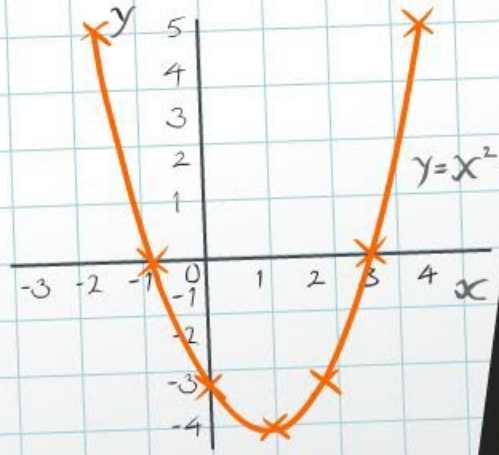
The equation of a straight line

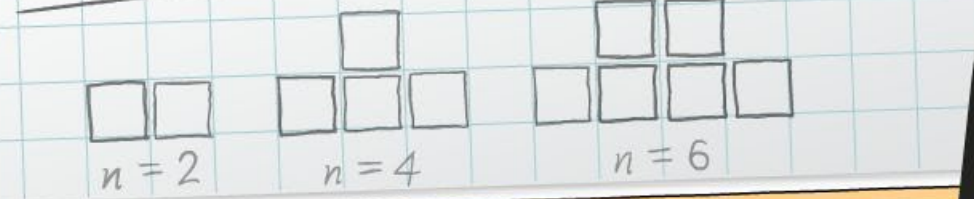
x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

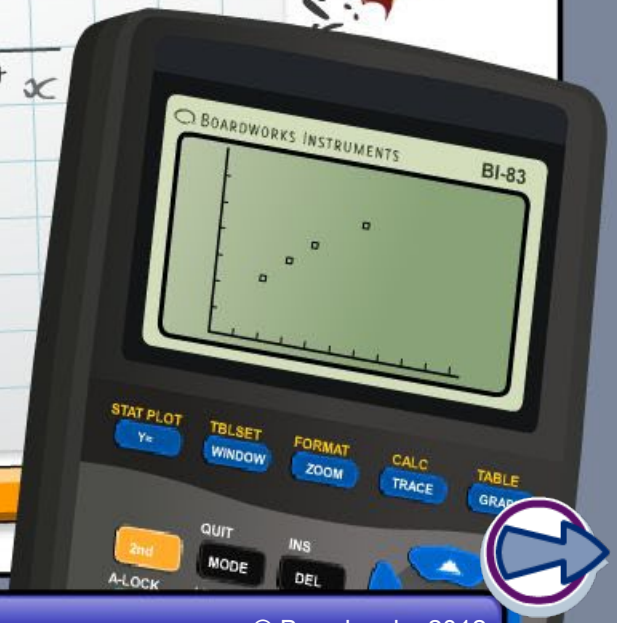
$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$







Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

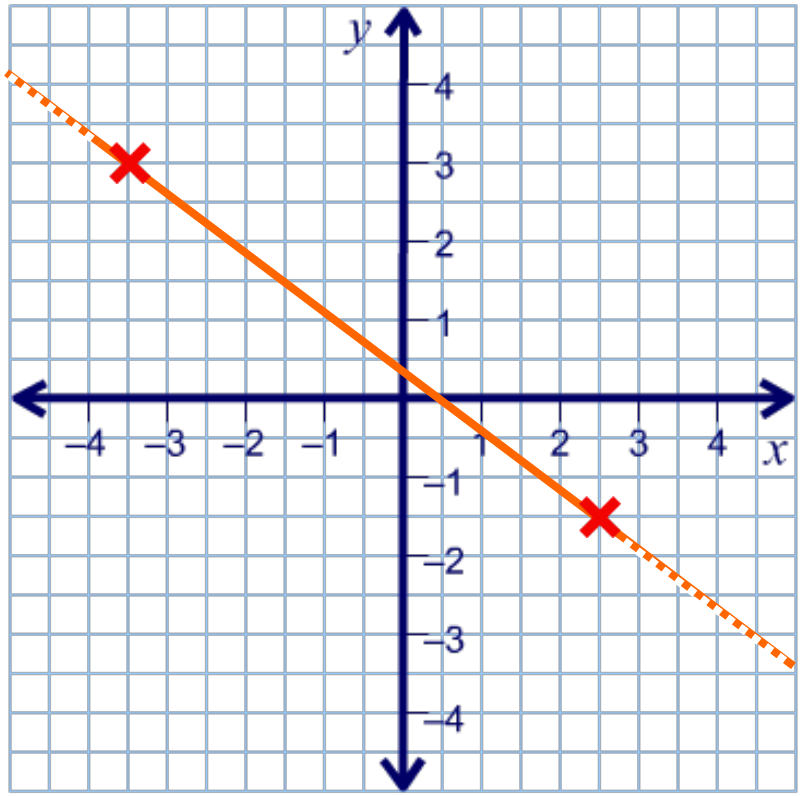
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

What is a line?

A line represents a geometrical object that is:

- **straight**
- **infinitely long**
- **infinitely thin**.

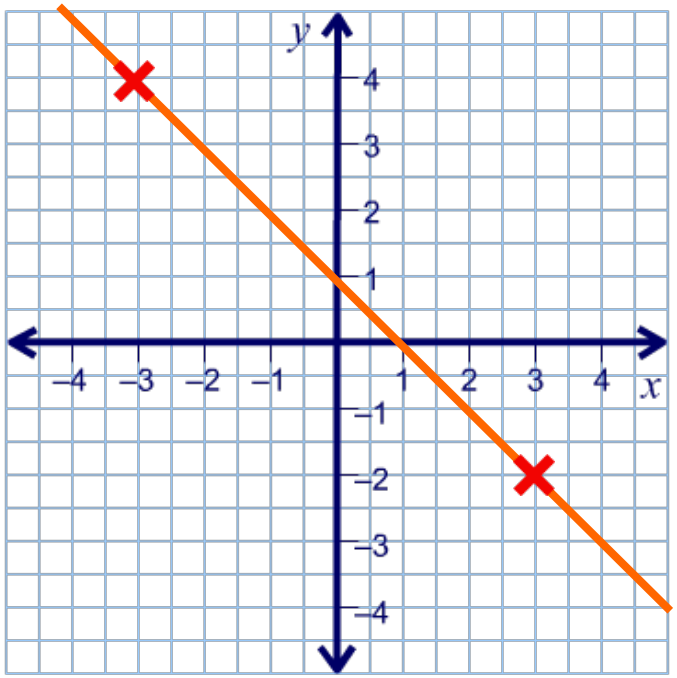


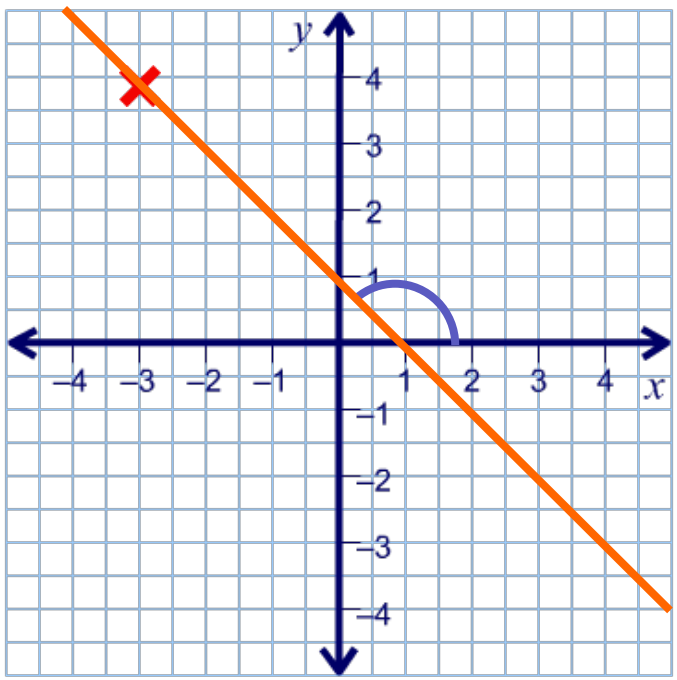
A **line segment** represents a geometrical object that is straight and infinitely thin, but has a **finite length**.

How to define a line

A line can be identified using two pieces of information:

- the coordinates of two points on the line
- one point on the line and the slope of the line.

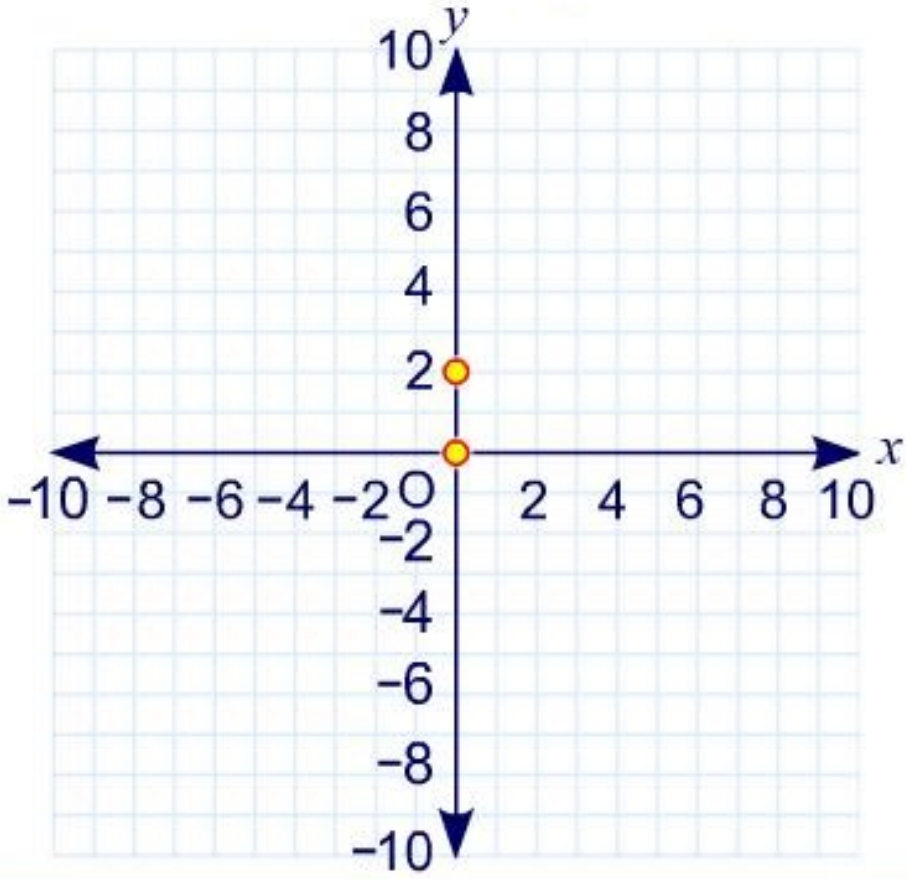




If you don't have at least two pieces of information, you cannot specify a unique line.

Understanding lines

Defining lines with points and slope



Set point 1

Set point 2

Set slope

An equation of a straight line

A line can be represented algebraically in three main ways.

slope-intercept form

$$y = mx + b$$

m is the **slope** of the line

b is the y-**intercept**

standard form

$$ax + by + c = 0$$

point-slope form

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) is a point on the line

Slope-intercept form

If we are given the slope of the line and a point on the y axis, we can write an equation of that line directly using $y = mx + b$.

For example:

A line passes through the point (0, –4) and has a slope of 3. Write an equation for that line.

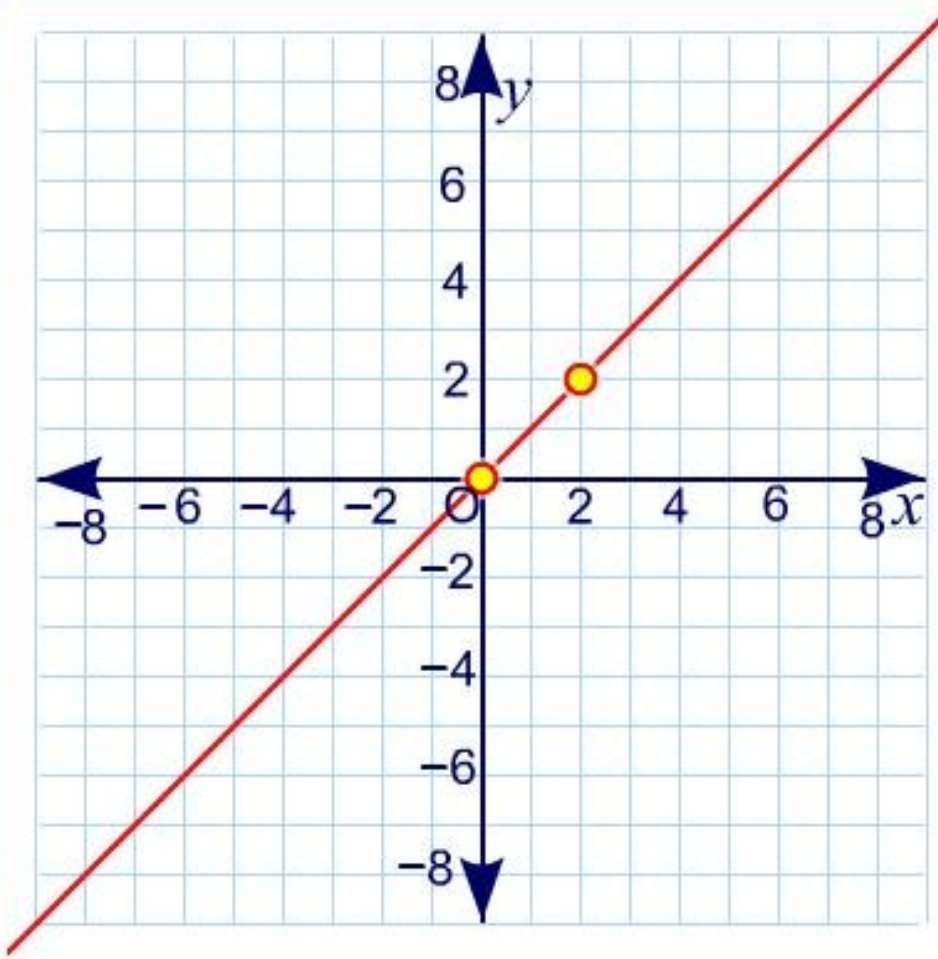
$y = mx + b$

$m = 3$

$b = -4$

$\mathbf{y = 3x - 4}$

Using slope-intercept form



Practice finding the equation of the line. Adjust the line by dragging the yellow dots.

y-intercept:

slope:

equation:

Equations, slopes and y-intercepts

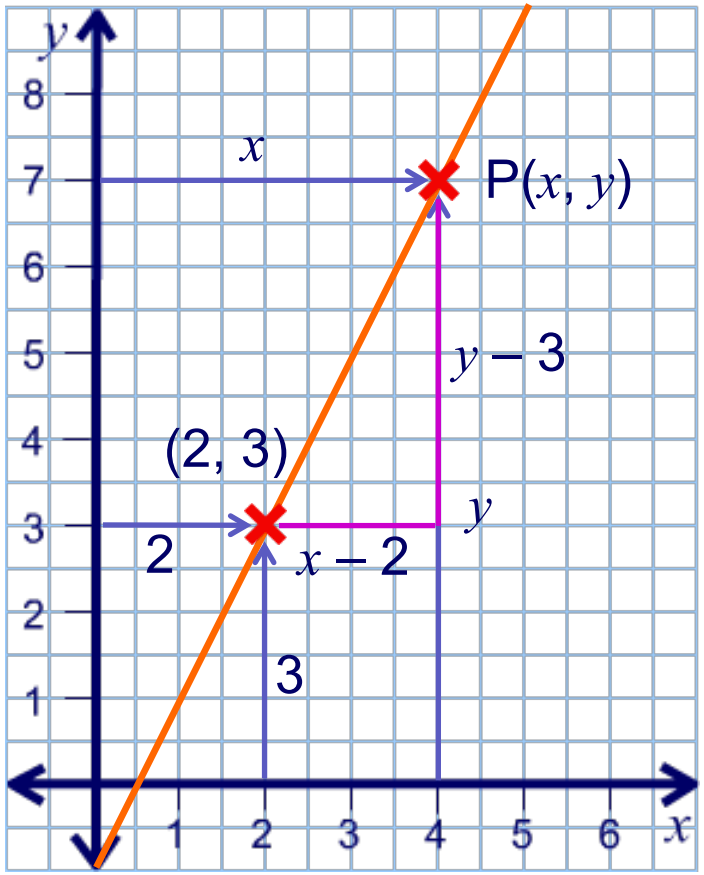
Link the equation with its slope and y-intercept.

equation	slope	y-intercept
$y = 5x - 3$	1	$(0, -3)$
$y = \frac{x}{2} - 5$	-3	$(0, 0)$
$y = 2 - 3x$	-2	$(0, -5)$
$y = x$	5	$(0, \frac{1}{2})$
$y = -2x + \frac{1}{2}$	$\frac{1}{2}$	$(0, 2)$

Using a point and the slope

Suppose we are given the slope of a line but that the point given is not the y-intercept.

A line passes through the point (2, 3) and has a slope of 2. Write an equation for that line.



Let P(x, y) be any point on the line.

The question tells us that slope = 2.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y - 3}{x - 2}$$

$$2 = \frac{y - 3}{x - 2}$$

Finding an equation of a line

A line passes through the point (2, 3) and has a slope of 2. Write an equation for that line.

$$2 = \frac{y - 3}{x - 2}$$

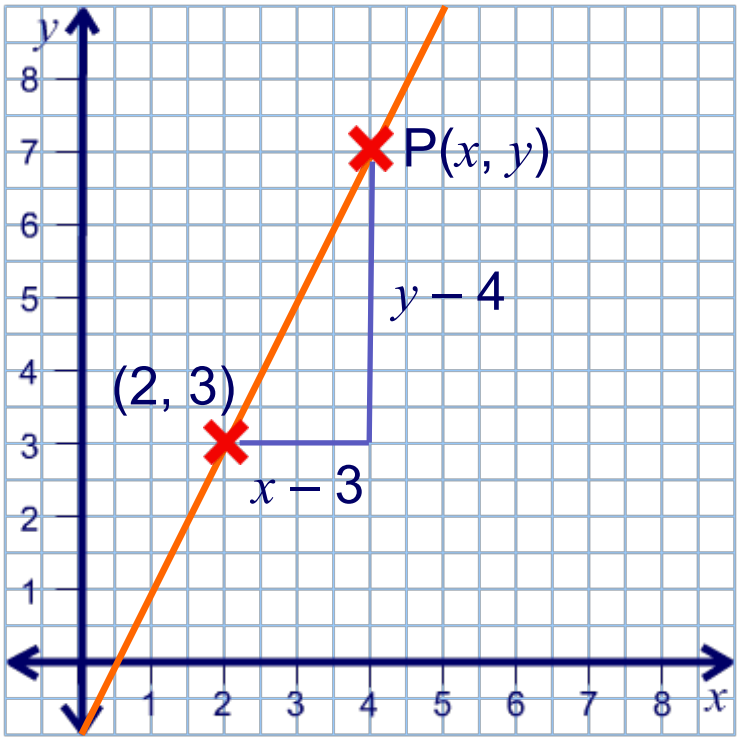
Rearranging:

$$2(x - 2) = y - 3$$

$$2x - 4 = y - 3$$

$$2x - 1 = y$$

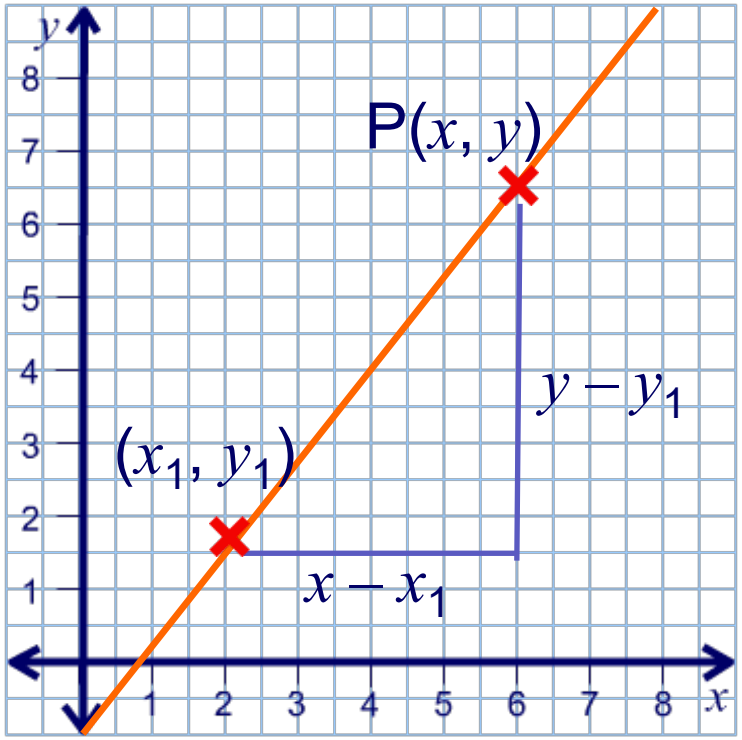
$$\mathbf{y = 2x - 1}$$



An equation of the line passing through the point (2, 3) with a slope of 2 is $y = 2x - 1$.

Point-slope form

In general, for a line that passes through (x_1, y_1) with slope m:



$$m = \frac{y - y_1}{x - x_1}$$

This can be rearranged to:

$$y - y_1 = m(x - x_1)$$

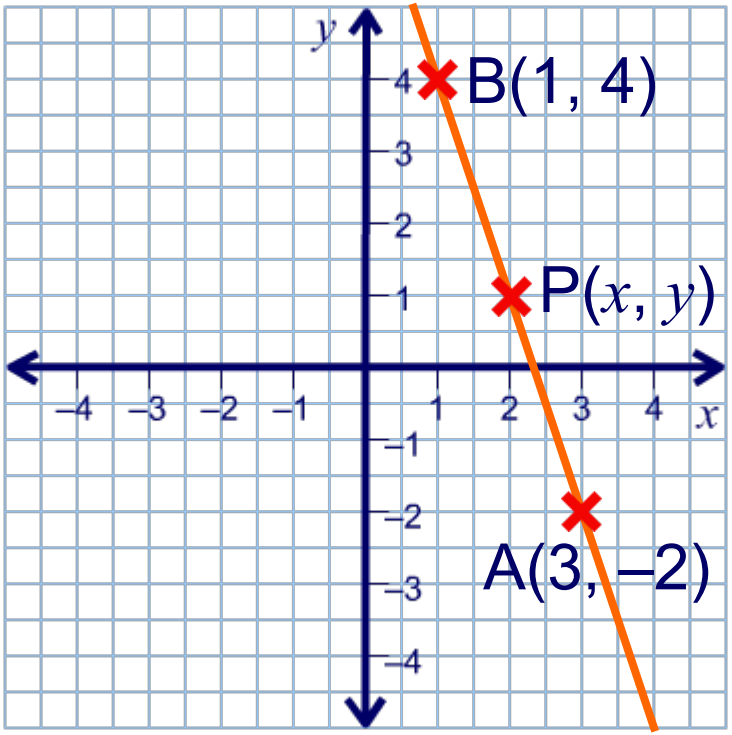
$$\mathbf{y - y_1 = m(x - x_1)}$$

This is called the **point-slope form** of a line.

Using two points

A line passes through the points A(3, –2) and B(1, 4). Write an equation for that line.

Let P(x, y) be any other point on the line.



The slope of AP, $m_{AP} = \dfrac{y-(-2)}{x-3}$

The slope of AB, $m_{AB} = \dfrac{4-(-2)}{1-3}$

AP and AB are parts of the same line so their slopes must be equal.

$$\frac{4-(-2)}{1-3} = \frac{y-(-2)}{x-3}$$

Using two points solution

$$\frac{4-(-2)}{1-3} = \frac{y-(-2)}{x-3}$$

Rearrange the equation:

$$\frac{6}{-2} = \frac{y+2}{x-3}$$

$$-3(x-3) = y+2$$

$$9-3x = y+2$$

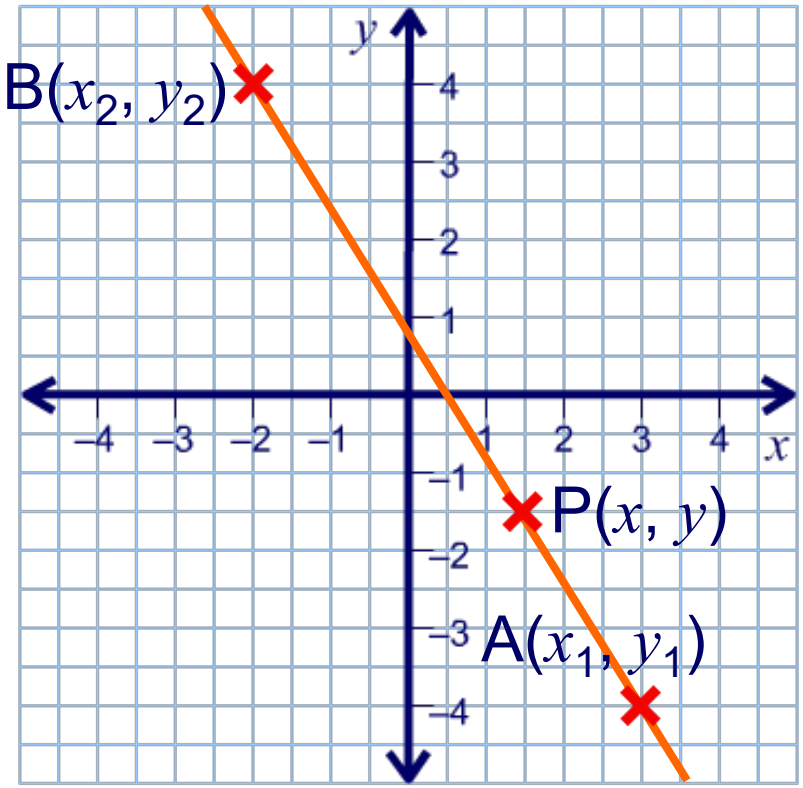
$$\mathbf{y = -3x + 7}$$

This can also be written as: $y = 7 - 3x$

An equation of the line passing through the points A(3, –2) and B(1, 4) is $\mathbf{y = -3x + 7}$.

The general case

A straight line passes through the points A(x_1, y_1) and B(x_2, y_2).



Let P(x, y) be any point on the line.

slope of AP = slope of AB.

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Rearranging: $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

An equation of a line through A(x_1, y_1) and B(x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

The standard form

The **standard form** of an equation of a line is:

$$ax + by + c = 0$$

b is different from the b in $y = mx + b$

When do you think this form would be more useful than $y = mx + b$?

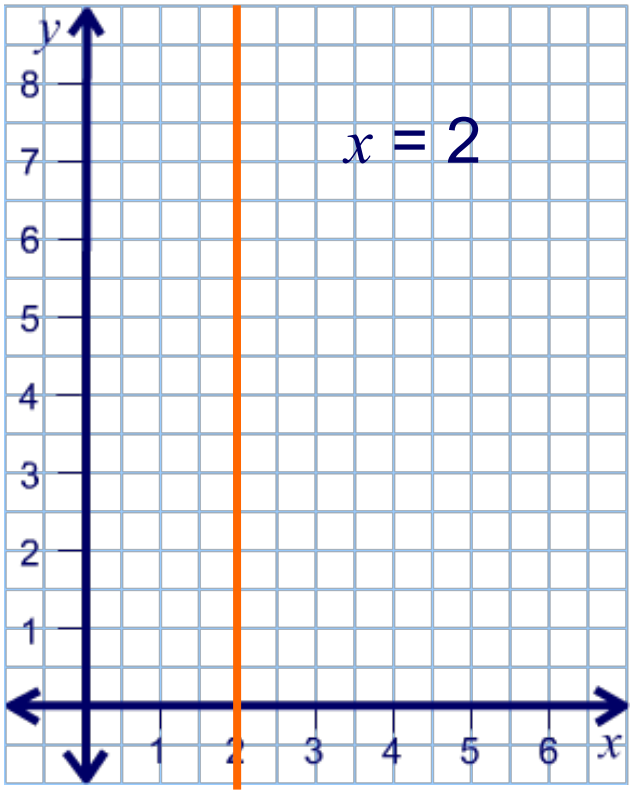
This form can be useful when the slope and y-intercept are fractions.

For example, the equation $y = \frac{3}{4}x - \frac{1}{2}$ can be rewritten as:

$$4y - 3x + 2 = 0$$

The standard form

Can you think of any lines that can be written in the form $ax + by + c = 0$ but cannot be written in the form $y = mx + b$?



Equations of the form $x = c$ can be written in the form $ax + by + c = 0$ but cannot be written in the form $y = mx + b$.

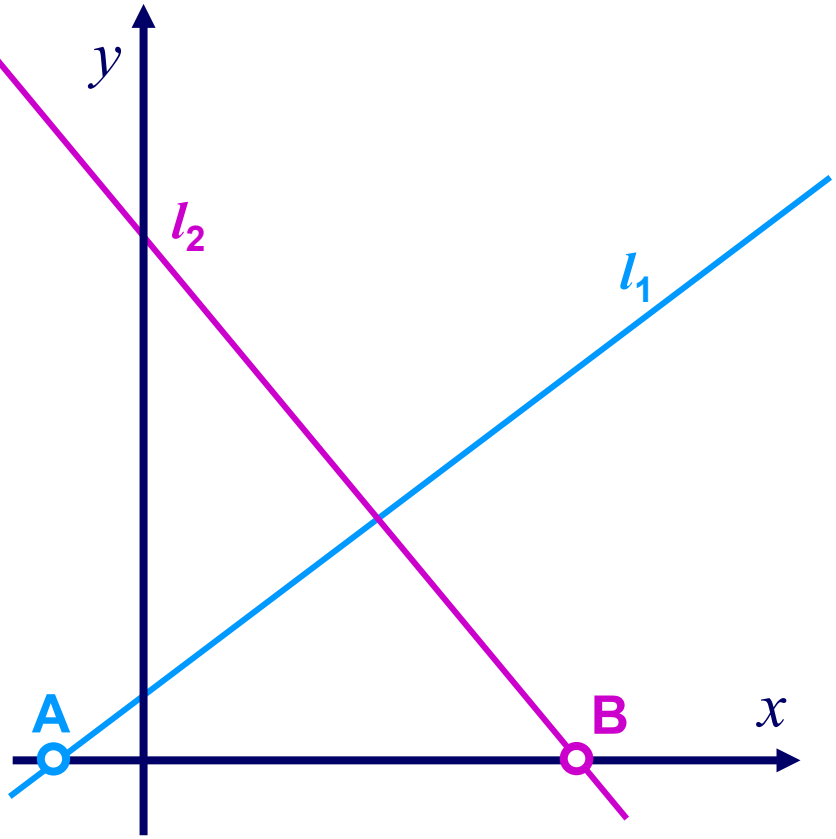
Any straight line can be written in the form $ax + by + c = 0$.

Comparing equations

Link the equations that describe the same line

slope-intercept form	point-slope form	standard form
$y = 2x + 1$	$y - 9 = 2(x - 4)$	$x - y - 4 = 0$
$y = \frac{x}{3} + 6$	$y + 2 = 1(x - 2)$	$5x + y - 3 = 0$
$y = x - 4$	$y + 22 = -5(x - 5)$	$x - 3y + 18 = 0$
$y = -5x + 3$	$y - 5 = \frac{1}{3}(x + 3)$	$x + 2y + 1 = 0$
$y = -\frac{x}{2} - \frac{1}{2}$	$y = -\frac{1}{2}(x + 1)$	$2x - y + 1 = 0$

Examination-style question



The line l_1 in the diagram has the equation:

$$3x - 4y + 6 = 0$$

The lines l_1 and l_2 cross the x-axis at the points A and B respectively.

The line l_2 is perpendicular to the line l_1 and passes through the point (2, 4).

a) Find an equation of the line l_2.

b) Find the length of AB.

Examination-style answers – part a)

Rearrange the equation of l_1 to the form $y = mx + b$:

$$3x - 4y + 6 = 0$$

$$4y = 3x + 6$$

$$y = \frac{3}{4}x + \frac{3}{2}$$

What is the slope of l_1? $\frac{3}{4}$

Since l_2 is perpendicular to l_1, what is the slope of l_2? $-\frac{4}{3}$

Substitute this slope and the point (2, 4) into $y - y_1 = m(x - x_1)$ to write the equation of l_2:

$$y - 4 = -\tfrac{4}{3}(x - 2)$$

$$3y - 12 = -4x + 8$$

$$\mathbf{4x + 3y - 20 = 0}$$

Examination-style answers – part b)

The point A lies on the line with equation $3x - 4y + 6 = 0$.

What is x when $y = 0$?

$3x + 6 = 0$

$x = -2$

So, what are the coordinates of point A? $(-2, 0)$

The point B lies on the line with equation $4x + 3y - 20 = 0$.

What is x when $y = 0$?

$4x - 20 = 0$

$x = 5$

So, what are the coordinates of point B? $(5, 0)$

What is the length of AB? $x_B - x_A = 5 - (-2)$ **$= 7$**

Practice activity

Straight line summary

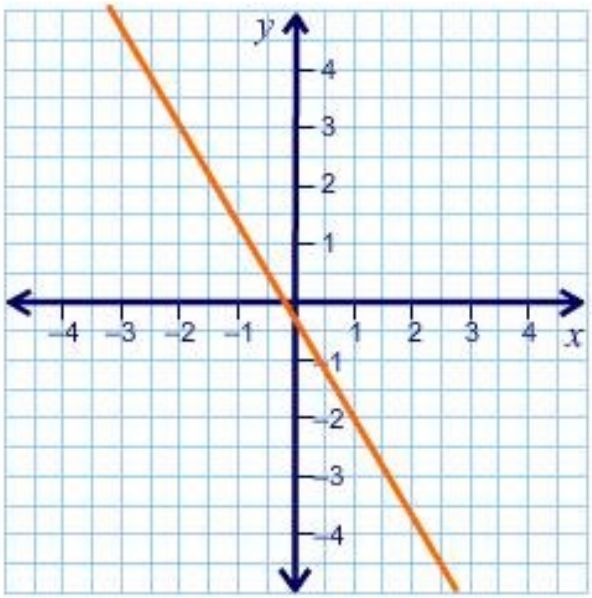
1) What is the definition of a line?

2) What is the difference between a line and line segment?

3) What are three forms of the equation of a straight line?

4) What is the equation of a line that passes through (x_1, y_1) with slope m?

5) What is the equation of a line through A(x_1, y_1) and B(x_2, y_2)?



press the questions to see the answers