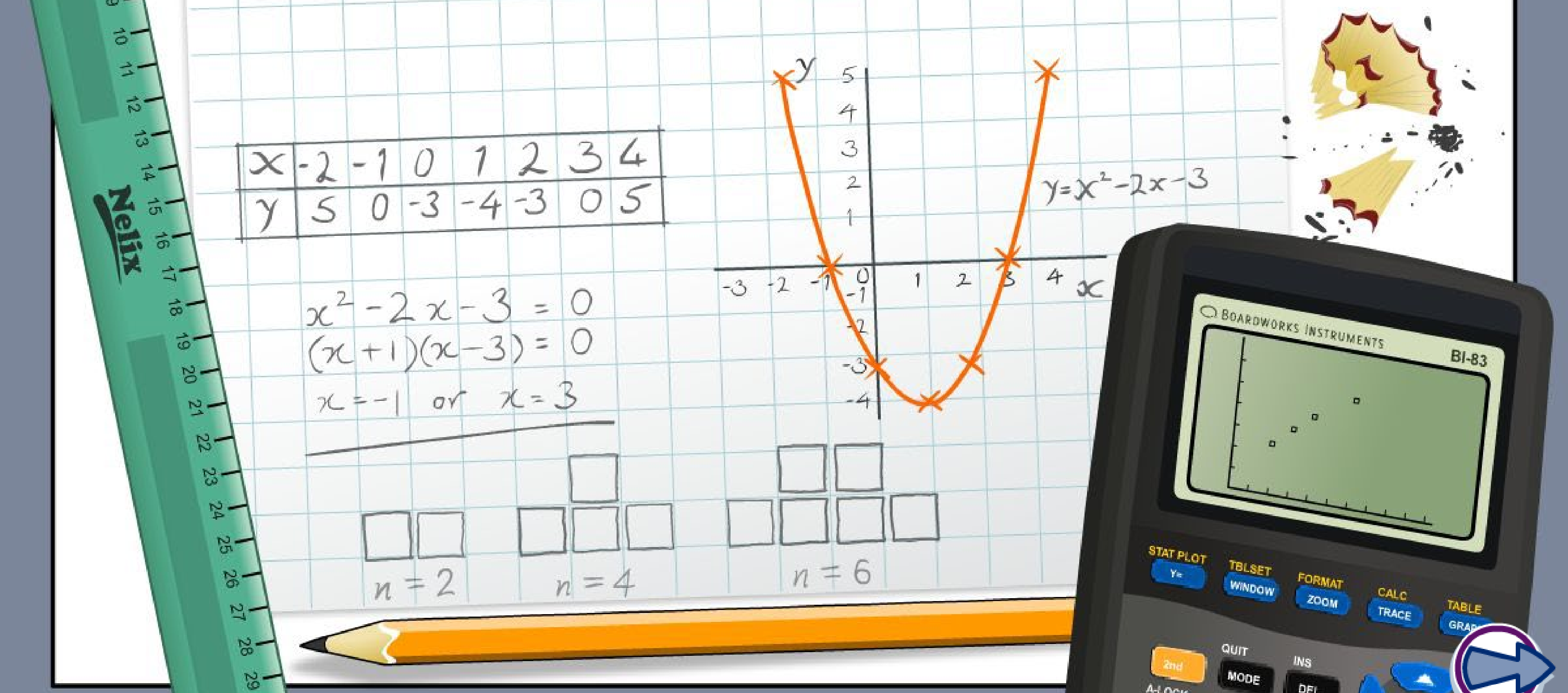
Systems of equations and the substitution method



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Satisfying equations

Suppose that $x + y = 3$.

How many pairs of values make this equation true?

$x = 1$ and $y = 2$, $x = 3$ and $y = 0$, $x = –2$ and $y = 5$… and so on.

Equations that contain two unknowns have an **infinite** number of solution pairs.

Suppose we have **two** of these sorts of linear equations in two variables, for example, $x + y = 3$ and $y – x = 1$.

How many pairs of values satisfy *both* equations?

There is just **one** solution pair: x **= 1** and y **= 2**.

Systems of equations

When we have two or more equations containing the same unknowns we call them a **system of equations**.

We know how to solve an equation in *one* variable; we use inverse operations.

Suppose that $y = 2x - 3$ and $x + 2y = 14$.
It is difficult to see the solution to these two equations.
Can you think of a method to help us find it?

One method we can use to easily solve a system of two equations is by **substituting** one equation into the other.

This gives us *one* equation in *one* variable, which we know how to solve, then we can easily find the second variable.

The substitution method

We can solve $y = 2x - 3$ and $x + 2y = 14$ by **substitution**.

Label the starting equations:

$y = 2x - 3$ (A)

$x + 2y = 14$ (B)

Equation (A) gives y in terms of x.

Substitute equation (A) into equation (B) to give *one* equation in x:

$$x + 2(\mathbf{2x - 3}) = 14$$

distributive property: $x + 4x - 6 = 14$

simplify: $5x - 6 = 14$

add 9 to both sides: $5x = 20$

divide both sides by 8: $\mathbf{x = 4}$

The substitution method

We now need to find the value of y when x **= 4**. Substitute this value into one of the original equations: $y = 2x - 3$.

$y = 2(4) - 3$

$\mathbf{y = 5}$

Our solution is: $\mathbf{x = 4, y = 5}$.

Why do you think it is important to substitute into the *other* equation to check your solution?

Check these answers by substituting $x = 4$ and $y = 5$ into the other equation: $x + 2y = 14$

$(4) + 2(5) = 14$

$4 + 10 = 14$ ✓

This is true, so we know that our solutions are correct.

How could the following system of equations be solved using substitution?

$$3x - y = 9 \quad (A)$$
$$8x + 5y = 1 \quad (B)$$

One of the equations needs to be arranged in the form $x = \ldots$ or $y = \ldots$ before it can be substituted into the other equation.

Rearrange equation (A):

	$3x - y = 9$
add y :	$3x = 9 + y$
subtract 9 :	$3x - 9 = y$
	$y = 3x - 9$

The substitution method

Now we have: $y = 3x - 9$ (A)

$8x + 5y = 1$ (B)

We can now substitute $y = \mathbf{3x - 9}$ into equation (B).

	$8x + 5(\mathbf{3x - 9}) = 1$
distributive property:	$8x + 15x - 45 = 1$
simplify:	$23x - 45 = 1$
add 45:	$23x = 46$
divide by 23:	$\mathbf{x = 2}$

Substitute $\mathbf{x = 2}$ into equation (A) to find the value of y.

	$3(\mathbf{2}) - y = 9$
simplify:	$6 - y = 9$
subtract 6:	$-y = 3$
multiply by –1:	$\mathbf{y = -3}$

The substitution method

Our equations: $y = 3x - 9$ (A)

$8x + 5y = 1$ (B)

Check the solutions $\boldsymbol{x} = \mathbf{2}$ and $\boldsymbol{y} = \mathbf{-3}$ by substituting them into equation (B).

$$8(2) + 5(-3) = 1$$

$$16 - 15 = 1$$ ✓

This is true, so we know that our solutions are correct.

Solve these systems of equations by substituting one into the other.

$$3x + 6y = -15$$

$$x + 10y = 11$$

$x =$

$y =$

Practice questions

Practice questions: the substitution method

1.	Solve $y = -x + 6$ and $2x + 5y = 9$.	? W
2.	Solve $y - x = 6$ and $3y + 5x = -6$.	? W
3.	Solve $2x - 3y = 4$ and $x + 2y = -5$.	? W
4.	Solve $3x - y = 6$ and $x + 5y = -14$.	? W
5.	Solve $4x + y = 13$ and $x + 3y = -5$.	? W

Special cases

Case 1

Case 2

BE CAREFUL!

Not all systems of equations have one solution.

Press on each of the tabs above to see an example of other cases to look out for.

Basketball quiz

Practice identifying solutions to systems of equations in this team quiz! Get into two teams: A and B.

Each team will be represented by a basketball player. If your team answers a question correctly, your player scores a point. The team with the highest score wins!

Press **start** to begin.

start