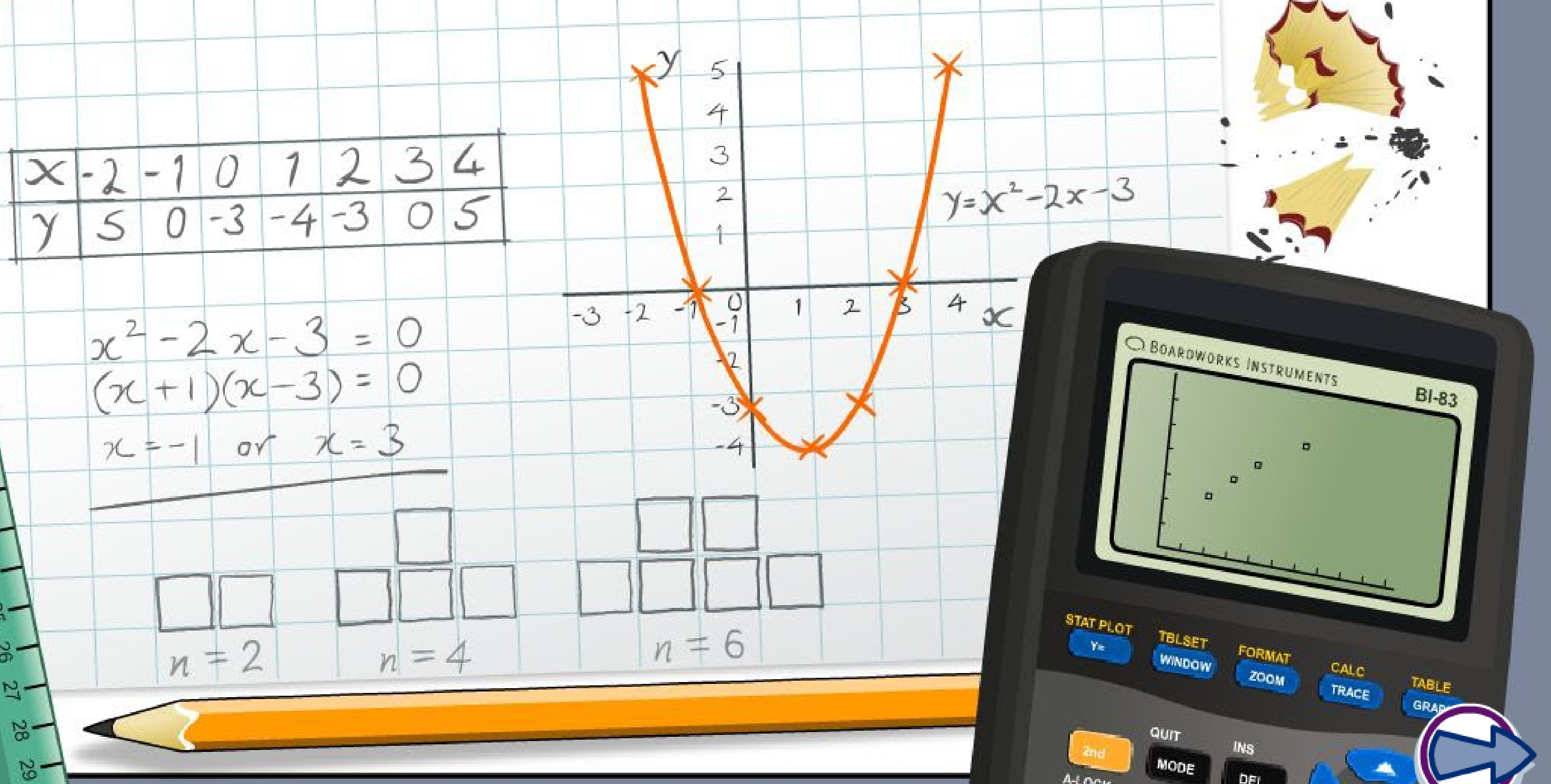
Solving linear equations



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Solving linear equations

Linear equations are the easiest type of equation to solve because the unknown is not raised to any power other than 1.

We can solve very simple linear equations by inspection.

For example:

$19 - x = 8$

$\boldsymbol{x = 11}$

We think of this as: "What number subtracted from 19 gives us an answer of 8?"

$7x = 42$

$\boldsymbol{x = 6}$

We think of this as: "What number multiplied by 7 gives us an answer of 42?"

Using inverse operations

For more tricky equations, use **inverse operations** to solve the equation in several steps.

Perform the same operations on both sides of the equals sign to keep the equation balanced. Aim to get the unknown (or "variable") on one side and a number on the other.

For example: $4x + 5 = 29$

subtract 5 from both sides: $-5 \quad -5$

$4x = 24$

divide both sides by 4: $\div 4 \quad \div 4$

$x = 6$

Check that 4 × 6 + 5 is equal to 29 in the original equation to verify your answer.

Unknowns on both sides

In some cases the unknown appears on both sides of the equals sign.

We must combine **like terms** by performing inverse operations as normal. Aim to get unknowns on the left-hand side of the equation and numbers on the right.

	unknowns		numbers
For example:	$8x - 2$	=	$2x + 1$

Here, we'll label the sides as "unknowns" and "numbers."

We always aim to finish with the unknown on its own on the left and a number on the right.

Unknowns on both sides

The equation can be solved by performing the same operations on both sides until the solution is found.

unknowns numbers

$$8x - 2 = 2x + 1$$

add 2 to both sides: **+ 2** **+ 2**

$$8x = 2x + 3$$

subtract $2x$ from both sides: **– $2x$** **– $2x$**

$$6x = 3$$

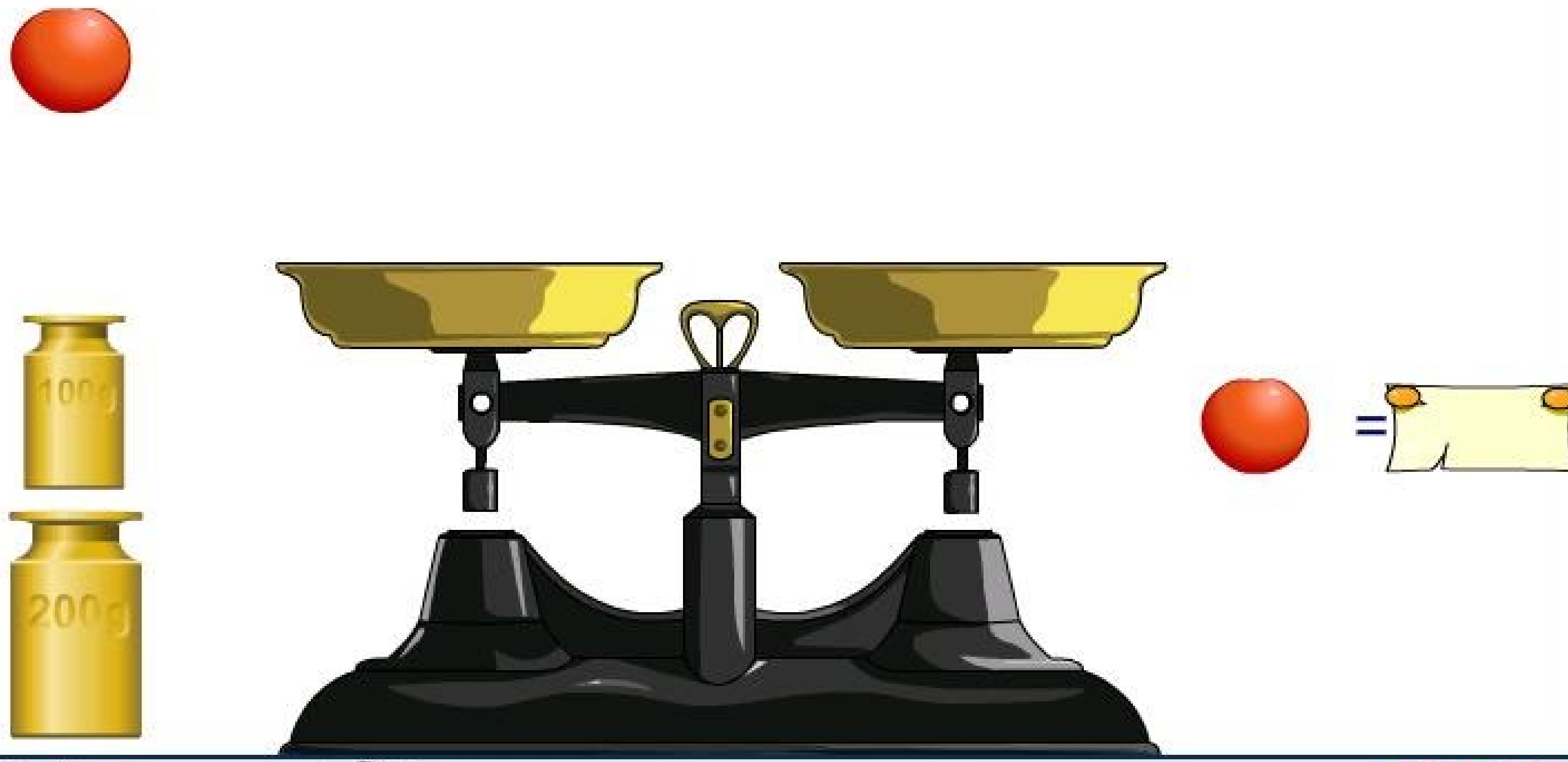
divide both sides by 6: **÷ 6** **÷ 6**

$$\boldsymbol{x = 0.5}$$

Check by substituting $x = 0.5$ into the original equation. Both sides are equal to 2, so the solution is correct.

Balance the scale

Use the scales to help find the mass of the tomatoes.



Careful, don't slip up!

Look at this equation: $(x - 1) = 2(x - 1)$

divide both sides by $(x - 1)$: $1 = 2$

This is false. What has gone wrong? Explain your answer.

Notice that the solution to this equation is $x = 1$, so when we divided by $(x - 1)$, we in fact divided by zero. This is not allowed.

Next time someone "proves" to you that 1 = 0, watch out! They probably divided by zero.

Equation solving 1

Use the red arrows to select the operation to apply to both sides of the given equation. Apply the operation by pressing the red button. Use the buttons underneath to multiply out parentheses, get a new equation, clear previous operations or swap the sides of the equation.

Each "type" of equation gets progressively harder, from Type 1 equations (involving a single step to solve) to Type 4 equations (involving multiple steps, fractions and parentheses).

Press **start** to begin.

start

Equations with parentheses

Equations can contain parentheses.

For example: $3(2x - 1) = 7x$

distributive property: $6x - 3 = 7x$

subtract $7x$ from both sides: $-x - 3 = 0$

add 3 to both sides: $-x = 3$

multiply by –1: $\boldsymbol{x = -3}$

Equations with parentheses

Example: $2(3x - 5) = 4x$

distributive property: $6x - 10 = 4x$

add 10 to both sides: $6x = 4x + 10$

subtract $4x$ from both sides: $2x = 10$

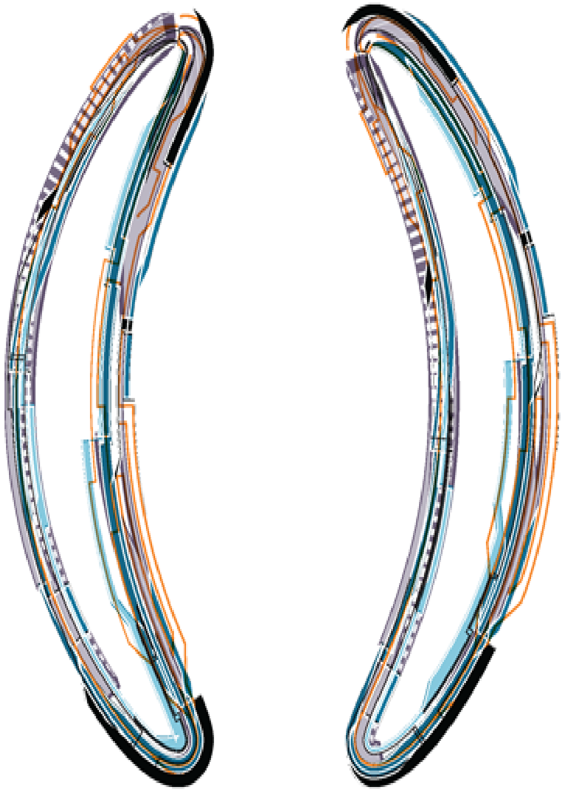
divide both sides by 2: $\boldsymbol{x = 5}$

Alternatively:

divide both sides by 2: $3x - 5 = 2x$

add 5 to both sides: $3x = 2x + 5$

subtract $2x$ from both sides: $\boldsymbol{x = 5}$



In this example, dividing first means that there are fewer steps.

We can only do this because both sides are divisible by 2.

Anna gets a $7 weekly allowance and already has $48. She spends $2 a week on candy.

Her brother Charlie also gets $7 a week. He has no money saved at the moment, but spends nothing each week from now on.

In how many weeks will Anna and Charlie both have the same amount of money?

How much will they each have at this point?

Equations with rational coefficients

Sometimes the coefficient of an unknown is a fraction.

For example: $\frac{3}{4}x - 5 = 9 - x$

We can remove the 4 from the denominator by multiplying both sides of the equation by 4.

$$4\left(\frac{3}{4}x - 5\right) = 4(9 - x)$$

expand the parentheses: $3x - 20 = 36 - 4x$

add $4x$ to both sides: $7x - 20 = 36$

add 20 to both sides: $7x = 56$

divide both sides by 7: $\mathbf{x = 8}$

Equations with rational coefficients

If an equation contains more than one fraction, these can be removed by multiplying throughout by the lowest common multiple of the two denominators.

For example: $$\frac{2}{3}x = \frac{1}{2}x + 1$$

The lowest common multiple of 3 and 2 is 6, so we need to multiply both sides by 6.

$$6\left(\frac{2}{3}x\right) = 6\left(\frac{1}{2}x + 1\right)$$

distributive property: $4x = 3x + 6$

subtract $3x$ from both sides: $\boldsymbol{x = 6}$

Equations with rational coefficients

Example: $\dfrac{2x + 7}{5} = x - 1$

In this example the whole of one side of the equation is divided by 5.

To remove the 5 from the denominator we first multiply both sides of the equation by 5.

	$2x + 7 = 5(x - 1)$
distributive property:	$2x + 7 = 5x - 5$
add 5 to both sides:	$2x + 12 = 5x$
subtract $2x$ from both sides:	$12 = 3x$
divide both sides by 3:	$\mathbf{4 = x}$

Equation solving 2

Use the red arrows to select the operation to apply to both sides of the given equation. Apply the operation by pressing the red button. Use the buttons underneath to multiply out parentheses, get a new equation, clear previous operations or swap the sides of the equation.

Each "type" of equation gets progressively harder, from Type 1 equations (involving a single step to solve) to Type 4 equations (involving multiple steps, fractions and parentheses).

Press **start** to begin.

start

I'm thinking of a number.

When I subtract 9 from the number and double it, I get the same answer as dividing the number by 5.

What number am I thinking of?

Let's call the unknown number n.

We can solve this problem by first writing this equation:

$$2(n - 9) = \frac{n}{5}$$

The number with 9 subtracted, then doubled **is the same as** **the number divided by 5.**

Equivalent expressions

Decide which cards show equations equivalent to this one.

$$7x+2 = 4(9-x)$$

$\frac{7x+2}{4(9-x)} = 1$	$7x+2 = \frac{4}{9-x}$
$4(7x+2) = 9-x$	$7x+2 = \frac{9-x}{4}$
$\frac{7x+2}{4} = 9-x$	$\frac{9-x}{7x+2} = \frac{1}{4}$
$\frac{7x+2}{9-x} = 4$	$\frac{4}{7x+2} = \frac{1}{9-x}$