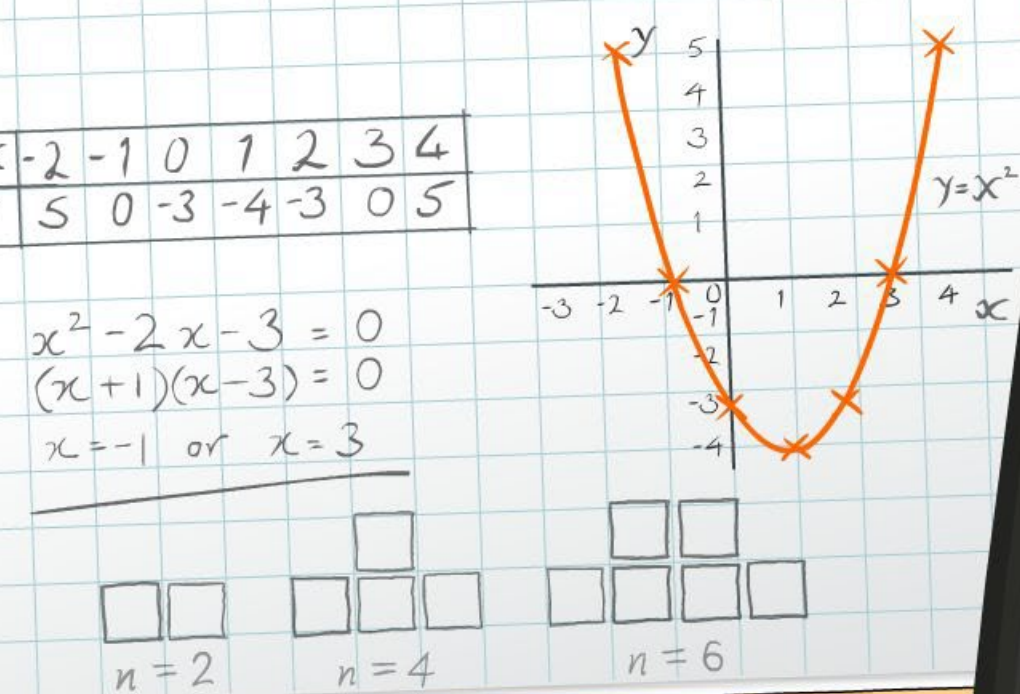
Simple inequalities

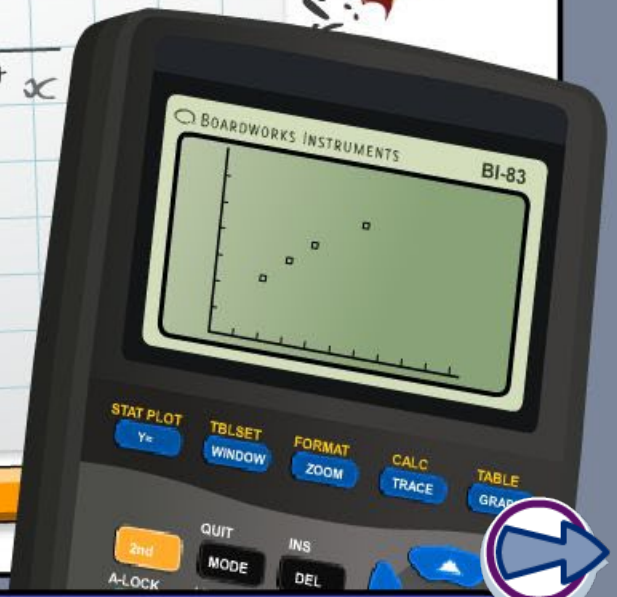
x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$





Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

What is an inequality?

>

An **inequality** is an algebraic statement involving the symbols

$>$, $<$, $\geq$ or $\leq$.

$x > 3$	means	"x is **greater than** 3."
$x < -6$	means	"x is **less than** –6."
$x \geq -2$	means	"x is **greater than or equal to** –2."
$x \leq 10$	means	"x is **less than or equal to** 10."

Inequality notation

You can remember the meanings of the inequality symbols by thinking of them as a crocodile's mouth.

The crocodile will always eat the bigger number!

3		5	**3 < 5**	3 is less than 5
5		3	**5 > 3**	5 is greater than 3

Think of the symbols ≤ and ≥ in the same way, just don't forget that they also mean "**or equal to**."

Reading inequalities

Inequalities can either be read from left to right or from right to left. Both ways have the same meaning.

For example: $5 > -3$

- can be read as "5 is greater than –3" by reading from left to right
- can also be read as "–3 is less than 5" by reading from right to left.

In general:

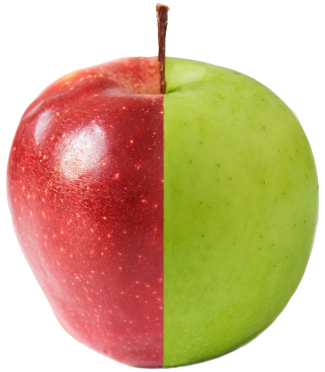
$x > y$ **is equivalent to** $y < x$

and

$x \geq y$ **is equivalent to** $y \leq x$

Combining inequalities

Two inequalities can be combined into one, using two inequality symbols in one expression.



Suppose that $x > 3$ **and** $x \leq 14$. This can be written in one inequality as: **$3 < x \leq 14$**

The maximum and minimum requirements for a new cargo container are given in the table below. Write one inequality to describe the volume (v) of this container.

	min	max
length (ft)	8	10
width (ft)	7	8
height (ft)	7	8

Min volume =

$8 \times 7 \times 7 = 392\ \text{ft}^3$

Max volume =

$10 \times 8 \times 8 = 640\ \text{ft}^3$

The inequality for the volume is: **$392 < v \leq 640$**

Constructing inequalities

Can you rearrange the inequality to be true?

Use this activity to practice creating inequalities. Drag the components into the correct order to make the inequality true before the timer runs out!

Press **start** to begin.

start

20

New York house prices

Press on the house price that fits the buyer's constraints.

10 New Yorkers are looking to buy a house. You will see a statement from each buyer, describing their price constraints. Press the one house price that meets their needs.

Write an inequality of each buyer's constraints.

Press **start** to begin.

start

Inequalities on number lines

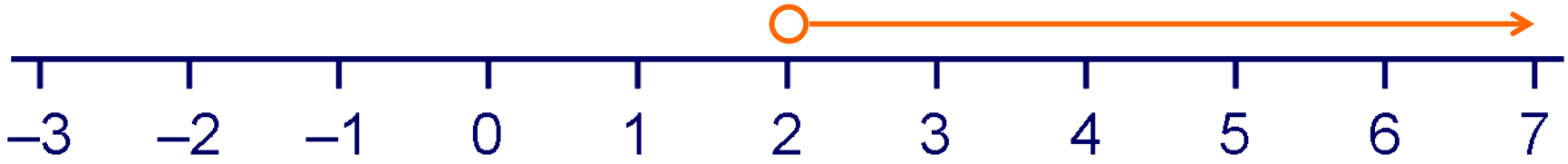
How many values of x satisfy the inequality $x > 2$?

There are infinitely many values that x could be.

x could be equal to 3, 7.3, $54\frac{3}{11}$, 18463.431 …

It would be impossible to write every solution down.

We can visualize the **solution set** on a number line:



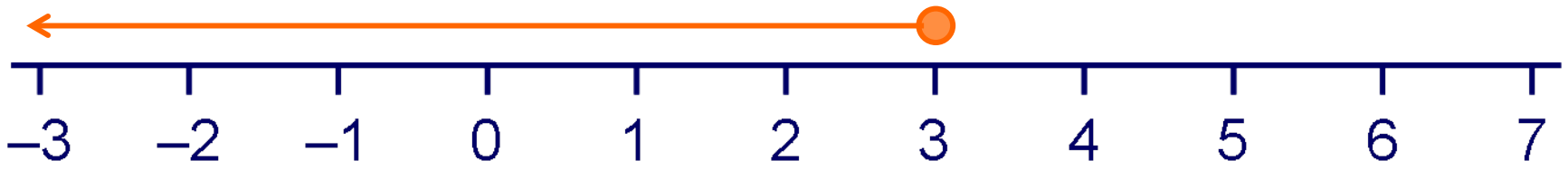
An open circle, ○, at 2 means that this number is not included. The arrow at the end of the line means that the solution set extends in the direction shown.

Inequalities on number lines

Look at the inequality $x \leq 3$. Again, there are infinitely many values that x could be.

x could be equal to 3, -1.4, $-94\frac{8}{17}$, -7452.802 …

We can represent the **solution set** on a number line:



A solid circle, ● , at 3 means that this number is included. The arrow at the end of the line means that the solution set extends in the direction shown.

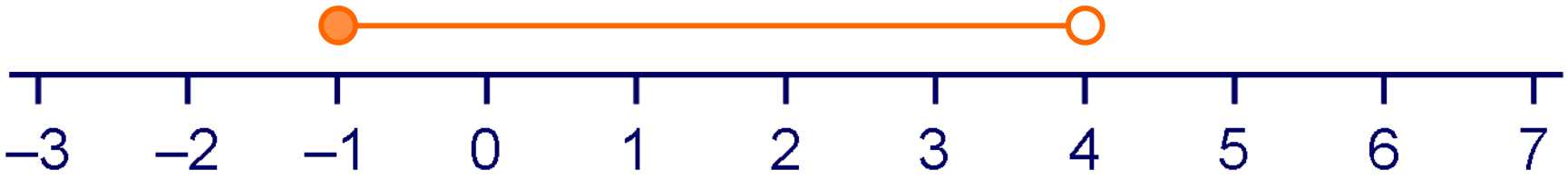
Inequalities on number lines

How many values of x satisfy the inequality $-1 \leq x < 4$? Justify your answer.

Although x is between two values, there are still infinitely many values that x could be.

x could be equal to 2, -0.7, $-3\frac{16}{17}$, 1.648953 …

We can represent the **solution set** on a number line:

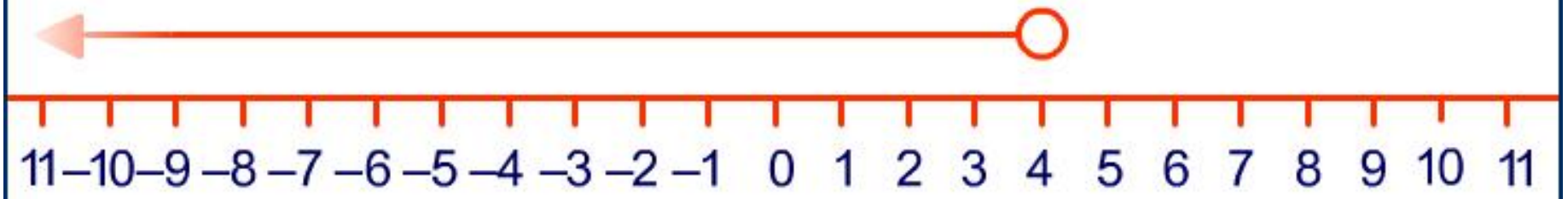


A solid circle, ●, is used at –1 because this value is included.
An open circle, ○, is at 4 because this value is not included.

The line represents all the values in between.

Reading number lines

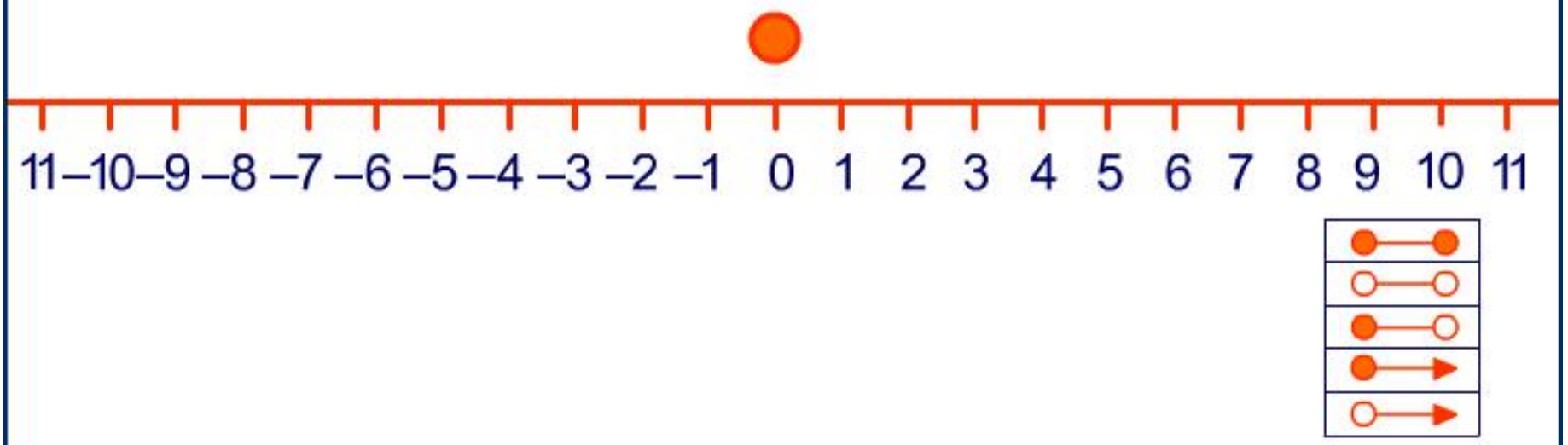
Write out the inequality shown on the number line.



Representing inequalities

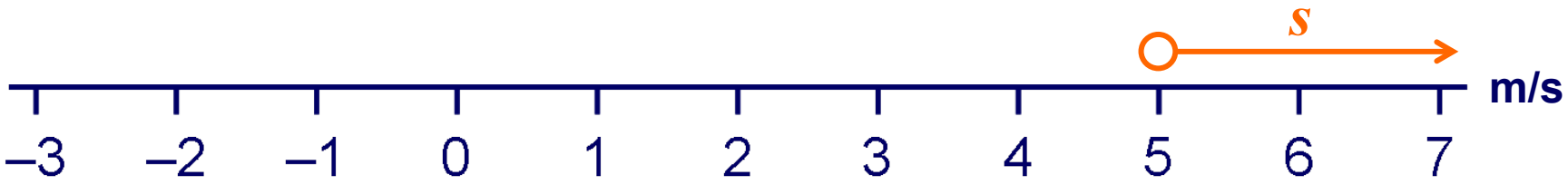
Represent this inequality on the number line:

$-7 < x < 2$



Inequalities in context

Sean runs a 100 m race in 20 seconds. We can draw a number line where s represents any running speed quicker than Sean's:



Do you think s could represent any speed greater than 5 m/s? Explain your answer.

In certain contexts, the value of x (or here, s) cannot realistically be *any* real number.

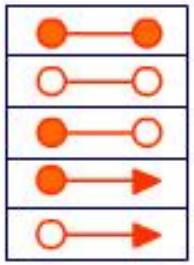
It would be very unrealistic for a human to run any faster than about 10.5 m/s.

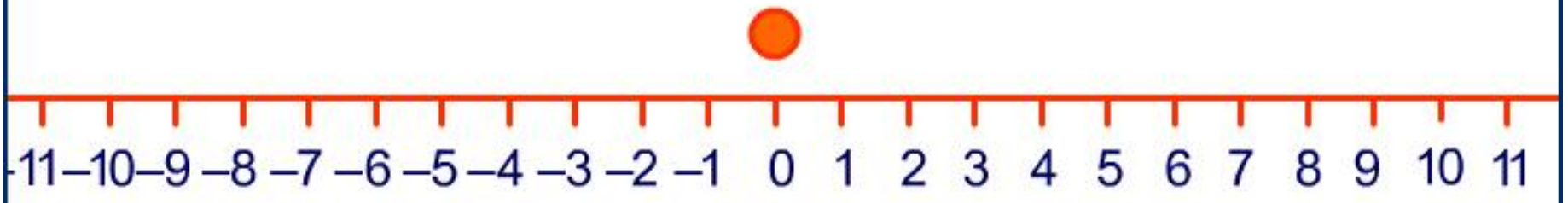
The world record 100 m time set by Usain Bolt is 9.58 s. That's an average speed of 10.4 m/s!

Representing inequalities

Represent this inequality on the number line:

v, the volume of any amount of water in a container with a maximum capacity of $5\ m^3$





Integer solutions

Sometimes we are told that x can only be an integer (a positive or negative whole number).

Consider the inequality $-3 < x \leq 5$. List the integer values that satisfy this inequality.

The integer values that satisfy this inequality are

$-2, \quad -1, \quad 0, \quad 1, \quad 2, \quad 3, \quad 4, \quad 5.$

Can you think of any real-life examples in which the solutions to an inequality must be integers?

Integer solutions

Write down an inequality that is obeyed by the following set of integers:

–4, –3, –2, –1, 0, 1.

There are four possible inequalities that give this solution set,

$-5 < x < 2$

$-4 \leq x < 2$

$-5 < x \leq 1$

$-4 \leq x \leq 1$

Remember that when we use < and > the values at either end are not included in the solution set.

Integer solutions