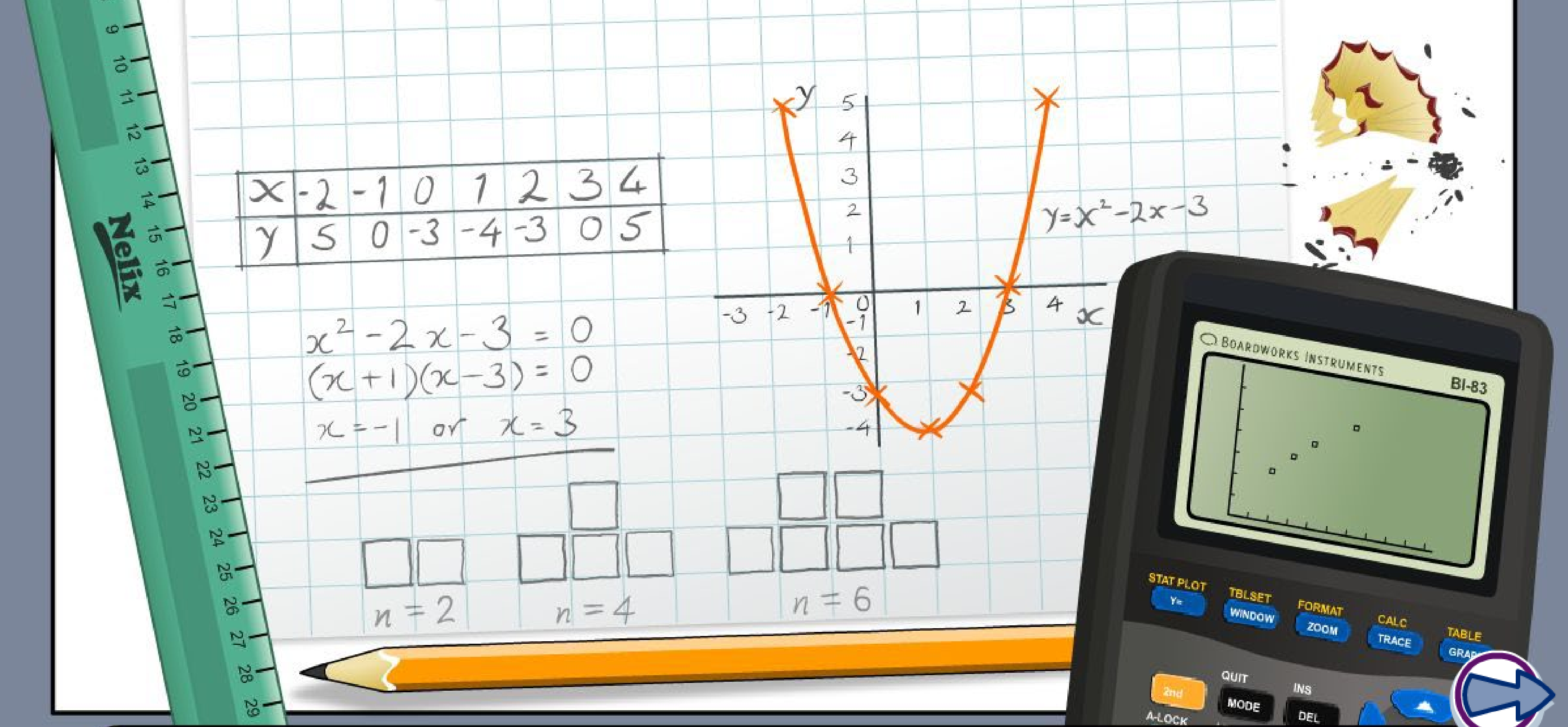
Sequences and rules



Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

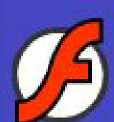
4) Model with mathematics.

5) Use appropriate tools strategically.

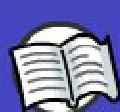
6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.



This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

In mathematics, a **sequence** is an ordered set of mathematical objects. Each object in a sequence is called a **term**.

2, 5, 8, 11, 14, 17, 20, 23, . . .

1st term (2) **6th term** (17)

If terms are next to each other, they are referred to as **consecutive terms**.

A sequence is a **function** in which the domain consists of the natural numbers and the range consists of the terms of the sequence.

In this example, the range is $2 \leq f(x) < \infty$

$f(1) = 2, f(2) = 5, f(3) = 8\ldots$

Predicting terms in a sequence

Sometimes, we can predict how a sequence will continue by looking for patterns.

What are the next two terms in the following sequence,
102, 95, 88, 81, 74, . . . ?

Look at the difference between each consecutive term.

102 95 88 81 74 **67** **60**

–7 –7 –7 –7 –7 –7

This sequence continues by subtracting 7 each time.
We can use this to find the next two terms.

Predicting terms in a sequence

A sequence begins 2, 4, 8, . . .

How could this sequence continue?

2 4 8 14 22 32 44

+2 +4 +6 +8 +10 +12

2 4 8 16 32 64 128

×2 ×2 ×2 ×2 ×2 ×2

If we are not given the rule for a sequence, or if it is not generated from a practical context, we cannot be certain how it will continue.

Defining sequences

Sequences can be defined in two ways:
using a **recursive definition** or an **explicit formula**.

1, 2, 3, 4, 5…

$a_1 = 1$
$a_{n+1} = a_n + 1$

$a_n = n$

Press the buttons below to find out about each one.

recursive definition

explicit formula

Using recursive definitions

Write the first five terms of each sequence using the recursive definition.

1st term	rule	first five terms
–7	add 3	**–7, –4, –1, 2, 5** (+3, +3, +3, +3)
100	subtract 8	**100, 92, 84, 76, 68** (– 8, – 8, – 8, – 8)
3	double	**3, 6, 12, 24, 48** (× 2, × 2, × 2, × 2)
111	divide by 5	**111, 22.2, 4.44, 0.888, 0.1776** (÷ 5, ÷ 5, ÷ 5, ÷ 5)

Recursive definitions

Match the recursive definition to the correct sequence.

→ 2, 4, 16, 256, 65536…

→ 1, –3, 9, –27, 81…

→ –1, 1, 1, 1, 1…

→ 1331, 121, 11, 1, 0.091…

→ $\pi, \frac{1}{\pi}, \pi, \frac{1}{\pi}, \pi\ldots$

$a_1 = 1331$
$a_{n+1} = a_n \div 11$

The Fibonacci sequence

Can you figure out the next three terms in this sequence?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55 . . .

1 + 1   1 + 2   2 + 3   3 + 5   5 + 8   8 + 13   13 + 21   21 + 34

This sequence starts 1, 1, … and then each term is found by adding together the two previous terms.

This sequence is called the **Fibonacci sequence** after an Italian mathematician.

It is a very important sequence because it often appears in nature. For example, in the arrangement of leaves on a stem.

The n^{th} term of the Fibonacci sequence

The Fibonacci sequence is a function that is defined **recursively** using the first two terms.

We can write:

$$f(1) = 1$$

$$f(2) = 1$$

$$f(3) = f(2) + f(1)$$

$$f(4) = f(3) + f(2)$$

$$f(5) = f(4) + f(3)$$

…

The function is defined recursively with the formula:

$$\boldsymbol{f(n + 1) = f(n) + f(n - 1)}$$

Explicit formulas

When a sequence is defined by an explicit formula it can sometimes help to put the terms in a table.

For example, the n^{th} term in a sequence is n^2, where n is the term's position in the sequence.

Position	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	…	n^{th}
Term	**1**	**4**	**9**	**16**	**25**	…	n^2

Each term can be found by squaring its position in the sequence.

What is the 20^{th} term in this sequence?

$20^2 =$ **400**

Using explicit formulas

Match the explicit formula to the correct sequence.

	Sequence
	–3, 0, 3, 6, 9…
	1, 3, 7, 15, 31…
	3, 7, 13, 21, 31…
	0, 8, 0, 32, 0…
	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \ldots$

$a_n = 2^n + (-2)^n$

Calculate the terms using the function definition.

$$a_n = f(n) = 1 - 5n$$

$a_1 =$

$a_2 =$

$a_3 =$

$a_4 =$

$a_{40} =$

Show

Fraction sequences

Fraction sequences

When the terms in a sequence are all fractions, look at the sequence formed by the numerators and the sequence formed by the denominators separately.

Press "**play**" to see an example.