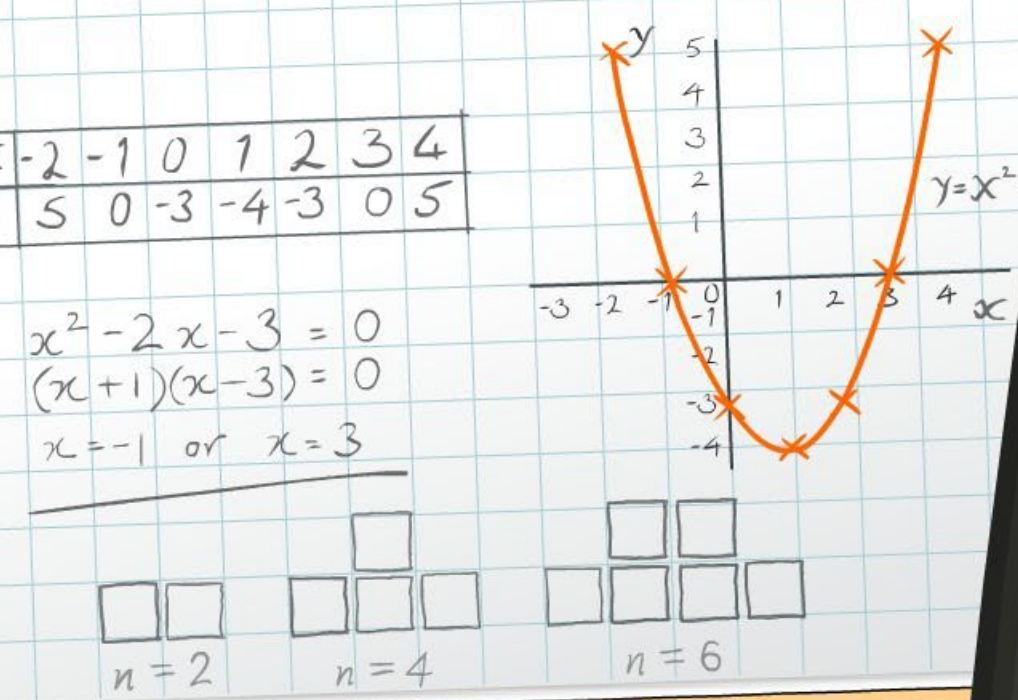
Relations and functions

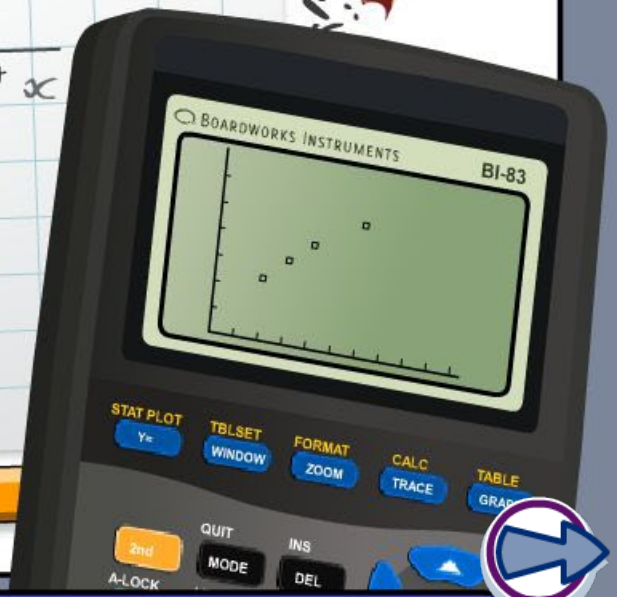
x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$





Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

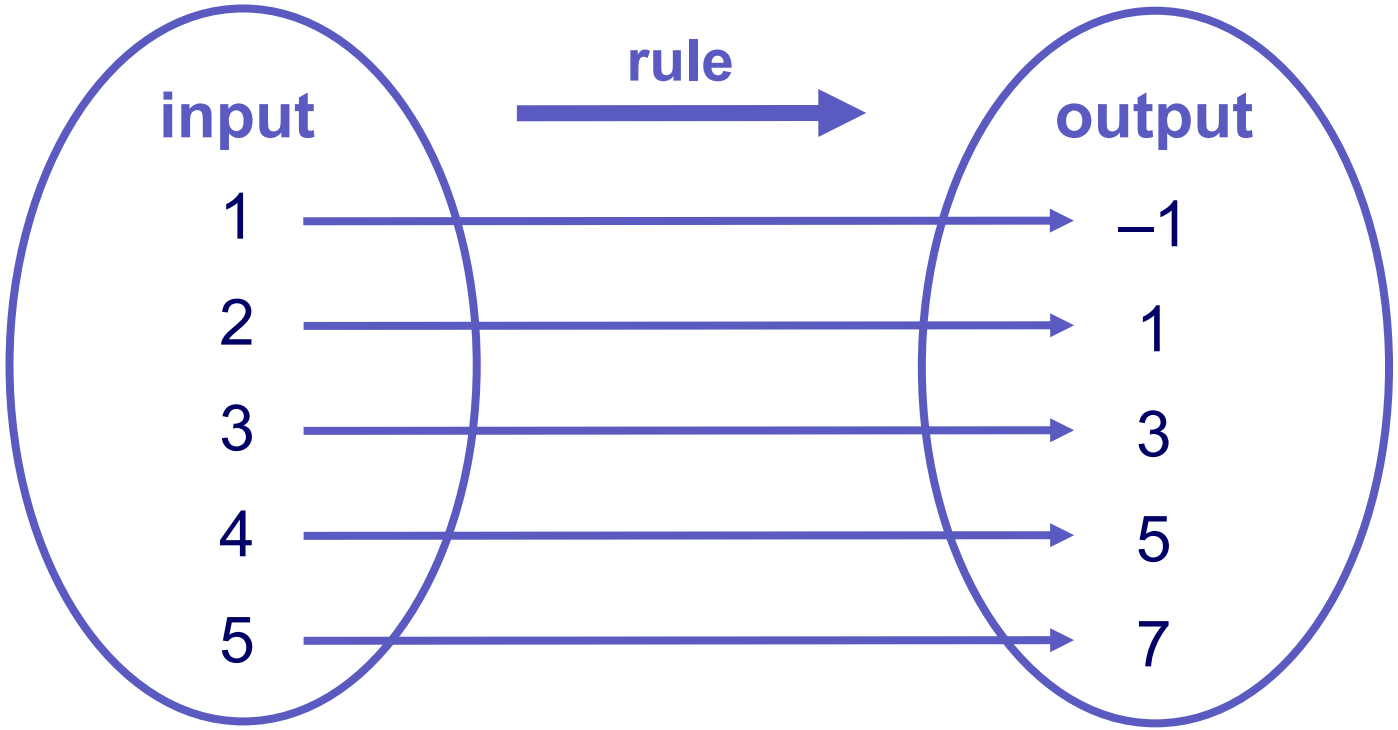
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Relations

A **relation** is a rule that links quantities. For every **input**, the relation associates one or more **outputs**.

For example: here is a diagram showing the rule **double and subtract 3**, with the input set {1, 2, 3, 4, 5}:



Types of relation

Relations can be categorized based on how many output values they give for each input and whether different input values can give the same output.

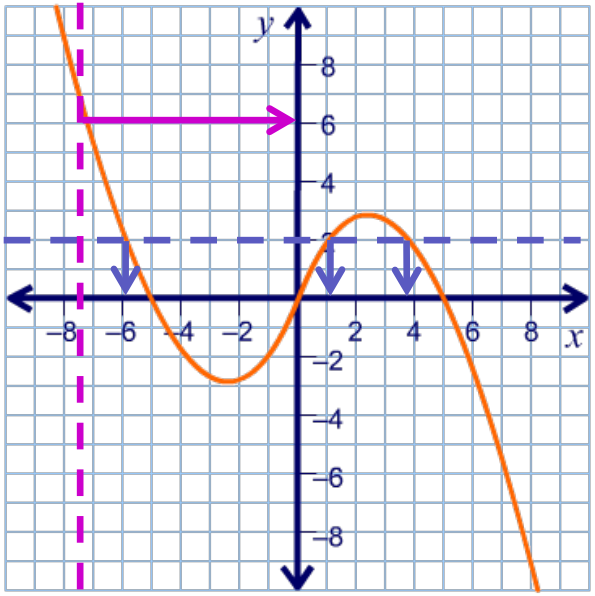
Press the buttons to find out about each type.

- one-to-one
- many-to-one
- one-to-many
- many-to-many

Graphs of relations I

We can graph relations by plotting the input on the horizontal axis and the output on the vertical axis.

What type of relation is shown here?



Many-to-one

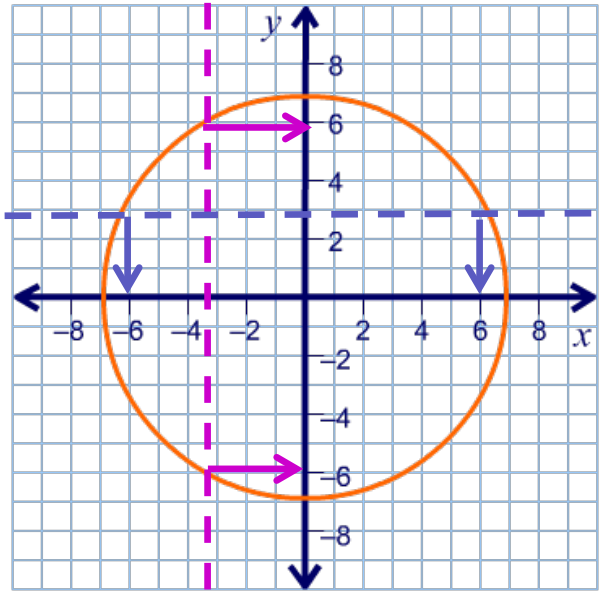
Draw horizontal lines and count how often the graph intersects each line. If any line intersects the graph more than once, it is a **many-to-something** relation.

Doing the same with vertical lines tells us whether it is a **something-to-one** or **something-to-many** relation.

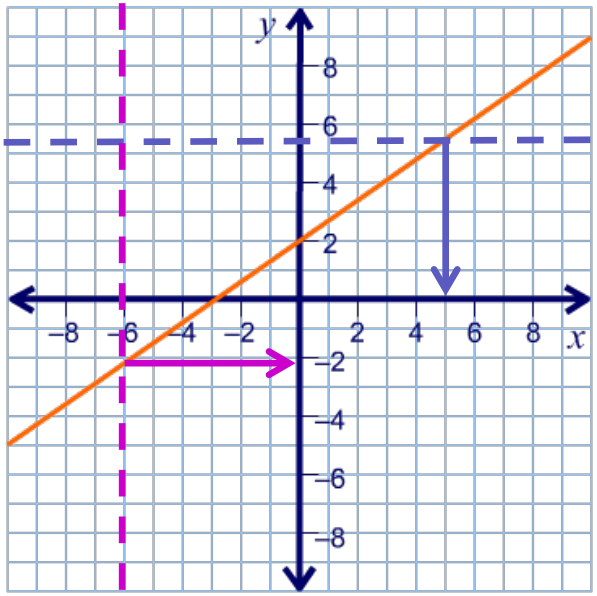
Graphs of relations II

What types of relations are shown here?

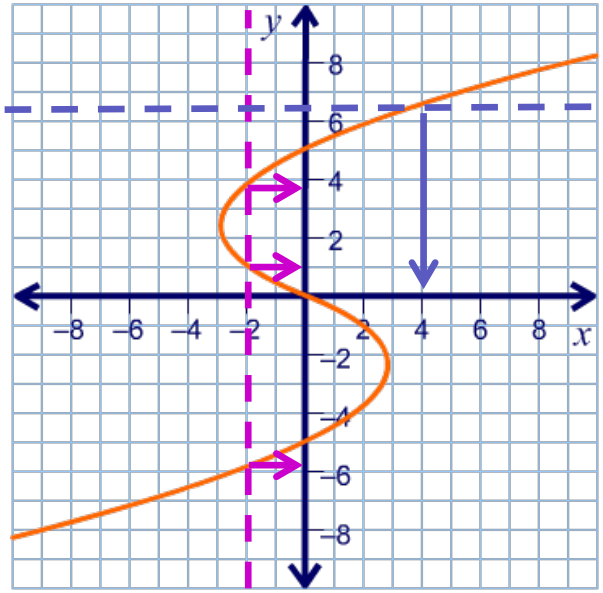
- Look at the horizontals to see if it is **one-to-…** or **many-to-…**
- Look at the verticals to see if it is **…-to-one** or **…-to-many**.



Many-to-many



One-to-one



One-to-many

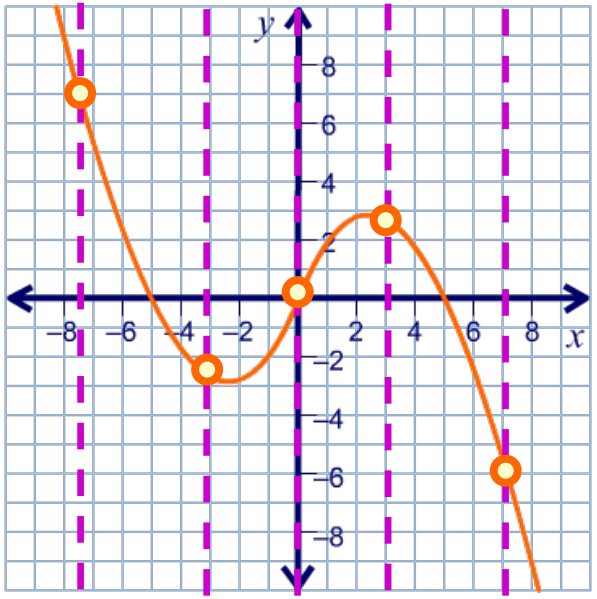
Functions

A **function** is a special type of relation that associates one, and only one, output to each input.

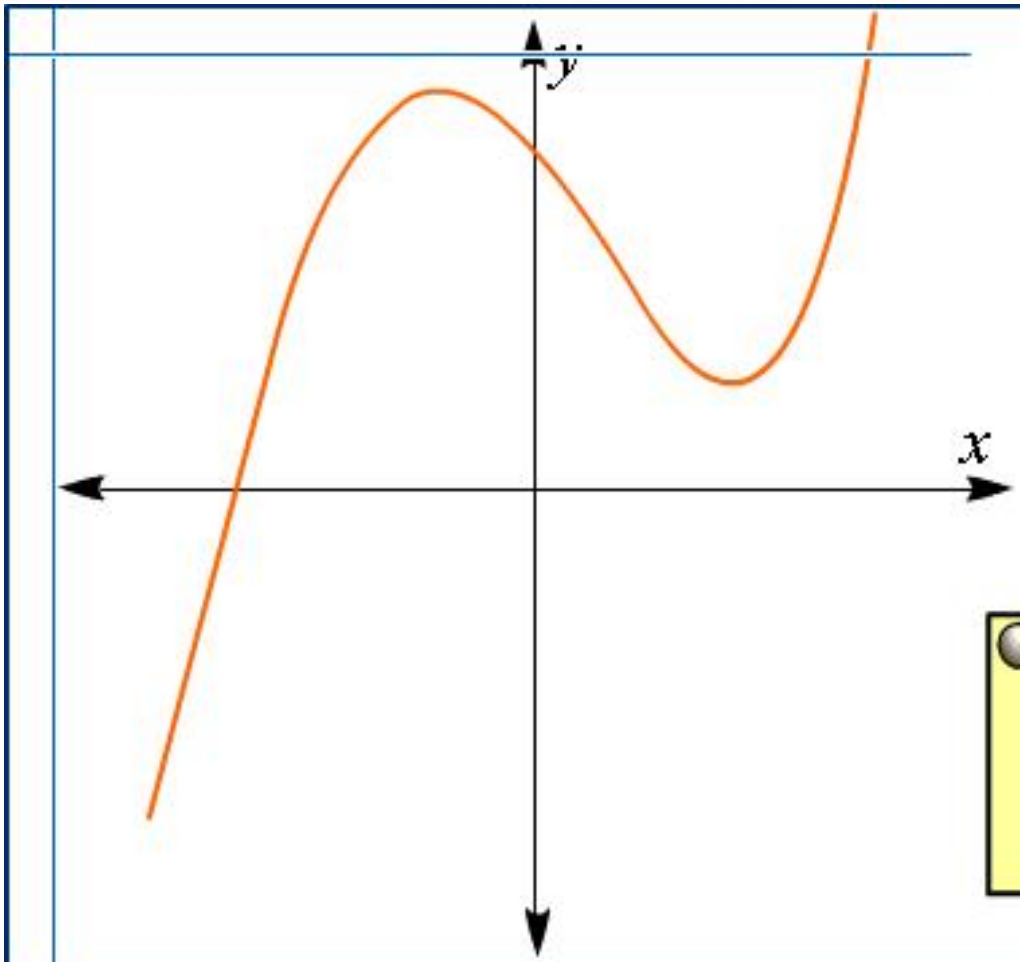
Only a one-to-one or a many-to-one relation can be called a function.

One-to-many and many-to-many relations are **not** functions.

A relation is a function if there is no vertical line that intersects the graph more than once.



Which graphs represent functions?



Function notation

We can **name** functions using function notation, just as we name variables with letters like x.

We usually use the letter *f* to represent a function and write $f(x)$ for the output of the function, f, at input value x.

For example, the function "square and add 5" can be written:

$$f(x) = x^2 + 5$$

"f of x equals x squared plus 5"

To find the output of a function for a particular input, for example at $x = -3$, we can write:

$$f(-3) = (-3)^2 + 5 = 14$$

Function notation

The set of all permissible inputs is called the **domain** of the function. If the domain has not been given, we assume that x belongs to the set of real numbers, $\mathbf{R}$.

The set of all corresponding outputs is called the **range** of the function.

What is the range of this function?

$f(x) = x^2 + 5$ **with domain** $x = \mathbf{R}$

The smallest value of $f(x)$ is 5, so:

$$f(x) \geq 5$$

Summary

Function or relation?

1.	Is the relation {(0, 1), (0, 2), (0, 3), (0, 4)} a function?	S
2.	Is the relation {(1, 0), (2, 0), (3, 0), (4, 0)} a function?	S
3.	What values of x would turn the relation {(7, 11), (11, 13), (x, 23), (27, 29)} into a function?	S
4.	The rule "squares to give" gives the outputs 1 **and** –1 for the input 1. How can we make this a function?	S
5.	For the relation with the rule "is a factor of", over what domain is it a function?	S