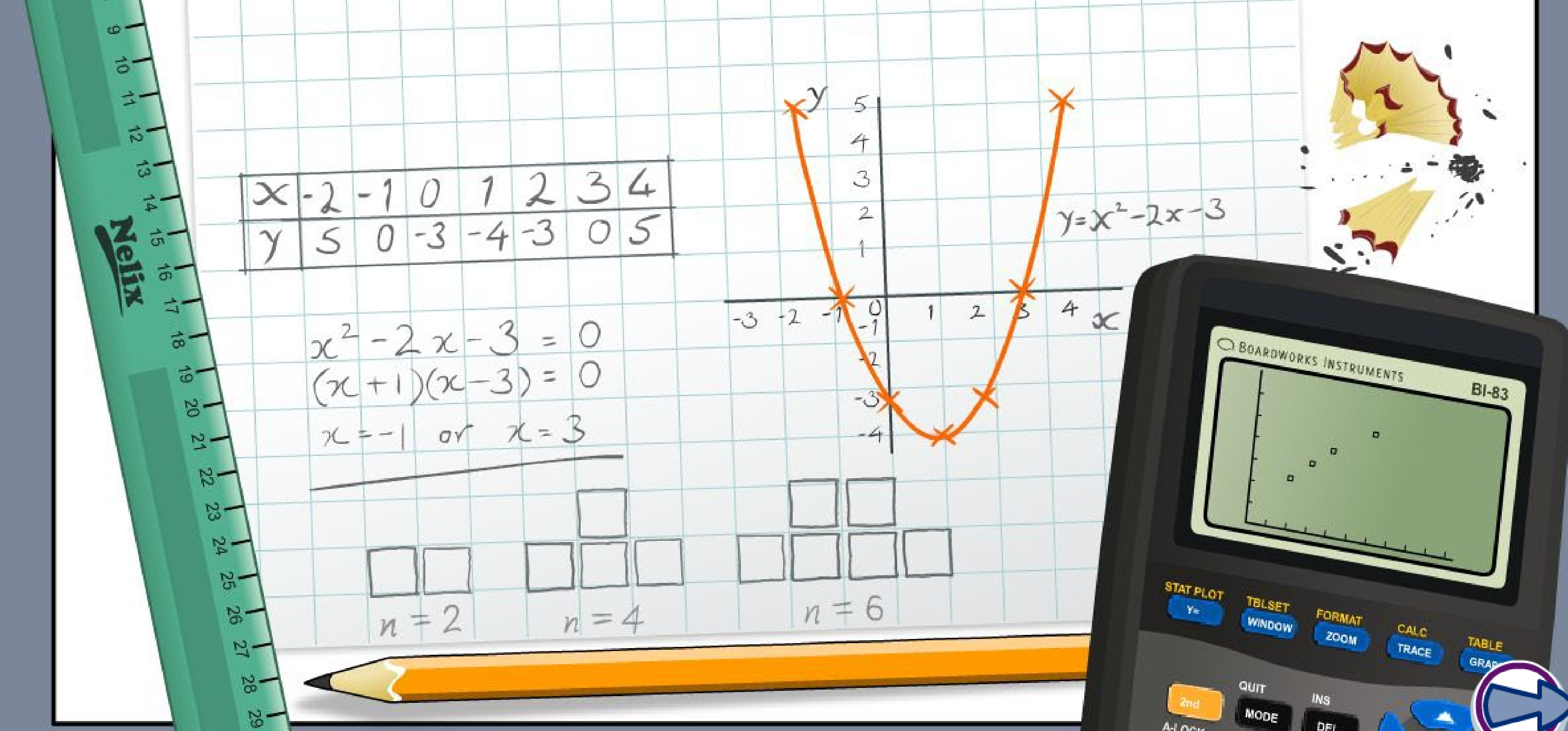
Radicals


x -2 -1 0 1 2 3 4
y 5 0 -3 -4 -3 0 5
y=x^2-2x-3
x^2-2x-3 = 0
(x+1)(x-3) = 0
x=-1 or x=3
n=2
n=4
n=6
BOARDWORKS INSTRUMENTS
BI-83

Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

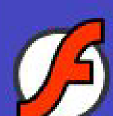
4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Radicals

The square roots of many numbers cannot be found exactly.

For example, the value of $\sqrt{3}$ cannot be written exactly as a fraction or a decimal.

$\sqrt{3}$ is an **irrational number**. It contains a **radical** which cannot be simplified to a rational number.

To keep the value in its exact form, we keep the square root sign and write the number as $\sqrt{3}$.

Which one of the following is not irrational?
$\sqrt{2}$, $\sqrt{6}$, $\sqrt{9}$ or $\sqrt{14}$

$\sqrt{9}$ is not irrational because it can be written as a whole number.

Do you know how to classify different numbers?

NATURAL NUMBERS

INTEGERS

Press on each type of number to reveal a description and some examples.

RATIONAL NUMBERS

IRRATIONAL NUMBERS

Multiplying radicals

How would you calculate the value of $\sqrt{4} \times \sqrt{4}$?

We can think of this as squaring the square root of four.

Squaring and square rooting are inverse operations so: $\sqrt{4} \times \sqrt{4} = \mathbf{4}$.

As a general rule, we can state that:

$$\sqrt{a} \times \sqrt{a} = a$$

What is the value of $\sqrt{4} \times \sqrt{4} \times \sqrt{4}$?

Multiplying radicals

Using your calculator, find the value of $\sqrt{4} \times \sqrt{7}$.

Can you identify a general rule for multiplying radicals?

$\sqrt{4} \times \sqrt{7} = \sqrt{28}$

As a general rule, $\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$

Use a calculator to find the value of $\sqrt{5} \times \sqrt{3}$ and the value of $\sqrt{3} \times \sqrt{12}$.

What do you notice?
Can you explain the results?

Find two other radicals that multiply to give a rational value.

Dividing radicals

Using your calculator, find the value of $\sqrt{30} \div \sqrt{5}$.

Can you identify a general rule for dividing radicals?

$\sqrt{30} \div \sqrt{5} = \sqrt{\mathbf{6}}$

As a general rule,

$$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

Use a calculator to find the value of $\sqrt{18} \div \sqrt{2}$ and the value of $\sqrt{21} \div \sqrt{3}$.

What do you notice?
Can you explain the results?

Find two other radicals that divide to give a rational value.

Simplifying radicals

In order to be simplified fully, some radicals need to be written in the form $a\sqrt{b}$.

We can do this using the fact that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

For example, simplify $\sqrt{50}$ by writing it in the form $a\sqrt{b}$.

Start by finding the largest perfect square factor of 50.

This is 25. We can use this to write:

$$\sqrt{50} = \sqrt{25 \times 2}$$

$$= \sqrt{25} \times \sqrt{2}$$

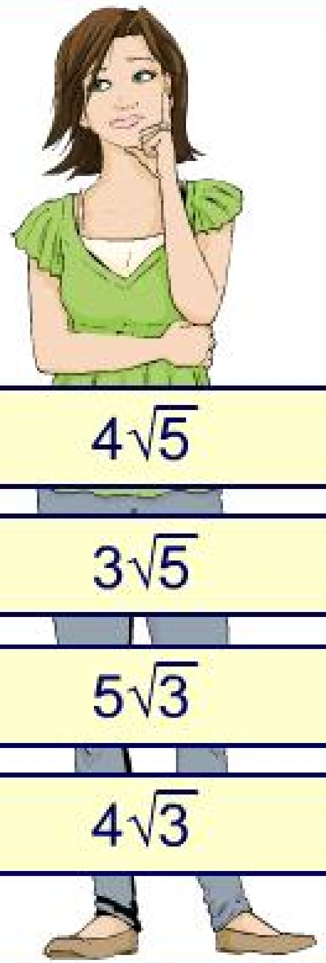
$$= \mathbf{5\sqrt{2}}$$

Notice that we don't write a multiplication sign. Compare this to writing multiples of variables in algebra, e.g. $5a$.

Simplifying radicals

Q1/3: Can you simplify $\sqrt{45}$ by writing it in the form $a\sqrt{b}$?

Press the "=" button to show the work step by step.



$4\sqrt{5}$

$3\sqrt{5}$

$5\sqrt{3}$

$4\sqrt{3}$

Match equivalent expressions to complete the radical laws.

$\sqrt{a} \times \sqrt{a} =$

$\sqrt{a} \times \sqrt{a} \times \sqrt{a} =$

$\sqrt{a} \times \sqrt{b} =$

$\sqrt{a} \div \sqrt{b} =$

$a\sqrt{a}$

$\sqrt{ab}$

$\sqrt{\frac{a}{b}}$

a

Complete these radical calculations

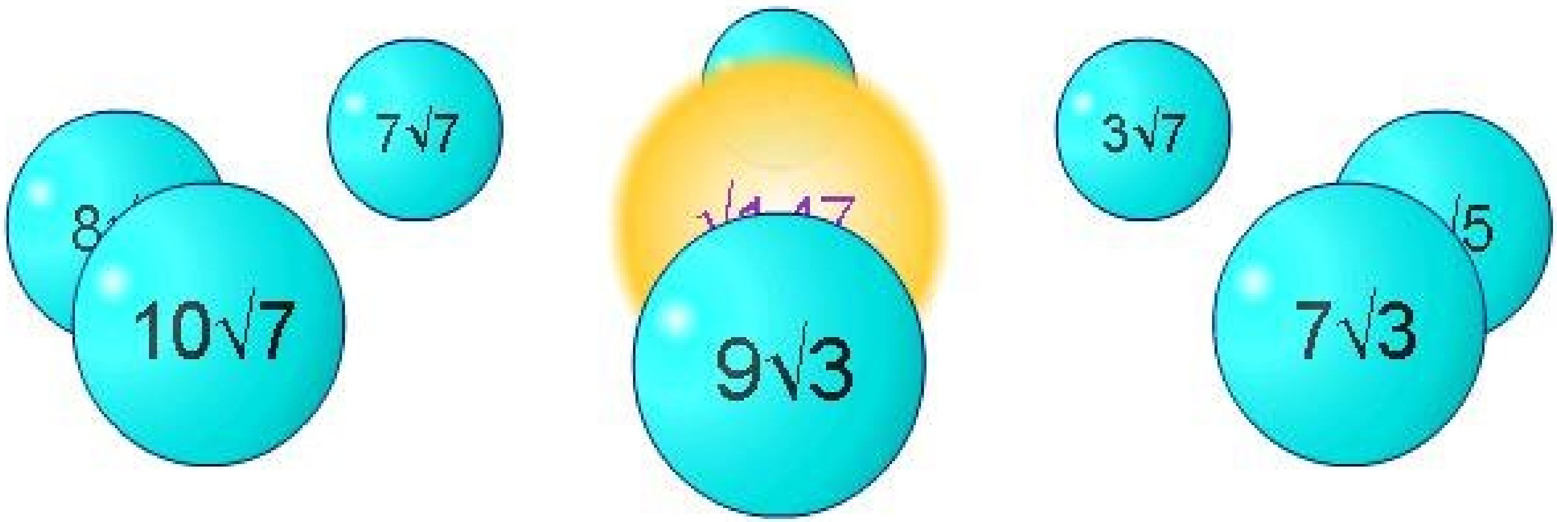
$\square \div \sqrt{8} = 3$	$\square \times \sqrt{2} = 6$
$\sqrt{32} \times \square = 8$	$\sqrt{48} \div \square = 2$
$\square \div \sqrt{5} = 5$	$\sqrt{27} \times \square = 9$

$\sqrt{18}$ $\sqrt{2}$ $\sqrt{72}$ $\sqrt{12}$ $\sqrt{125}$ $\sqrt{3}$

S C ?

Simplifying radicals

Press the orbiting radical that is equivalent to the center radical.



Adding and subtracting radicals

How can we calculate $\sqrt{45} + \sqrt{80}$?

Start by writing $\sqrt{45}$ and $\sqrt{80}$ in their simplest forms.

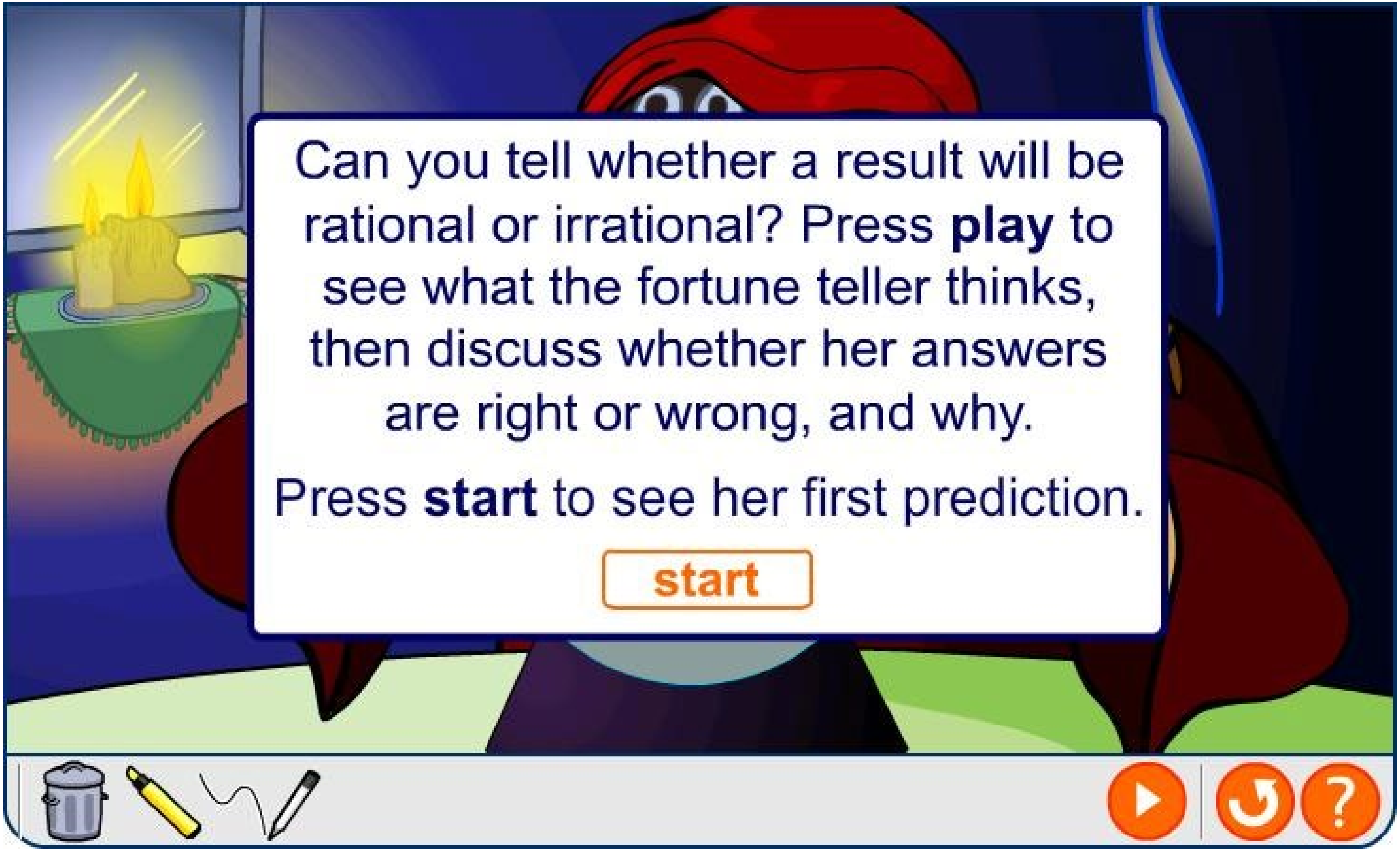
$$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$$

$$\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$$

$$\sqrt{45} + \sqrt{80} = 3\sqrt{5} + 4\sqrt{5} = \mathbf{7\sqrt{5}}$$

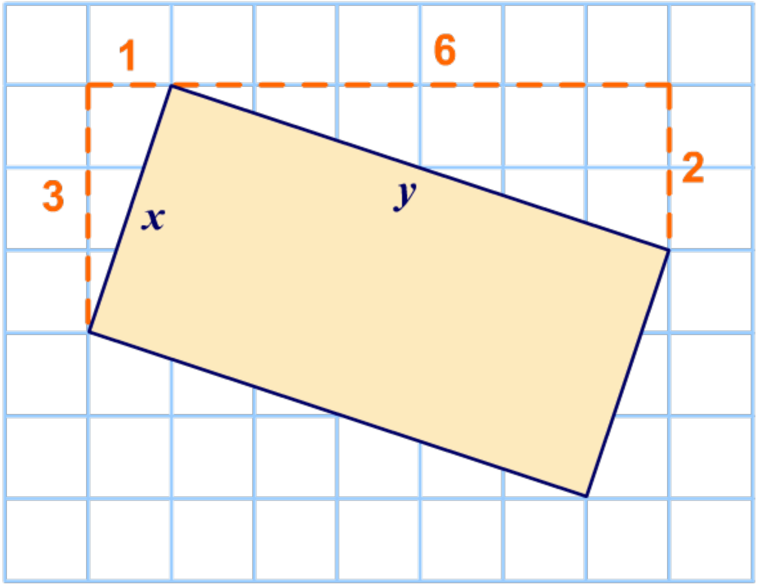
Radicals can be added or subtracted if the number under the radical sign is the same.

Rational or irrational?



Shed dimensions

The plan for a shed has been drawn on the meter grid below.



Use the given lengths to find the length (y) and width (x) of the shed. Show your work.

Find its perimeter and area in radical form.

Use the Pythagorean Theorem.

Width (x) = $\sqrt{(3^2 + 1^2)}$ = $\sqrt{(9 + 1)}$ = $\sqrt{10}$ units

Length (y) = $\sqrt{(6^2 + 2^2)}$ = $\sqrt{(36 + 4)}$ = $\sqrt{40}$ = $2\sqrt{10}$ units

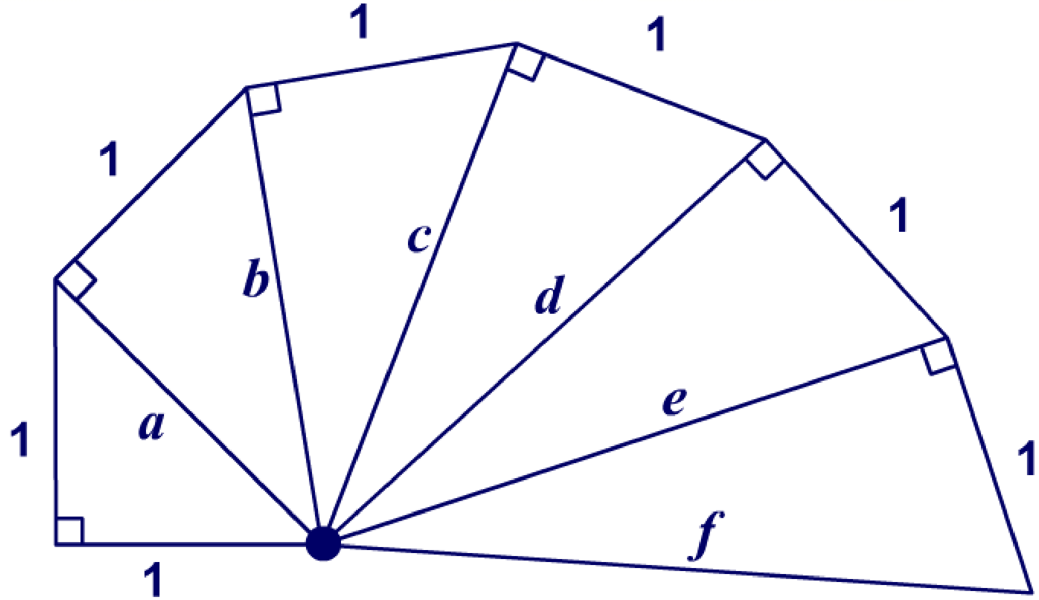
Perimeter = $\sqrt{10} + 2\sqrt{10} + \sqrt{10} + 2\sqrt{10}$ = **$6\sqrt{10}$ units**

Area = $\sqrt{10} \times 2\sqrt{10} = 2 \times \sqrt{10} \times \sqrt{10} = 2 \times 10 =$ **20 units²**

The Pythagorean spiral

An architect is working on the design for a spiral staircase.

He thinks he can use the Pythagorean Theorem to calculate the missing lengths.



Can you calculate the width of the sections marked *a*, *b*, *c*, *d*, *e* and *f*? Express your answers in radical form wherever necessary.

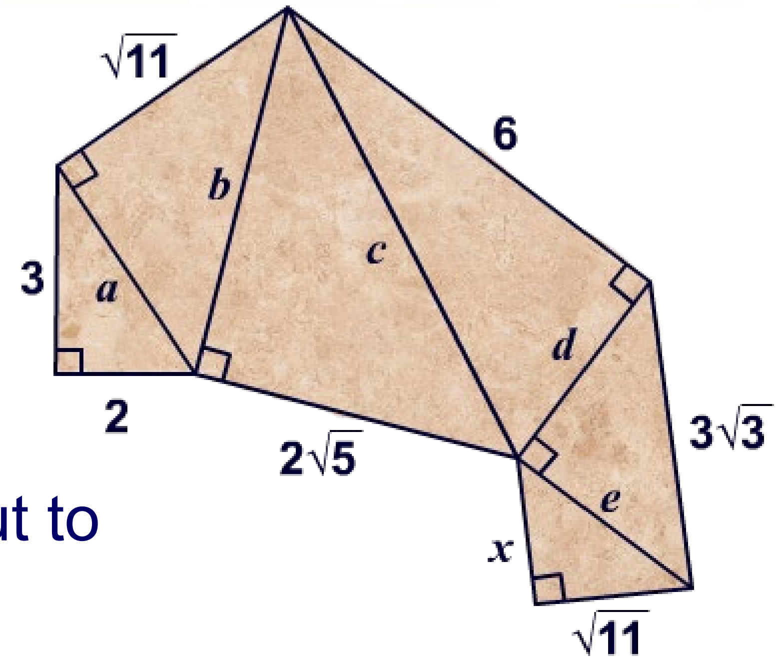
Draw the diagram to measure the lengths and see how accurate you are.

Laying paving slabs

A landscape gardener has drawn up plans for laying paving slabs through a yard.

The plans have not been drawn to scale.

The slabs need to be accurately cut to fit within the pattern perfectly.



Without using a calculator, work out the lengths: a, b, c, d, and e to help you find x.

Make sure all your answers are in radical form and show your work.

When a fraction contains a radical as the denominator we usually rewrite it so that the denominator is a rational number.

This is called **rationalizing the denominator**.

Simplify the fraction $\frac{5}{\sqrt{2}}$.

In this example we rationalize the denominator by multiplying the numerator and the denominator by $\sqrt{2}$:

$$\frac{5}{\sqrt{2}} \overset{\times\sqrt{2}}{=} \frac{5\sqrt{2}}{2}$$

Rationalizing the denominator

Q1/3: Can you simplify $\frac{2}{\sqrt{3}}$ by rationalizing the denominator?

Press the "=" button to show the work step by step.

$\frac{2\sqrt{3}}{3}$

$\frac{6}{3}$

$\frac{\sqrt{6}}{3}$

Rationalizing the denominator

When the denominator is in the form $\sqrt{a} \pm b$ we can rationalize the denominator using the difference of squares:

$$(a - b)(a + b) = a^2 - b^2$$

For example, simplify $\dfrac{1}{\sqrt{5} - 2}$

$$\frac{1}{\sqrt{5} - 2} = \frac{1}{\sqrt{5} - 2} \times \frac{\sqrt{5} + 2}{\sqrt{5} + 2}$$

$$= \frac{\sqrt{5} + 2}{5 - 4} = \mathbf{\sqrt{5} + 2}$$

The denominators multiply to give the difference of squares:
$(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 2^2$
This removes the radical.

In general: $$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$$

Can you apply the distributive property to rationalize the expressions containing radicals?

$$(8+\sqrt{3})\,(8-\sqrt{3}) =$$

$$=$$

$$=$$

How can we simplify the fraction $\dfrac{5}{6+\sqrt{11}}$?

$$\frac{5}{6+\sqrt{11}} \times \frac{6-\sqrt{11}}{6-\sqrt{11}}$$

The denominators will multiply to give the difference of squares:
$(6+\sqrt{11})(6-\sqrt{11}) = 6^2 - 11$
This removes the radical.

$$= \frac{30-5\sqrt{11}}{36-11}$$

$$= \frac{30-5\sqrt{11}}{25}$$

Express the fraction in its simplest form. Be sure to cancel any common factors.

$$= \frac{5(6-\sqrt{11})}{25} = \mathbf{\frac{6-\sqrt{11}}{5}}$$

Rationalizing the denominator

Rationalizing the denominator

Q1/3: Can you simplify $\frac{12}{4 - \sqrt{2}}$ by rationalizing the denominator?

Press the "=" button to show the work step by step.

$$\frac{24 + 12\sqrt{2}}{7}$$

$$\frac{24 - 6\sqrt{2}}{14}$$

$$\frac{24 + 6\sqrt{2}}{7}$$