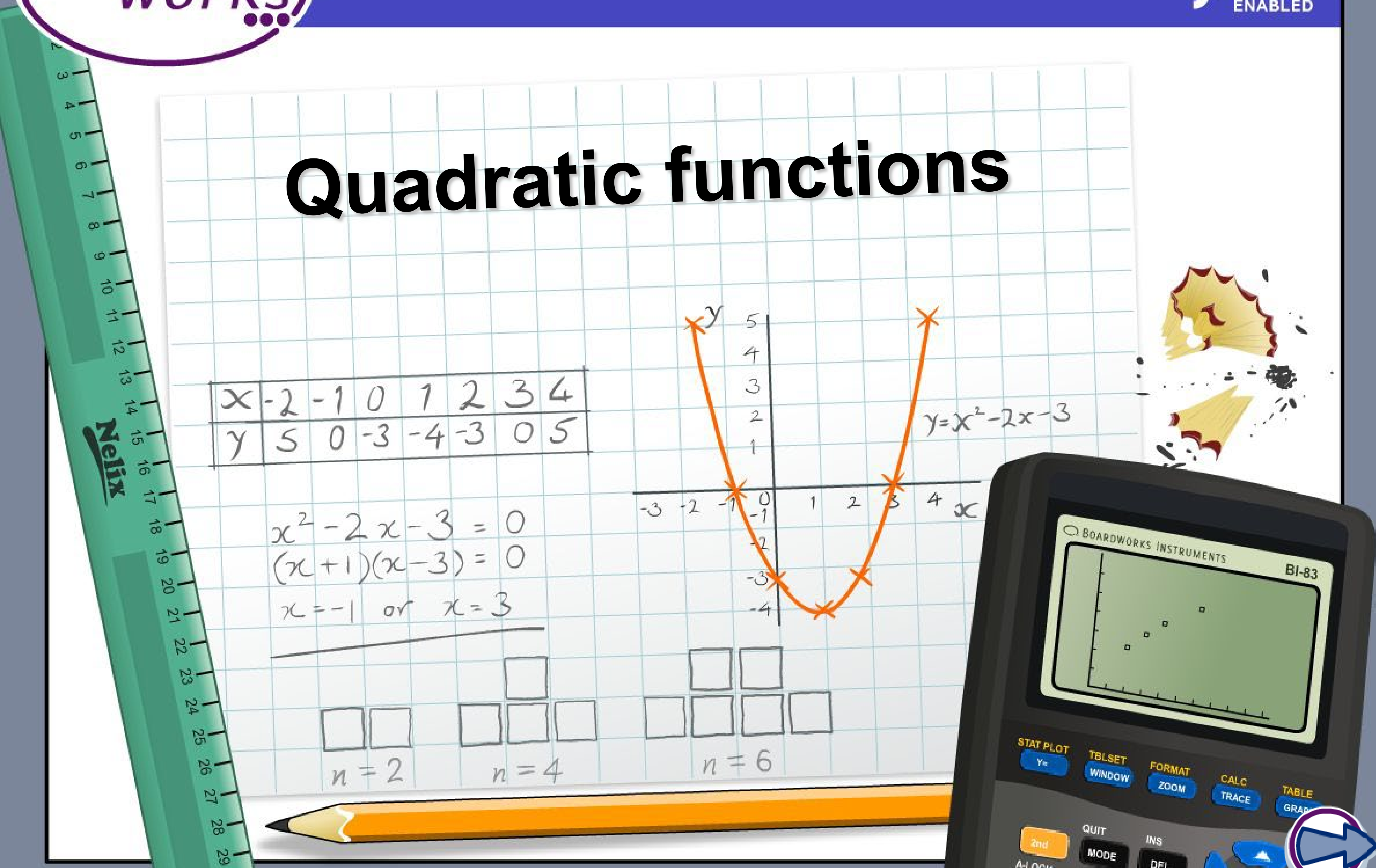
Quadratic functions



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

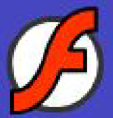
4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

What is a quadratic function?

Linear functions are those in the form $y = mx + b$, where the highest power of x is 1.

A **quadratic function** has x^2 as its highest power of x. They can therefore all be written in the form:

$$y = ax^2 + bx + c \qquad \text{(where } a \neq 0\text{)}$$

So, what do the graphs of these functions look like?

We can draw the graph of a quadratic function by making a table of values then plotting the points.

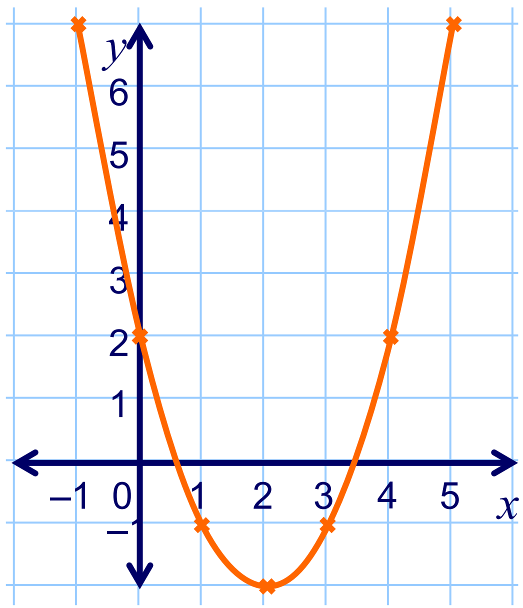
Plot the graph of $y = x^2 - 4x + 2$ for $-1 \leq x \leq 5$.

Plotting graphs of quadratic functions

Draw a table of values for $y = x^2 - 4x + 2$ for $-1 \leq x \leq 5$.

x	–1	0	1	2	3	4	5
$y = x^2 - 4x + 2$	**7**	**2**	**–1**	**–2**	**–1**	**2**	**7**

$y = (-1)^2 - 4(-1) + 2$
$= 1 + 4 + 2 = \mathbf{7}$

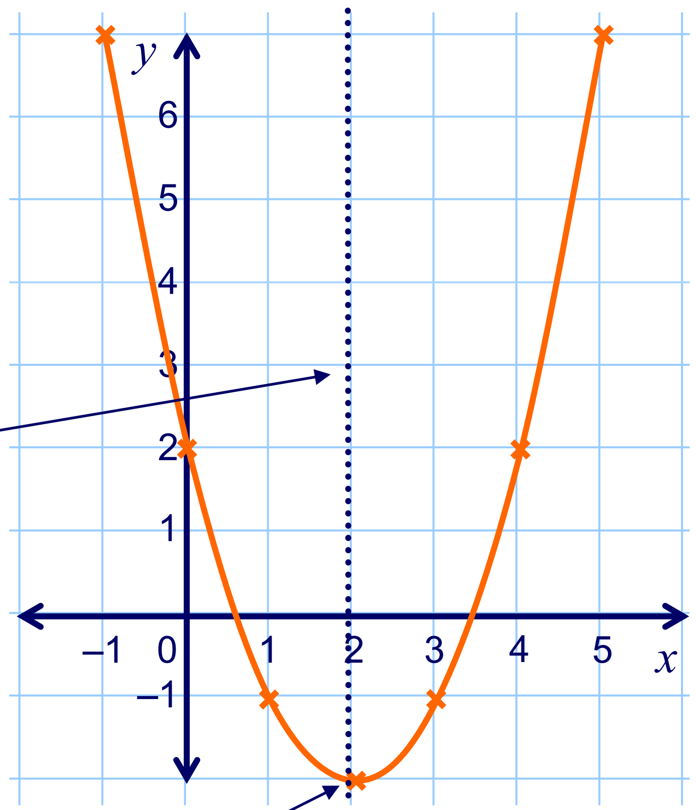


The pairs of coordinates in the table can now be plotted and joined with a smooth curve.

Examine the graph and discuss its shape.

The shape of this curve is called a **parabola**.

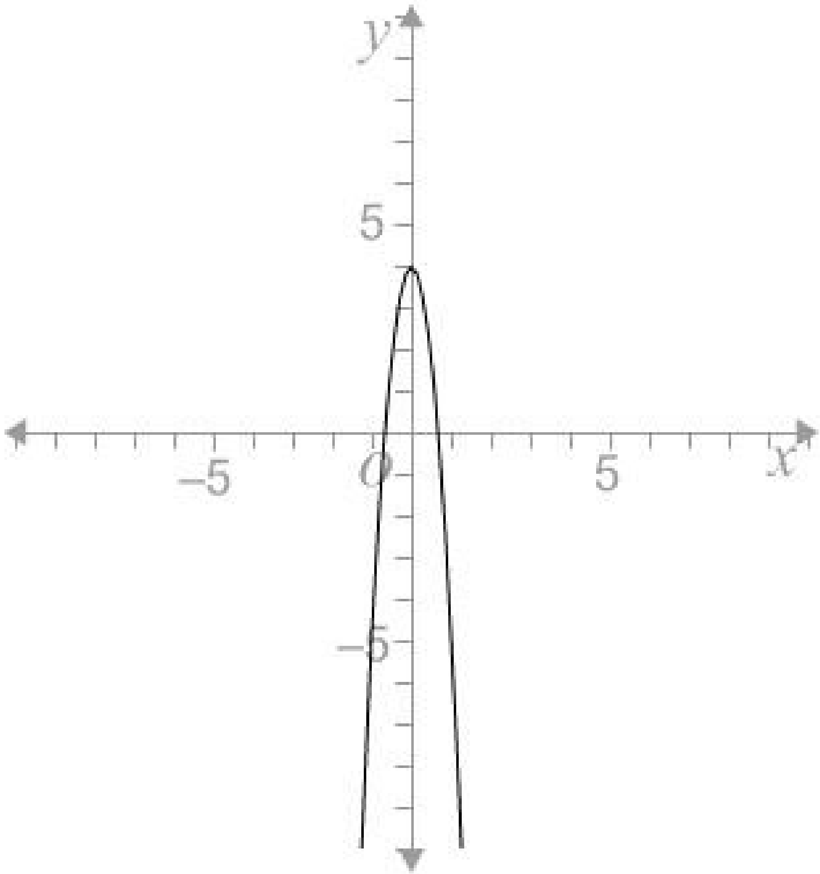
It is characteristic of a quadratic function.



Parabolas are **symmetrical** about a vertical axis.

This particular parabola is a U-shape.

Parabolas all have a **turning point** called the **vertex**.

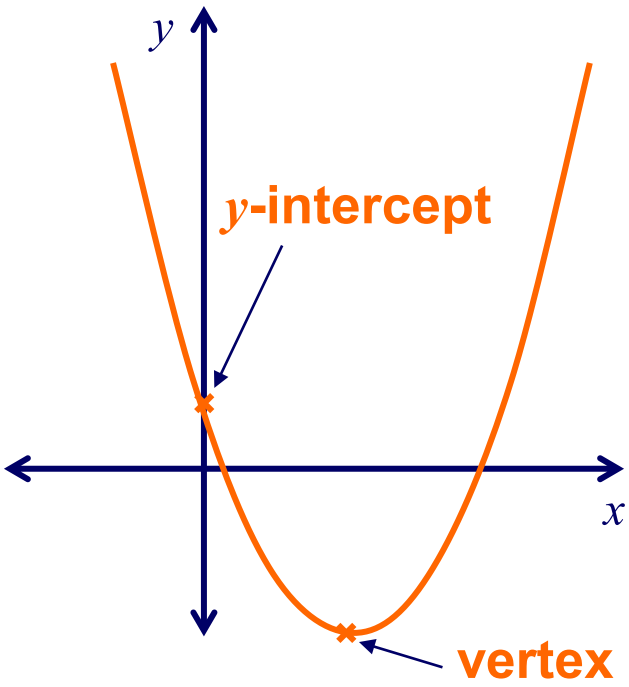


Change the coefficient of x^2.
What do you notice?

Change the constant value.
What do you notice?

$y = 0x^2 \; 0x \; 0$

Characteristics of parabolas



Things to observe from the graph of a quadratic function:

When the coefficient of x^2 is **positive**, the vertex is a **minimum** point and the graph is $\cup$-shaped.

When the coefficient of x^2 is **negative**, the vertex is a **maximum** point and the graph is $\cap$-shaped.

The constant term determines the **y-intercept** of the curve.

What are the coordinates of the y-intercept of the quadratic $y = ax^2 + bx + c$?

The y-intercept

The function $y = ax^2 + bx + c$ crosses the y-axis at $(0, c)$.

At what point does the curve of the function $y = x^2 + 6x$ cross the y-axis?

This function has no constant value (i.e. $c = 0$), so it will cross the x-axis at (0, 0), which is the **origin**.

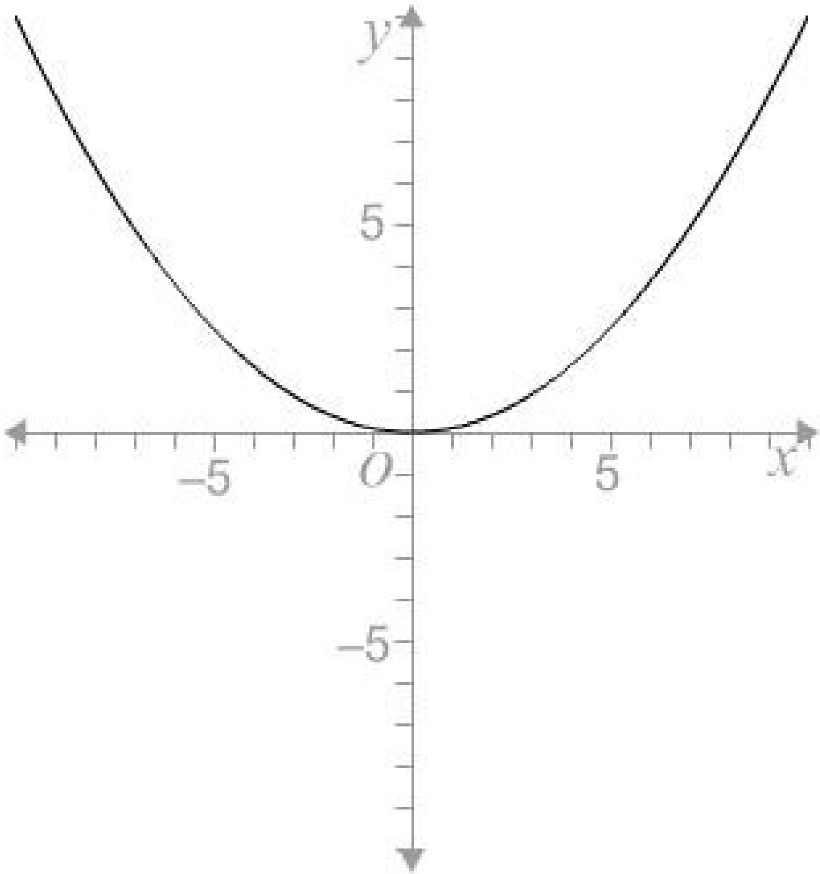
We can also find where the function crosses the y-axis by substituting $x = 0$ into the function.

$$y = x^2 + 6x$$

$$y = (0)^2 + 6(0)$$

$$y = 0$$

So the function crosses the y-axis at the point (0, 0).



Change the values within the parentheses. What do you notice?

Change the value in front of the parentheses. What do you notice?

$y = 0(x + 0)(x + 0)$

The zeros of a quadratic function

Observations:

When a quadratic function is in factored form, the values inside the parentheses relate to where it crosses the x-axis.

These points are called the **zeros** of the function.

The value in front of the parentheses stretches the curve vertically but the zeros remain the same.

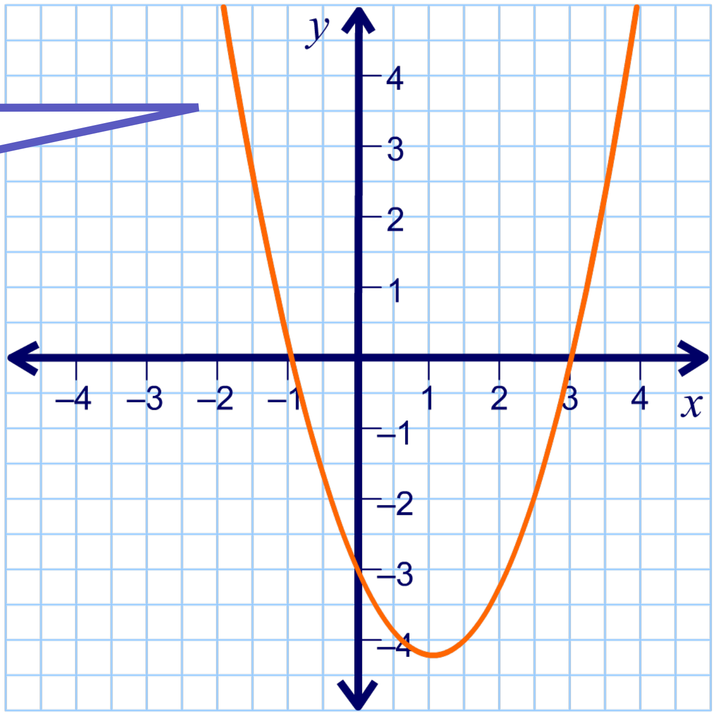
In general:

When a quadratic function is written in the form $y = a(x - p)(x - q)$, it will intersect the x-axis at the points $(p, 0)$ and $(q, 0)$.

p and q are the **zeros** of the quadratic function.

The zeros of a quadratic function

This is the graph of the function $y = x^2 - 2x - 3$.



What is the factored form of this function?

Use the fact that the function crosses the x-axis when $y = 0$ to write a mathematical proof of how a function's factored form tells us its zeros.

Finding the zeros of a quadratic

When a quadratic function factors, we can use its factored form to find where it crosses the x-axis. Here's how:

The function $y = x^2 – 2x – 3$ crosses the x-axis when $y = 0$.

set equal to zero: $x^2 – 2x – 3 = 0$

factor: $(x + 1)(x – 3) = 0$

If the product of two terms is 0, one of the terms must be zero.

This means that either $(x + 1) = 0$ or $(x – 3) = 0$:

$x + 1 = 0$ or $x – 3 = 0$

subtract 1: $\boldsymbol{x = –1}$ add 3: $\boldsymbol{x = 3}$

So the function crosses the x-axis at $(–1, 0)$ and $(3, 0)$.

When we solve a quadratic equation for $y = 0$, we are in fact finding the values of its zeros.

What are the zeros of the quadratic function $y = 3x^2 + 4x - 4$?

This function can be solved by factoring:

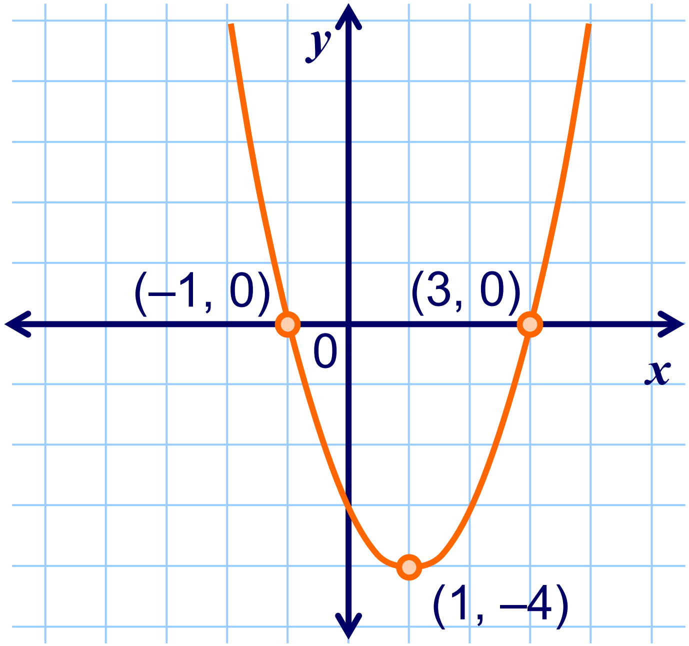
$$y = (3x - 2)(x + 2)$$

substitute $y = 0$: $$0 = (3x - 2)(x + 2)$$

Zero-Product Property: $$3x - 2 = 0 \quad \text{or} \quad x + 2 = 0$$

Therefore, the zeros are $\mathbf{\frac{2}{3}}$ and **–2**.

The vertex of a quadratic



The function $y = x^2 - 2x - 3$ has zeros $x = -1$ and $x = 3$. Where will its vertex be?

A parabola is **symmetrical**, so the x-coordinate of the vertex is half-way between the zeros. Here, it is between –1 and 3:

$$x = \frac{-1 + 3}{2} = 1$$

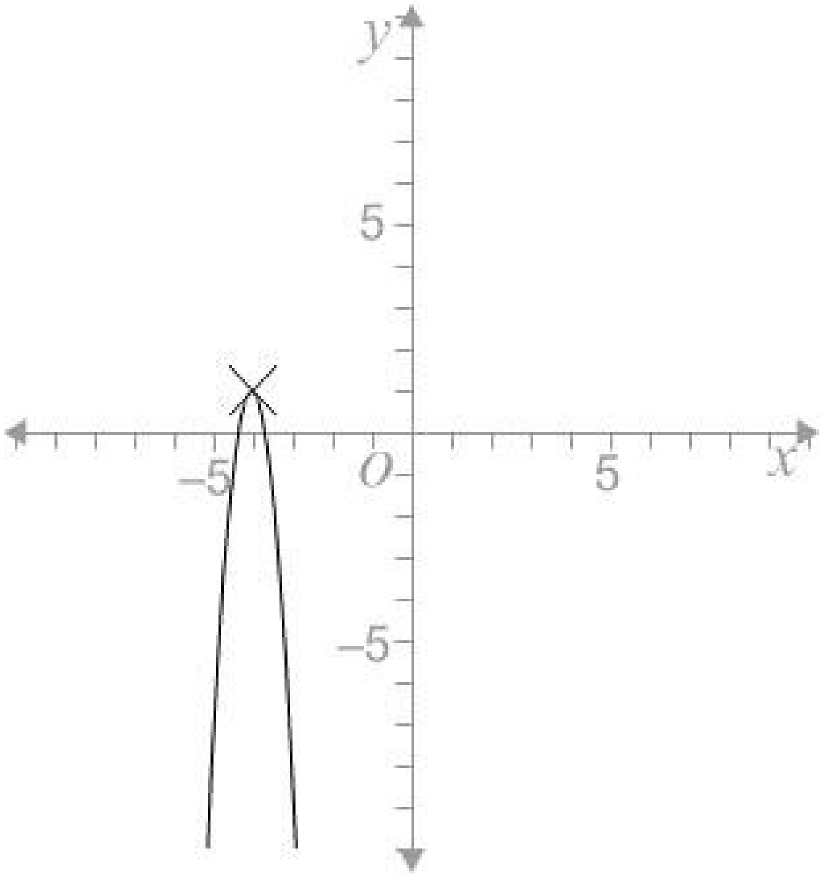
Substitute $x = 1$ back into the function:

$$y = (1)^2 - 2(1) - 3$$

$$y = -4$$

So the coordinates of the vertex are **(1, –4)**.

Graphs of the form $y = a(x - h)^2 + k$



This function is given in the form $y = a(x - h)^2 + k$.

Change the value of $-h$, then k, then a.

What do you notice about the graph when you change each of these values?

$y = 0(x\ 0)^2\ 0$

Vertex form observations

The quadratic function $y = ax^2 + bx + c$ can be written in **vertex form** (or **completed square form**) as $a(x - h)^2 + k$.

Observations:

- The coordinates of the vertex are $\boldsymbol{(h, k)}$.
- The axis of symmetry has the equation $\boldsymbol{x = h}$.
- If $a > 0$ (h, k) will be the **minimum** point.
- If $a < 0$ (h, k) will be the **maximum** point.

When a function does not factor, we write it in vertex form to find the coordinates of the vertex.

Sketching quadratic functions

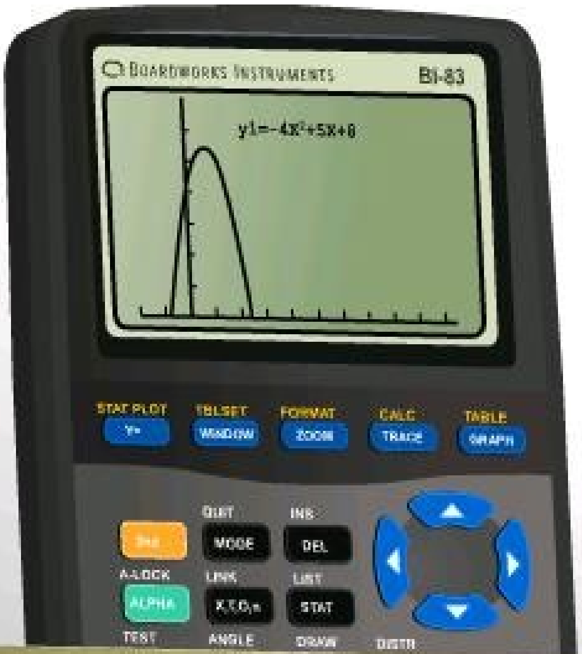
Given the equation of a quadratic function, we can sketch its graph.

Press **start** to see how.

start

Using a calculator

zeros | vertex | y-intercept



You can use your graphing calculator to help you find the zeros, vertex and y-intercept of a quadratic function.

Press on each of the tabs above to see how to find each feature.

The flight path of a ball

Mr. Jackson's physics class runs an experiment where they launch a ball from a catapult across level ground.

In class, the students learn that the height of the ball in feet at any time during its flight can be calculated from the quadratic equation $h = -16t^2 + 50t + 2$.

1) How high is the ball from the ground when it is launched?

2) After how long does the ball hit the ground?

3) Find the ball’s maximum height.

The flight path of a ball

The path of the ball is a parabola, so we can use our knowledge of quadratics to answer the questions.

1) How high is the ball off the ground when it is launched?

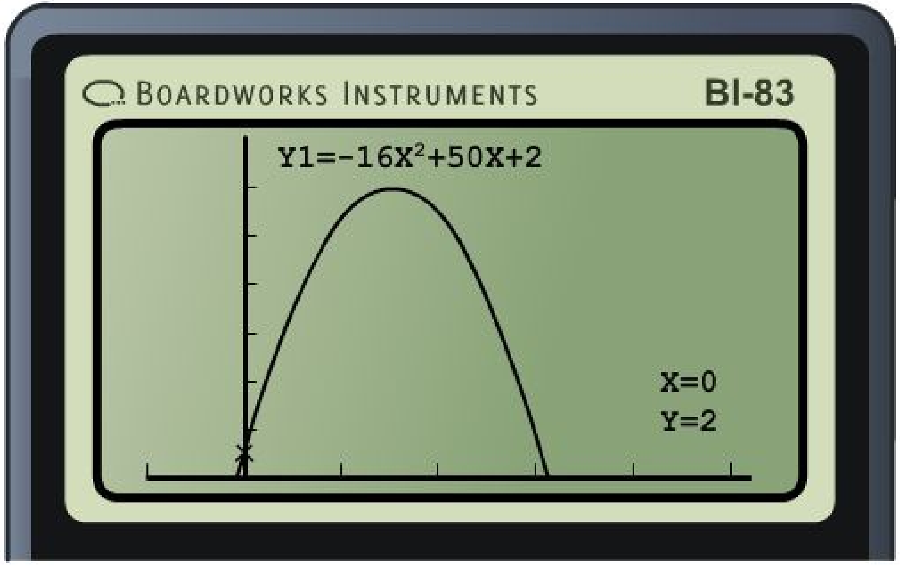
When the ball is launched, the time is $t = 0$.

Substitute $t = 0$ into the quadratic equation $h = –16t^2 + 50t + 2$:

$h = –16(0)^2 + 50(0) + 2$

$\boldsymbol{h = 2 \text{ ft}}$

This can be seen by graphing the function:



2) After how long does the ball hit the ground?

When the ball hits the ground, $h = 0$ ft: $0 = -16t^2 + 50t + 2$.

Find the roots using the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-50 \pm \sqrt{50^2 - 4(-16)(2)}}{2(-16)}$$

$$= \frac{-50 \pm \sqrt{2628}}{-32}$$

$$t = \frac{-50 + \sqrt{2628}}{-32} = -0.04\text{ s} \quad \text{and} \quad t = \frac{-50 - \sqrt{2628}}{-32} = 3.16\text{ s}$$

The ball hits the ground after **3.16 seconds**.

3) Find the ball's maximum height.

The greatest height of the ball occurs at the vertex.

$$h = -16t^2 + 50t + 2$$

factor: $h = -16(t^2 - 3.125t - 0.125)$

complete the square: $h = -16[(t - 1.5625)^2 - (1.5625)^2 - 0.125]$

distributive property: $h = (t - 1.5625)^2 + 16(1.5625)^2 + 16(0.125)$

simplify: $h = (t - 1.5625)^2 + 41.0625$

The vertex is at (1.5625, 41.0625) so the ball's maximum height is approximately **41.1 ft**.

How much have you learned about quadratics?

Split into two teams and choose which character will represent each team. Take it in turns to roll the dice to determine how many squares you will move, then answer the questions correctly to move around the board. Whoever reaches the end first wins the game! Watch out for the colored squares…you might get in trouble with the teacher!

start